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AXISYMMETRIC ELASTICITY PROBLEM OF CUBIC QUASICRYSTAL*

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A method for analyzing the elasticity problem of cubic quasicrystal is developed. The axisymmetric elasticity problem of cubic quasicrystal is reduced to a single higher-order partial differential equation by introducing a displacement function. As an example, the solutions of elastic field of cubic quasicrystal with a penny-shaped crack are obtained, and the stress intensity factor and strain energy release rate are determined.

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I. INTRODUCTION

Quasicrystal as a new structure of solid matter was discovered around 1984[1,2], it brings profound new ideas to the traditional condensed matter physics and encourages considerable theoretical and experimental studies on the physical and mechanical properties of the material, including the elasticity theory of the quasicrystal.[3,4]

Cubic quasicrystal is one of important three-dimensional quasicrystals, Feng *et al.*[5] reported cubic quasicrystals with cubic symmetry(432) observed in a VNiSi alloy cooled rapidly, Wang *et al.*[6] analyzed the structure of the material by using projective approach.

Defects such as dislocation, disclination and stacking fault etc. in the material were observed soon after the discovery of the quasicrystal.[7,8] Crack is one of the important defects, its existence greatly influences the physical and mechanical properties of quasicrystals. The study on crack in quasicrystals is significant subjects.

To describe the behaviour of defects in the material, elasticity theory of quasicrystals is necessary. In addition, appropriate mathematical methods are also needed for this purpose. The practice shows that the procedure of eliminating variables is powerful[9−12] which simplifies extremely the partial differential equations of elasticity of quasicrystals. Based on this, we further develop the Fourier analyses, and exact solutions for complicated defect problems of quasicrystals were found.

In this paper, the formulation for the axisymmetric elasticity of cubic quasicrystals is suggested. By introducing a displacement function, the problem is reduced to solution of a single higher-order partial differential equation. As an example, the elastic field of a penny-shaped crack in the material is determined, as well as the stress intensity factor and the strain energy release rate, which provides some useful information for studying deformation and fracture of the quasicrystaline material.

II. AXISYMMETRIC PROBLEM AND DISPACEMENT FUNCTION

The stress-strain relations(the generalized Hooke's law) of cubic quasicrystal can be expressed as follows under an axisymmetric condition(about z-axis symmetry) by using cylindrical coordinates(r, θ, z).

$$\sigma_{rr} = C_{11}\varepsilon_{rr} + C_{12}(\varepsilon_{\theta\theta} + \varepsilon_{zz}) + R_{11}E_{rr} + R_{12}(E_{\theta\theta} + E_{zz}), \quad (1a)$$

$$\sigma_{\theta\theta} = C_{11}\varepsilon_{\theta\theta} + C_{12}(\varepsilon_{rr} + \varepsilon_{zz}) + R_{11}E_{\theta\theta} + R_{12}(E_{rr} + E_{zz}), \quad (1b)$$

$$\sigma_{zz} = C_{11}\varepsilon_{zz} + C_{12}(\varepsilon_{\theta\theta} + \varepsilon_{rr}) + R_{11}E_{zz} + R_{12}(E_{\theta\theta} + E_{rr}), \quad (1c)$$

$$\sigma_{zr} = \sigma_{rz} = 2C_{44}\varepsilon_{rz} + 2R_{44}E_{rz}, \quad (1d)$$

$$H_{rr} = R_{11}\varepsilon_{rr} + R_{12}(\varepsilon_{\theta\theta} + \varepsilon_{zz}) + K_{11}E_{rr} + K_{12}(E_{\theta\theta} + E_{zz}), \quad (1e)$$

$$H_{\theta\theta} = R_{11}\varepsilon_{\theta\theta} + R_{12}(\varepsilon_{rr} + \varepsilon_{zz}) + K_{11}E_{\theta\theta} + K_{12}(E_{rr} + E_{zz}), \quad (1f)$$

$$H_{zz} = R_{11}\varepsilon_{zz} + R_{12}(\varepsilon_{\theta\theta} + \varepsilon_{rr}) + K_{11}E_{zz} + K_{12}(E_{\theta\theta} + E_{rr}), \quad (1g)$$

$$H_{zr} = H_{rz} = 2R_{44}\varepsilon_{rz} + 2K_{44}E_{rz}, \quad (1h)$$

where σ_{ij} are the phonon stress components, H_{ij} the phason stress components; ε_{ij} the phonon strain components, E_{ij} the phason strain components; C_{ij} the elastic constants in phonon field, K_{ij} the elastic con-

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stants in phason field, R_{ij} the phonon-phason coupling elastic constants.

