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A METHOD OF CONTROLLING SPATIOTEMPORAL CHAOS IN COUPLED MAP LATTICES*

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We propose a method that allows one to control spatiotemporal chaos by applying pulses proportional to the system variables and compressing the phase space of strange attractor in nonlinear system. The method is illustrated by the coupled map lattices at different strengths of coupling. Various numerical results are given. The advantage of this method is that it does not need to know any previous knowledge of the system dynamics.

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I. INTRODUCTION

Chaos as a complicated nonlinear phenomenon has received increasing interest during the last two decades. The inherent feature, known as the "butterfly effect", is often troublesome or even unwanted in many cases of practical applications. Chaos control is, therefore, important for both chaos utilization and elimination.

Ott, Grebogi, and Yorke (OGY)[1] proposed an effective method to stabilize unstable periodic orbits (UPOs) embedded in chaotic attractors by introducing small parameter perturbations. From then on, a series of methods of chaos control have been proposed one after another.[2−6]

Recently, control of spatiotemporal chaos (STC) in spatially extended system has become one of the fundamental questions and an important subject in chaos control theories.[7,8] This is because the behaviors in spatially extended systems are extremely rich.[9] Spatiotemporal chaos appears in vast diverse phenomena, such as plasma fusion, hydrodynamical and optical instabilities, chemical reactions, storage of memory in the brain, and so on. For theoretical investigations, spatially extended systems are often modeled by partial differential equations, coupled map lattices (CMLs) and cellular automata. Among all the above models, the CMLs are the simplest and have been found valuable in revealing many important features of spatiotemporal chaos. In this paper, we will use the CML as a model system for clarifying our idea and method.

II. METHOD

In Ref.[3], Guemez and Matias presented a method for controlling deterministic chaos by the stabilization of one of the underlying unstable periodic orbits. This method works by applying pulses proportional to the system variables (PPSV), i.e., the variables of the map, every Δn iteration. For example, let us consider the logistic map,

$$x_{n+1} = 1.0 - \mu x_n^2. \tag{1}$$

Their control algorithm consists of applying every Δn iteration of a feedback to the variable x, having the form

$$x'_n = x_n(1 + G), \tag{2}$$

where G represents the strength of the feedback. In other words, the map is controlled by changing x_n in such a way that a proportional feedback is applied in the form of pulses. Various numerical results showed that this strategy of control is effective for controlling chaos in a discrete and continuous nonlinear system. In Ref.[5], Luo presented a method to control deterministic chaos and hyperchaos by compressing phase space of the system (CPSS). The control method can be described as follows.

Let us consider an n-dimensional autonomous dynamical system described by the differential equation

$$\dot{X} = F(X), \tag{3}$$

where the state vector $X \in R^n$ is an n-dimensional vector with components $x_1, x_2, ..., x_n$. If the solution

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of Eq.(3) is chaos or hyperchaos, the orbits of it are limited in a space $V(R^n)$. Let us select a nonzero subset space $W(R^n)$ in the space V, then variables $x_i (i = 1, 2, ..., n)$ in Eq.(3) are controlled according to the following equation,

$$x_i' = \begin{cases} x_i, & x_{i\min} < x_i < x_{i\max}, \\ x_{i\max}, & x_i \geq x_{i\max}, \\ x_{i\min}, & x_i \leq x_{i\min}, \end{cases} \qquad i = 1, 2, ..., n. \tag{4}$$

Here $x_{i\min}$ and $x_{i\max}$ are boundaries of space W. Various numerical simulation results showed that this method is capable of stablizing the unstable periodic orbits of chaotic and hyperchaotic systems. In this work, our research results show that the methods mentioned above are not capable of controlling spatiotemporal chaos in coupled map lattices. However, the method of cascade of PPSV[3] and CPSS[5] is effective for controlling spatiotemporal chaos in CML. The block diagram of the method is presented in Fig.1.

