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GENERAL SYNCHRONIZATION OF CHAOS IN A SYSTEM OF TWO NON-IDENTICAL LASERS*

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General synchronization of the chaotic intensity fluctuations of two spatially coupled Nd:YAG lasers with pump modulation is investigated theoretically when the losses of the two lasers are different. It is shown that the chaotic synchronization still exists for medium coupling even though the two lasers are different. For strong coupling the system shows periodic motion of laser intensities. While for weak coupling, the two lasers oscillate independently. It is obvious that the increase of difference between the two lasers reduces the degree of chaotic synchronization.

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I. INTRODUCTION

Due to the possibility of practical applications of chaotic synchronization, experimental and theoretical investigations of this fundamental phenomenon in coupled nonlinear systems have attracted much attention in recent years. Several papers have studied the synchronization of chaotic signals in the context of electronic circuits[1,2], secure communication[3], chemical and biological systems[4] and laser dynamics.[5,6] The experimental measurements on synchronization of two chaotic lasers were performed recently.[6]

From a practical point of view, no two systems can be perfectly identical due to some defects in constructing the two systems. It is necessary to investigate the generalized synchronization of chaos in two non-identical systems.

In this paper, the synchronization of two spatially coupled, chaotic Nd: YAG lasers with pump modulations is investigated theoretically when the losses of the two lasers are different. The numerical studies reveal that the degree of chaotic synchronization depends on the deviations of the two lasers. This can explain the phenomenon that there is no perfect chaotic synchronization existing in the experiment.[6] It is also shown that the existence of chaos in the laser system depends on the coupling between the two lasers.

II. THEORETICAL EQUATIONS

The equations describing the time evolution of the complex, slowly varying electric field E_j and gain G_j of laser j of two spatially coupled, pump-modulated single-mode Class B lasers are as follows:

$$\begin{aligned}
\frac{\mathrm{d}E_1}{\mathrm{d}t} &= \frac{1}{r_\mathrm{c}}[(G_1-\alpha_1)E_1-\kappa E_2]+\mathrm{i}w_1E_1,\\
\frac{\mathrm{d}G_1}{\mathrm{d}t} &= \frac{1}{\tau_\mathrm{f}}\{p_1[1+M\sin(\Omega t)]-G_1(1+|E_1|^2)\},\\
\frac{\mathrm{d}E_2}{\mathrm{d}t} &= \frac{1}{\tau_\mathrm{c}}[(G_2-\alpha_2)E_2-\kappa E_1]+\mathrm{i}w_2E_2,\\
\frac{\mathrm{d}G_2}{\mathrm{d}t} &= \frac{1}{\tau_\mathrm{f}}\{p_2[1+M\sin(\Omega t)]-G_2(1+|E_2|^2)\}. \quad (1)
\end{aligned}$$

In these equations, τ_c is the cavity round trip time, τ_f is the fluorescence time of the upper lasing level of the Nd^{3+} ion, p_j, α_j, and ω_j are the pump parameters, losses, and detunings from a common cavity mode respectively, M is the modulation depth of the pump beams, and $\Omega = 2\pi f$ is the angular frequency of modulation. The modulation frequency f is chosen to be near the relaxation frequency of the lasers.[7]

The lasers are coupled linearly to each other with strength κ that is assumed to be small. For laser beams of Gaussian intensity profile with a beam radius ω_0 at $I/I_0 = \dfrac{1}{e^2}$, the coupling strength is defined as

$$\kappa = \exp\left(\frac{-d^2}{2\omega_0^2}\right), \quad (2)$$

where d is the distance between the two lasers. Figure 1 shows the spatial coupling between lasers 1 and 2 schematically.

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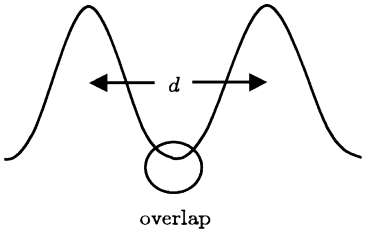


Fig.1. Schematic diagram of the spatial coupling between two laser intensities.