The cubic quasicrystal is one of specific ones in which the phonon field $\boldsymbol{u}$ and phason field $\boldsymbol{w}$ have the same irreducible representation, which are different from those of the elasticity of other quasicrystal system studied by Bak[13] and Socolar *et al.*[14] For this reason, we denote its phason strain tensor by E_{ij} rather than w_{ij} in the following. In addition, the present paper is devoted to the study of the axisymmetric deformation of quasicrystals, in the case of symmetry with respect to z-axis, all the stress, strain and displacement components are independent of θ, and $u_\theta = w_\theta = 0$, so the strain field are defined as

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \varepsilon_{\theta\theta} = \frac{u_r}{r}, \varepsilon_{zz} = \frac{\partial u_z}{\partial z}, \varepsilon_{rz} = \varepsilon_{zr} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right), \tag{2a}$$

$$E_{rr} = \frac{\partial w_r}{\partial r}, E_{\theta\theta} = \frac{w_r}{r}, E_{zz} = \frac{\partial w_z}{\partial z}, E_{rz} = E_{zr} = \frac{1}{2}\left(\frac{\partial w_r}{\partial z} + \frac{\partial w_z}{\partial r}\right), \tag{2b}$$

the rest strain components are vanishing. The equilibrium equations in the absence of body forces are

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0, \frac{\partial \sigma_{zr}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{zr}}{r} = 0, \tag{3a}$$

$$\frac{\partial H_{rr}}{\partial r} + \frac{\partial H_{rz}}{\partial z} + \frac{H_{rr} - H_{\theta\theta}}{r} = 0, \frac{\partial H_{zr}}{\partial r} + \frac{\partial H_{zz}}{\partial z} + \frac{H_{zr}}{r} = 0, \tag{3b}$$

Substituting Eqs.(1) and (2) into Eqs.(3) leads to

$$\begin{aligned}
&C_{11}\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{1}{r^2}u_r\right) + (C_{12} + C_{44})\frac{\partial^2 u_z}{\partial r \partial z} + C_{44}\frac{\partial^2 u_r}{\partial z^2} \\
&+ R_{11}\left(\frac{\partial^2 w_r}{\partial r^2} + \frac{1}{r}\frac{\partial w_r}{\partial r} - \frac{1}{r^2}w_r\right) + (R_{12} + R_{44})\frac{\partial^2 w_z}{\partial r \partial z} + R_{44}\frac{\partial^2 w_r}{\partial z^2} = 0,
\end{aligned} \tag{4a}$$

$$\begin{aligned}
&C_{44}\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r}\right) + (C_{12} + C_{44})\left(\frac{\partial^2 u_r}{\partial r \partial z} + \frac{1}{r}\frac{\partial u_r}{\partial z}\right) + C_{11}\frac{\partial^2 u_z}{\partial z^2} \\
&+ R_{44}\left(\frac{\partial^2 w_z}{\partial r^2} + \frac{1}{r}\frac{\partial w_z}{\partial r}\right) + (R_{12} + R_{44})\left(\frac{\partial^2 w_r}{\partial r \partial z} + \frac{1}{r}\frac{\partial w_r}{\partial z}\right) + R_{11}\frac{\partial^2 w_z}{\partial z^2} = 0,
\end{aligned} \tag{4b}$$

$$\begin{aligned}
&R_{11}\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{1}{r^2}u_r\right) + (R_{12} + R_{44})\frac{\partial^2 u_z}{\partial r \partial z} + R_{44}\frac{\partial^2 u_r}{\partial z^2} \\
&+ K_{11}\left(\frac{\partial^2 w_r}{\partial r^2} + \frac{1}{r}\frac{\partial w_r}{\partial r} - \frac{1}{r^2}w_r\right) + (K_{12} + K_{44})\frac{\partial^2 w_z}{\partial r \partial z} + K_{44}\frac{\partial^2 w_r}{\partial z^2} = 0,
\end{aligned} \tag{4c}$$