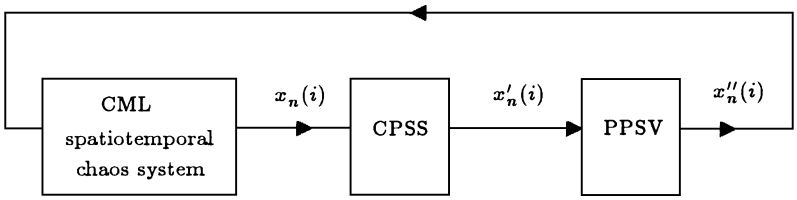


Fig.1. Block diagram of the method.

Ⅲ. CONTROL MODEL AND NUMERICAL RESULTS

A. Control model

Now consider the coupled map lattice which can be defined as

$$x_{n+1}(i) = (1-\varepsilon) f(x_n(i)) + \varepsilon/2[f(x_n(i-1)) + f(x_n(i+1))] \qquad i = 1, 2, \cdots, L. \tag{5}$$

Here $f(x_n(i))$ describes the internal dynamics of the ith site when couplings on other sites are absent. In this work, we take $f(x_n) = 1 - \mu x_n^2$, which is the one-dimensional logistic map behaving chaotically in the range of μ =1.414–2.0. In Eq.(5), i and n denote discrete space and time variables, respectively, and L is the total number of sites, ε is the coupling strength of space neighbouring site. The dynamical behavior of CML depends on the parameter μ, the coupling strength ε and boundary conditions. We have used periodic boundary conditions in this work. Using the controlling method described in Fig.1, the controlled CML then becomes

$$\begin{cases} x_i' = \begin{cases} x_i, & x_{i\min} < x_i < x_{i\max}, \\ x_{i\max}, & x_i \geq x_{i\max}, \\ x_{i\min}, & x_i \leq x_{i\min}, \end{cases} \\ x_n''(i) = x_n'(i)(1+G) \qquad i = 1, 2, ..., L, \\ x_{n+1}(i) = (1-\varepsilon) f(x_n''(i)) + \varepsilon/2(f(x_n''(i-1) + (f(x_n''(i+1)). \end{cases} \tag{6}$$

Here $x_{\max}$ and $x_{\min}$ are the boundaries of space W.

B. Control results

We obtain a series of stable spatiotemporal patterns and periodic orbits in space and in time based on the control of Eq.(6). Some typical results are presented in Fig.3. Figure 2 represents a typical spatiotemporal chaos behavior without any control.

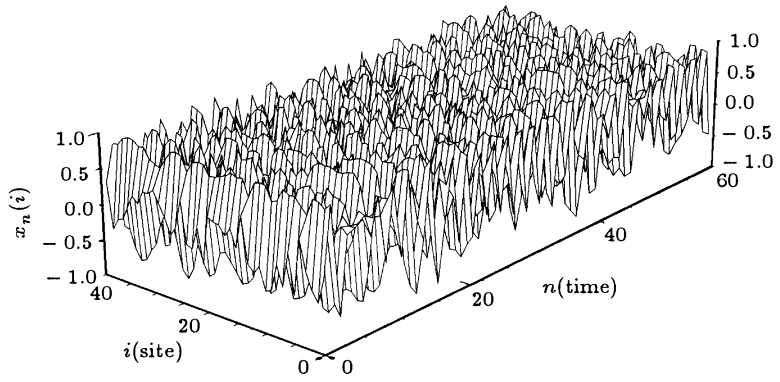


Fig.2. Spatiotemporal chaos in CML without any control, with $\varepsilon = 0.3$, $\mu = 1.9$.