Ⅲ. NUMERICAL SIMULATIONS

The numerical simulations are based on integration of Eqs.(1) and (2). In the numerical calculations, the parameters $\omega_1, \omega_2, p_1, p_2, \tau_c$ and τ_f are fixed constants with $\omega_1 = 0.0, \omega_2 = 5.0 \times 10^5, p_1 = 0.02, p_2 = 0.02, \tau_c = 0.12\,\text{ns}$, and $\tau_f = 0.24\,\text{ms}$. Other parameters are varied according to the requirement of chaotic synchronization. The losses, α_1 and α_2, of the two lasers are different in the simulations with the relative difference defined as

$$\delta = \frac{\Delta\alpha}{\alpha_1} = \frac{\alpha_2 - \alpha_1}{\alpha_1}. \tag{3}$$

Figures 2–5 are the time series and X-Y plot of intensities I_1 and I_2 of lasers 1 and 2 for $\delta = 0.0\%, \delta = 0.1\%, \delta = 1.0\%$, and $\delta = 10.0\%$, respectively.

Figure 2 shows a high degree of chaotic synchronization for $\delta = 0.0\%, f = 100\text{kHz}$, and $d = 0.94\,\text{mm}$. Figure 3 shows a good degree of chaotic synchronization for $\delta = 0.1\%, f = 95.3\,\text{kHz}$, and d=0.93999 mm. There are some data slightly scattered along the straight line of I_2 versus I_1. Figure 4 shows some degree of chaotic synchronization for $\delta = 1.0\%, f = 95.5\,\text{kHz}$, and $d = 0.941\,\text{mm}$. There are more data with large amount of scattering along the straight line of I_2 versus I_1. Figure 5 shows that the chaotic synchronization has been lost for $\delta = 10.0\%, f = 95.5\,\text{kHz}$, and $d = 0.94\,\text{mm}$. It seems that the amount of scattering is similar to that in Fig.4. However, the curve of I_2 versus I_1 is not a straight line but like a parabola. Also the average intensity I_1 of laser 1 is smaller than that of laser 2.

Figures 6–9 are the bifurcation diagrams of laser intensities of I_1 and I_2 with the same parameters as those in Figs.2–5. It is shown that all the lasers are in the chaotic regime for $\delta = 0.0\%, d = 0.94\,\text{mm}$ (Fig.6); $\delta = 0.1\%, d = 0.93999\,\text{mm}$ (Fig.7); $\delta = 1.0\%, d = 0.94\,\text{mm}$ (Fig.8); and $\delta = 10.0\%, d = 0.94\,\text{mm}$ (Fig.9)

From Figs.6–9, it is also shown that the lasers are operated from independent oscillation for weak coupling ($d \geq 1.0\,\text{mm}$) to chaotic motion for medium coupling ($0.9\,\text{mm} \leq d \leq 1.0\,\text{mm}$) to periodic motion for strong coupling ($d \leq 0.9\,\text{mm}$). The exact boundary of independent oscillation, chaotic motion and periodic motion depends on the parameters f, d, etc.