$$\begin{aligned}
&R_{44}\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r}\right) + (R_{12} + R_{44})\left(\frac{\partial^2 u_r}{\partial r \partial z} + \frac{1}{r}\frac{\partial u_r}{\partial z}\right) + R_{11}\frac{\partial^2 u_z}{\partial z^2} \\
&+ K_{44}\left(\frac{\partial^2 w_z}{\partial r^2} + \frac{1}{r}\frac{\partial w_z}{\partial r}\right) + (K_{12} + K_{44})\left(\frac{\partial^2 w_r}{\partial r \partial z} + \frac{1}{r}\frac{\partial w_r}{\partial z}\right) + K_{11}\frac{\partial^2 w_z}{\partial z^2} = 0.
\end{aligned} \tag{4d}$$

These are named the equilibrium equations expressed by displacements.

To simplify the equations, a new unknown function $F(r, z)$ is introduced, such as

$$u_r = -\frac{\partial^2}{\partial r \partial z}\left[A_1\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^2 - A_2\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\frac{\partial^2}{\partial z^2} + A_3\frac{\partial^4}{\partial z^4}\right]F, \tag{5a}$$

$$u_z = -\left[B_1\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^3 - B_2\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^2\frac{\partial^2}{\partial z^2} + B_3\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\frac{\partial^4}{\partial z^4} - B_4\frac{\partial^6}{\partial z^6}\right]F, \tag{5b}$$

$$w_r = -\frac{\partial^2}{\partial r \partial z}\left[C_1\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^2 - C_2\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\frac{\partial^2}{\partial z^2} + C_3\frac{\partial^4}{\partial z^4}\right]F, \tag{5c}$$

$$w_z = -\left[D_1\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^3 - D_2\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^2\frac{\partial^2}{\partial z^2} + D_3\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\frac{\partial^4}{\partial z^4} - D_4\frac{\partial^6}{\partial z^6}\right]F, \tag{5d}$$

where A_i, B_i, C_i and D_i are known constants composed of the elastic constants (given in the Appendix), then Eqs.(4) are automatically satisfied if $F(r,z)$ satisfies the following partial differential equation,

$$\left[\frac{\partial^8}{\partial z^8} - b\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\frac{\partial^6}{\partial z^6} + c\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)^2\frac{\partial^4}{\partial z^4} - d\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)^3\frac{\partial^2}{\partial z^2} + e\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)^4\right]F = 0, \tag{6}$$

where b, c d and e are known constants composed of the elastic constants (given in theAppendix). Substituting Eqs.(5) into Eqs.(1),the stress components σ_{ij} and H_{ij} can be expressed by $F(r,z)$. Because these expressions are quite lengthy, they are omitted here. The axisymmetric elasticity problem of cubic quasicrystal is reduced to solving Eq.(6).

Ⅲ. HANKEL TRANSFORM AND THE INTEGRAL EXPRESSIONS OF THE SOLUTIONS

Introducing the Hankel transform of zero order $\overline{F}(\xi,z)=\int_0^{+\infty} rF(r,z)J_0(\xi r)\mathrm{d}r$, where $J_0(\xi r)$ is the zero order Bessel function of first kind. Performing the Hankel transform on Eq.(6) yields

$$\left[\frac{\mathrm{d}^8}{\mathrm{d}z^8} + b\xi^2\frac{\mathrm{d}^6}{\mathrm{d}z^6} + c\xi^4\frac{\mathrm{d}^4}{\mathrm{d}z^4} + d\xi^6\frac{\mathrm{d}^2}{\mathrm{d}z^2} + e\xi^8\right]\overline{F} = 0, \tag{7}$$

in which ξ represents the Hankel transform parameter.