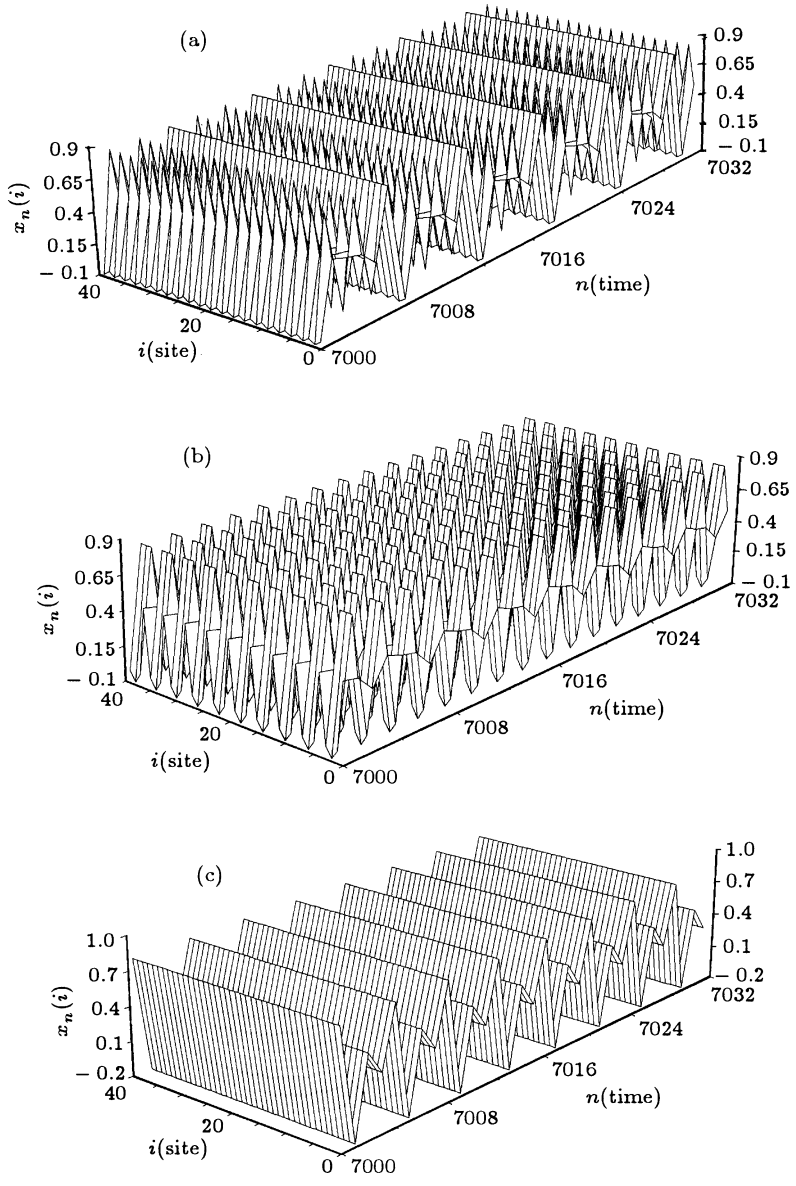


Fig.3. Some three-dimensional patterns generated by using the method described by Eq.(6), with $(x_{\min}, x_{\max}) = (-0.71749251361514, 0.84355140696884)$. (a) Stable spatial pattern and period-6 in time with $\varepsilon = 0.2$, $\mu = 1.52$, $G = -0.396, \Delta n = 3, \Delta i = 2$. (b) Stable spatiotemporal pattern with $\varepsilon = 0.2$, $\mu = 1.52$, $\Delta n = 4$, $\Delta i = 4$, $G = -0.396$. (c) The period-4 in time and fixed point in space with $\varepsilon = 0.3$, $\mu = 1.9$, $G = -0.396$, $\Delta n = 4$, $\Delta i = 1$.

IV. CONCLUSION

We have studied the control of spatiotemporal chaos in CML using the control method of cascade. It is shown that the method is effective for controlling STC and does not require any preliminary information about the UPOs embedded in STC. In this work, the boundaries of space W and the srtength of feedback pulses G are not changed. Our further work show that some encouraging stable spatiotemporal patterns are obtained when these control parameters mentioned above are changed.

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