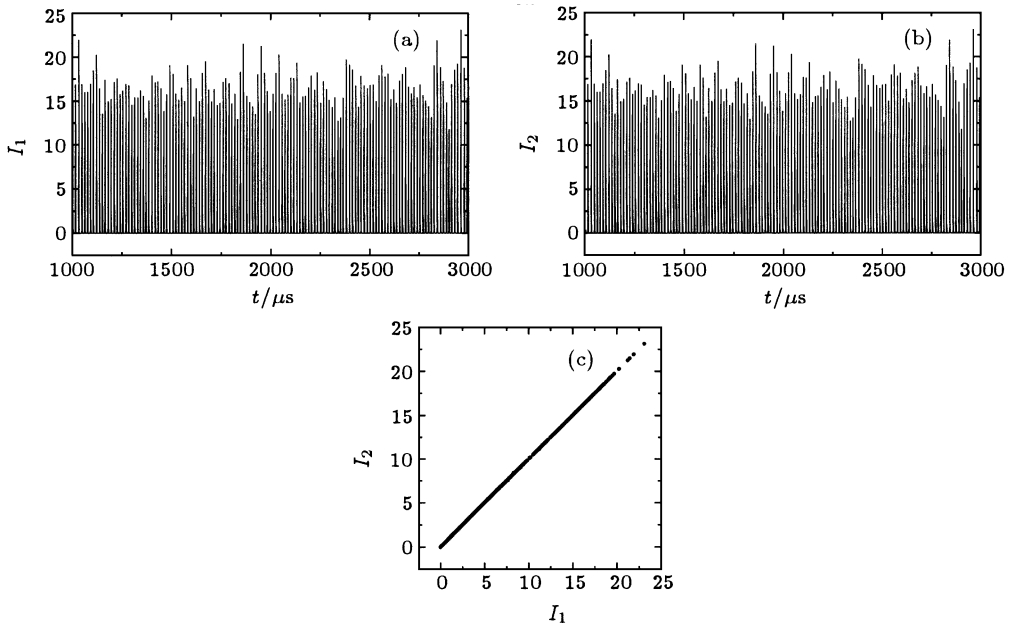


Fig.2. The time series of laser intensities I_1, I_2, and the phase diagram for $\delta = 0.0\%, f = 100\,\text{kHz}$, and $d = 0.94\,\text{mm}$.

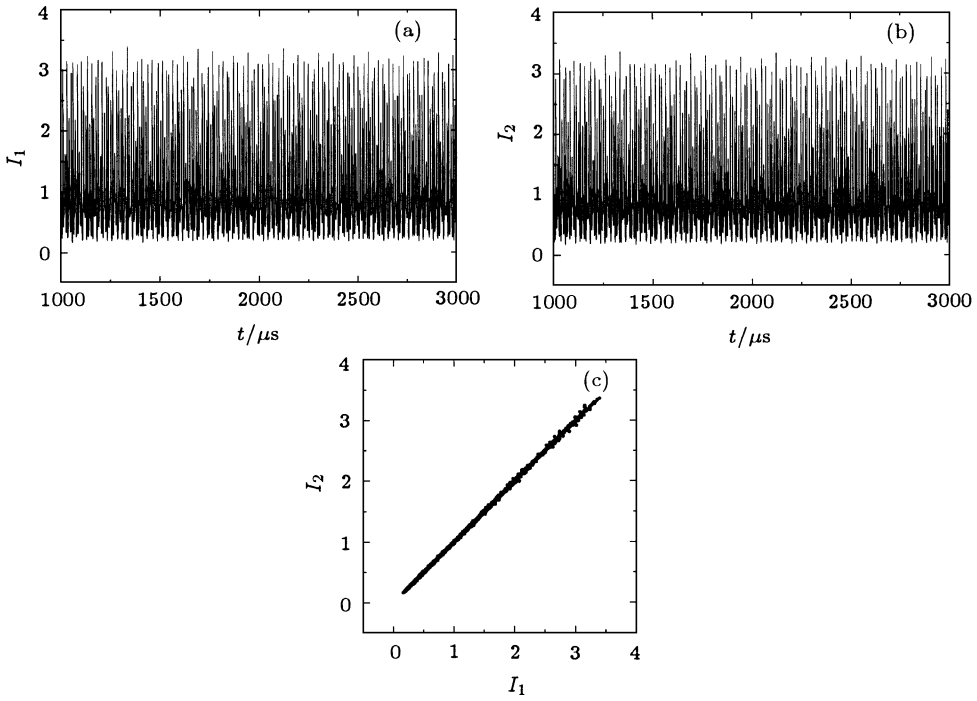


Fig.3. The time series of laser intensities I_1, I_2, and the phase diagram for $\delta = 0.1\%, f = 95.3\,\mathrm{kHz}$, and $d = 0.93999\,\mathrm{mm}$.