Using the relations among stress, displacement and function $F(r,z)$, the stress and displacement components can be expressed by $\overline{F}(\xi,z)$

$$u_r = \int_0^{+\infty}\left(A_1\xi^6\frac{\mathrm{d}}{\mathrm{d}z} + A_2\xi^4\frac{\mathrm{d}^3}{\mathrm{d}z^3} + A_3\xi^2\frac{\mathrm{d}^5}{\mathrm{d}z^5}\right)\overline{F}(\xi,z)J_1(\xi r)\mathrm{d}\xi, \tag{8a}$$

$$u_z = \int_0^{+\infty}\left(B_1\xi^7 + B_2\xi^5\frac{\mathrm{d}^2}{\mathrm{d}z^2} + B_3\xi^3\frac{\mathrm{d}^4}{\mathrm{d}z^4} + B_4\xi\frac{\mathrm{d}^6}{\mathrm{d}z^6}\right)\overline{F}(\xi,z)J_0(\xi r)\mathrm{d}\xi, \tag{8b}$$

$$w_r = \int_0^{+\infty}\left(C_1\xi^6\frac{\mathrm{d}}{\mathrm{d}z} + C_2\xi^4\frac{\mathrm{d}^3}{\mathrm{d}z^3} + C_3\xi^2\frac{\mathrm{d}^5}{\mathrm{d}z^5}\right)\overline{F}(\xi,z)J_1(\xi r)\mathrm{d}\xi, \tag{8c}$$

$$w_z = \int_0^{+\infty}\left(D_1\xi^7 + D_2\xi^5\frac{\mathrm{d}^2}{\mathrm{d}z^2} + D_3\xi^3\frac{\mathrm{d}^4}{\mathrm{d}z^4} + D_4\xi\frac{\mathrm{d}^6}{\mathrm{d}z^6}\right)\overline{F}(\xi,z)J_0(\xi r)\mathrm{d}\xi, \tag{8d}$$

$$\sigma_{rr} = \int_0^{+\infty}\left(E_1\xi^7\frac{\mathrm{d}}{\mathrm{d}z} + E_2\xi^5\frac{\mathrm{d}^3}{\mathrm{d}z^3} + E_3\xi^3\frac{\mathrm{d}^5}{\mathrm{d}z^5} + E_4\xi\frac{\mathrm{d}^7}{\mathrm{d}z^7}\right)\overline{F}(\xi,z)J_0(\xi r)\mathrm{d}\xi + \frac{1}{r}\int_0^{+\infty}\left(E_5\xi^6\frac{\mathrm{d}}{\mathrm{d}z} + E_6\xi^4\frac{\mathrm{d}^3}{\mathrm{d}z^3} + E_7\xi^2\frac{\mathrm{d}^5}{\mathrm{d}z^5}\right)\overline{F}(\xi,z)J_1(\xi r)\mathrm{d}\xi, \tag{8e}$$

$$\sigma_{\theta\theta} = \int_0^{+\infty}\left(F_1\xi^7\frac{\mathrm{d}}{\mathrm{d}z} + F_2\xi^5\frac{\mathrm{d}^3}{\mathrm{d}z^3} + F_3\xi^3\frac{\mathrm{d}^5}{\mathrm{d}z^5} + F_4\xi\frac{\mathrm{d}^7}{\mathrm{d}z^7}\right)\overline{F}(\xi,z)J_0(\xi r)\mathrm{d}\xi - \frac{1}{r}\int_0^{+\infty}\left(E_5\xi^6\frac{\mathrm{d}}{\mathrm{d}z} + E_6\xi^4\frac{\mathrm{d}^3}{\mathrm{d}z^3} + E_7\xi^2\frac{\mathrm{d}^5}{\mathrm{d}z^5}\right)\overline{F}(\xi,z)J_1(\xi r)\mathrm{d}\xi, \tag{8f}$$

$$\sigma_{zz} = \int_0^{+\infty}\left(G_1\xi^7\frac{\mathrm{d}}{\mathrm{d}z} + G_2\xi^5\frac{\mathrm{d}^3}{\mathrm{d}z^3} + G_3\xi^3\frac{\mathrm{d}^5}{\mathrm{d}z^5} + G_4\xi\frac{\mathrm{d}^7}{\mathrm{d}z^7}\right)\overline{F}(\xi,z)J_0(\xi r)\mathrm{d}\xi, \tag{8g}$$

$$\sigma_{rz} = \sigma_{zr} = \int_0^{+\infty}\left(H_1\xi^8 + H_2\xi^6\frac{\mathrm{d}^2}{\mathrm{d}z^2} + H_3\xi^4\frac{\mathrm{d}^4}{\mathrm{d}z^4} + H_4\xi^2\frac{\mathrm{d}^6}{\mathrm{d}z^6}\right)\overline{F}(\xi,z)J_1(\xi r)\mathrm{d}\xi, \tag{8h}$$