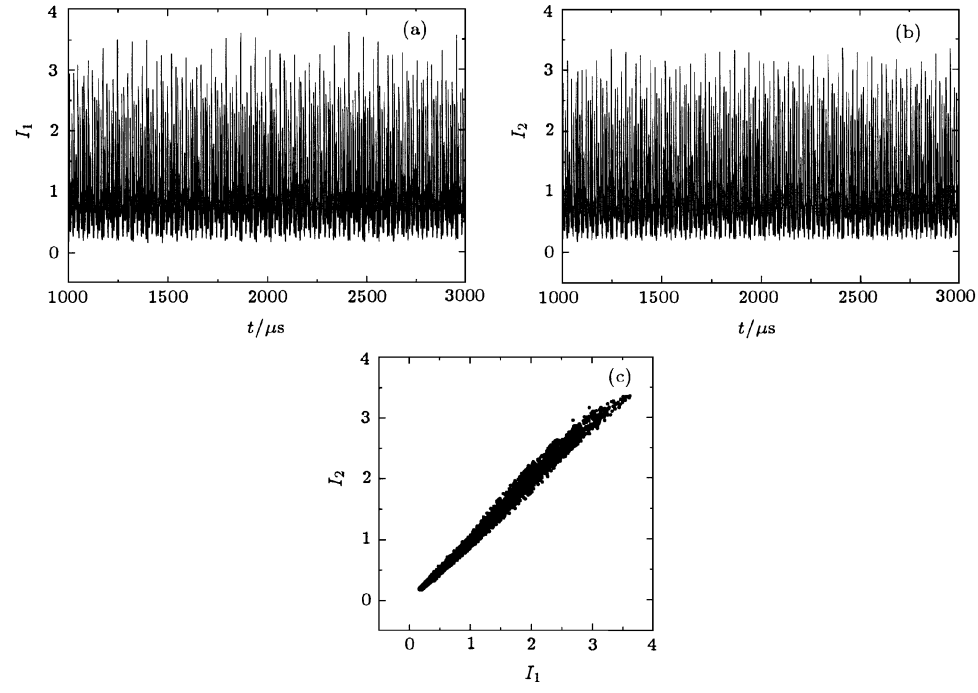


Fig.4. The time series of laser intensities I_1, I_2, and the phase diagram for $\delta = 1.0\%, f = 95.5\,\mathrm{kHz}$, and $d = 0.941\,\mathrm{mm}$.

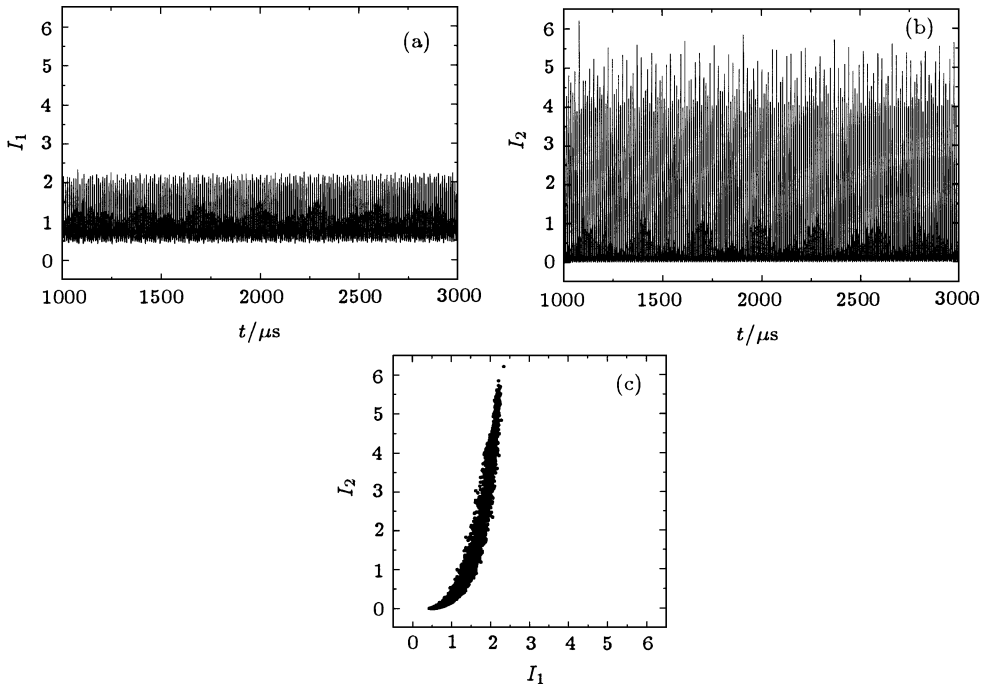


Fig.5. The time series of laser intensities I_1, I_2, and the phase diagram for $\delta = 10.0\%, f = 95.5\,\mathrm{kHz}$, and $d = 0.94\,\mathrm{mm}$.

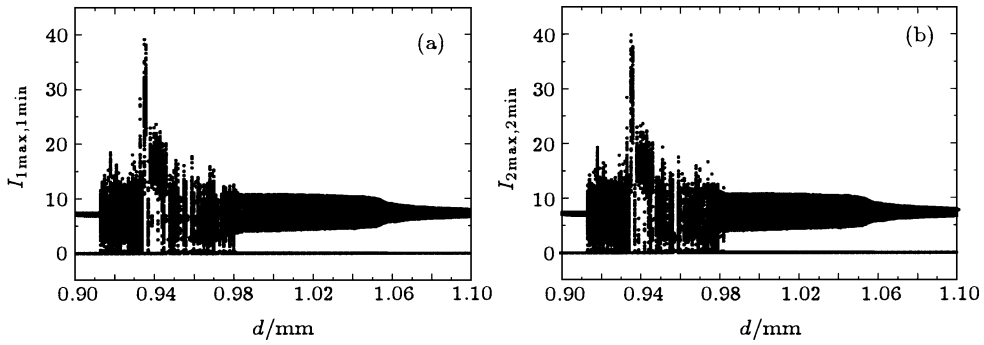


Fig.6. The bifurcation diagram of laser intensities I_1 and I_2 for $\delta = 0.0\%, f = 100\,\mathrm{kHz}$, and $d = 0.94\,\mathrm{mm}$.

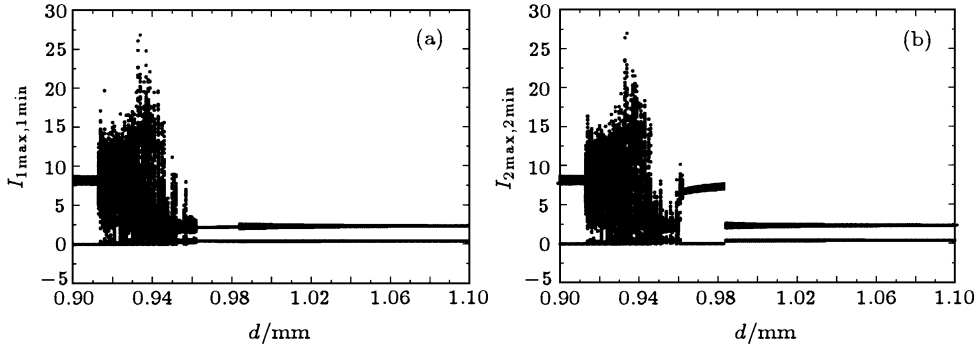


Fig.7. The bifurcation diagram of laser intensities I_1 and I_2 for $\delta = 0.1\%, f = 95.3\,\mathrm{kHz}$, and $d = 0.93999\,\mathrm{mm}$.

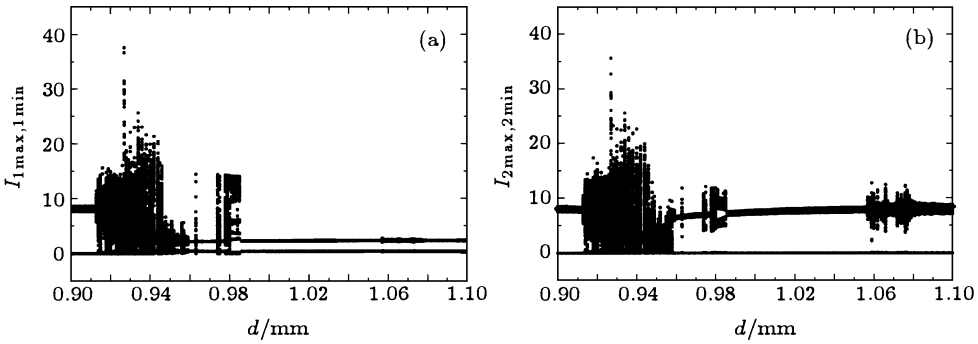


Fig.8. The bifurcation diagram of laser intensities I_1 and I_2 for $\delta = 1.0\%, f = 95.5\,\mathrm{kHz}$, and $d = 0.941\,\mathrm{mm}$.

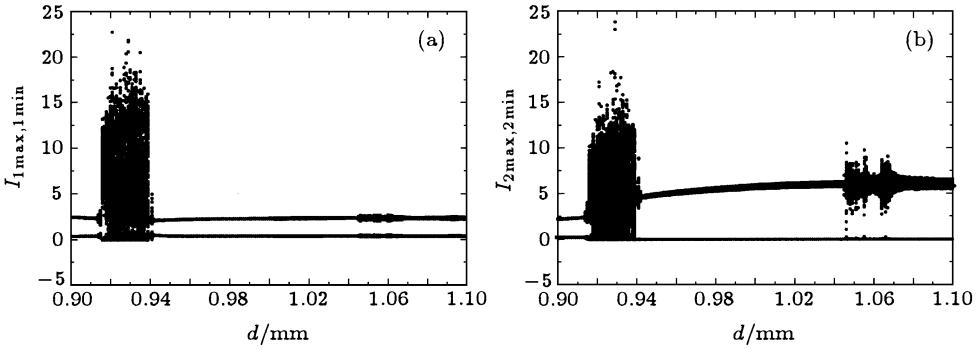


Fig.9. The bifurcation diagram of laser intensities I_1, I_2 for $\delta = 10.0\%, f = 95.5\,\mathrm{kHz}$, and $d = 0.94\,\mathrm{mm}$.

IV. DISCUSSION

It is necessary to estimate the degree of general synchronization of chaotic laser intensities for different δ since the curve of chaotic synchronization of I_2 versus I_1 is like an ellipse with long axis b and short axis a. If the maximum width of the curve is a and the maximum length is b, the degree of chaotic synchronization e is defined as the ratio of a and b with

$$e = a/b. \tag{4}$$

It is seen from Figs.2–5 that $e = 0.0$ for $\delta = 0.0, e \approx 0.0025$ for $\delta = 0.001$, and $e \approx 0.009$ for $\delta = 0.01$. It seems that small amount of difference between two lasers can induce large intensity fluctuations in chaotic synchronization. In the experiment[6], the curve of chaotic synchronization of I_2 versus I_1 looks a thin ellipse not a straight line. This is mainly due to the non-identical working conditions of the two lasers.

REFERENCES

[1] L. M. Pecora, T. L. Carroll, *Phys. Rev. Lett.*, **64**(1990), 821; *Phys. Rev.*, **A 44**(1991), 2374.

[2] K. M. Cuomo, A. V. Oppenheim, *Phys. Rev. Lett.*, **71**(1993), 65.

[3] L. Kocarev, U. Parilz, *Phys. Rev. Lett.*, **74**(1995), 5028.

[4] S. K. Han, C. Kurrer, Y. Kuramoto, *Phys. Rev. Lett.*, **75**(1995), 3190.

[5] H. G. Winful, L. Rahman, *Phys. Rev. Lett.*, **65**(1990), 1575; R. K. DeFreez *et al.*, in Nonlinear Dynamics in Optical System, ed. by N. B. Abraham, E. M. Germire, P. Mandel(Optical Society of America, Washington, D. C., 1991), p. 106.

[6] R. Roy, K. S. Thornburg, Jr., *Phys. Rev. Lett.*, **72**(1994), 2009.

[7] K. S. Thornburg, Jr., *et al.*, *Phys. Rev.*, **E55**(1997), 3865.