$$H_{rr} = \int_0^{+\infty}\left(I_1\xi^7\frac{\mathrm{d}}{\mathrm{d}z} + I_2\xi^5\frac{\mathrm{d}^3}{\mathrm{d}z^3} + I_3\xi^3\frac{\mathrm{d}^5}{\mathrm{d}z^5} + I_4\xi\frac{\mathrm{d}^7}{\mathrm{d}z^7}\right)\overline{F}(\xi,z)J_0(\xi r)\mathrm{d}\xi + \frac{1}{r}\int_0^{+\infty}\left(I_5\xi^6\frac{\mathrm{d}}{\mathrm{d}z} + I_6\xi^4\frac{\mathrm{d}^3}{\mathrm{d}z^3} + I_7\xi^2\frac{\mathrm{d}^5}{\mathrm{d}z^5}\right)\overline{F}(\xi,z)J_1(\xi r)\mathrm{d}\xi, \tag{8i}$$

$$H_{\theta\theta} = \int_0^{+\infty}\left(J_1\xi^7\frac{\mathrm{d}}{\mathrm{d}z} + J_2\xi^5\frac{\mathrm{d}^3}{\mathrm{d}z^3} + J_3\xi^3\frac{\mathrm{d}^5}{\mathrm{d}z^5} + J_4\xi\frac{\mathrm{d}^7}{\mathrm{d}z^7}\right)\overline{F}(\xi,z)J_0(\xi r)\mathrm{d}\xi - \frac{1}{r}\int_0^{+\infty}\left(I_5\xi^6\frac{\mathrm{d}}{\mathrm{d}z} + I_6\xi^4\frac{\mathrm{d}^3}{\mathrm{d}z^3} + I_7\xi^2\frac{\mathrm{d}^5}{\mathrm{d}z^5}\right)\overline{F}(\xi,z)J_1(\xi r)\mathrm{d}\xi, \tag{8j}$$

$$H_{zz} = \int_0^{+\infty}\left(K_1\xi^7\frac{\mathrm{d}}{\mathrm{d}z} + K_2\xi^5\frac{\mathrm{d}^3}{\mathrm{d}z^3} + K_3\xi^3\frac{\mathrm{d}^5}{\mathrm{d}z^5} + K_4\xi\frac{\mathrm{d}^7}{\mathrm{d}z^7}\right)\overline{F}(\xi,z)J_0(\xi r)\mathrm{d}\xi, \tag{8k}$$

$$H_{rz} = H_{zr} = \int_0^{+\infty}\left(L_1\xi^8 + L_2\xi^6\frac{\mathrm{d}^2}{\mathrm{d}z^2} + L_3\xi^4\frac{\mathrm{d}^4}{\mathrm{d}z^4} + L_4\xi^2\frac{\mathrm{d}^6}{\mathrm{d}z^6}\right)\overline{F}(\xi,z)J_1(\xi r)\mathrm{d}\xi, \tag{8l}$$

where $E_i, F_i, G_i, H_i, I_i, J_i, K_i$ and L_i are known constants composed of the elastic constants (shown in the Appendix), $J_1(\xi r)$ is the first order Bessel function of first kind.

As a typical example of axisymmetric problem, the elastic problem of cubic quasicrystal with a penny-shaped crack under the action of axisymmetric stress is studied below.

IV. SOLUTIONS OF A PENNY-SHAPED CRACK PROBLEM

Assume a penny-shaped crack with radius a in the center of the cubic quasicrystal material, the size of the crack is much smaller than the solid, so that the size of the material can be considered as infinite, at the infinity, the quasicrystal material is subjected to a tension p in z-direction. The origin of the Coordinate system is at the center of the crack (as shown in Fig 1).

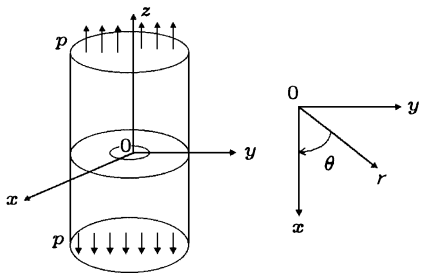


Fig.1. The penny-shaped crack in a cubic quasicrystal and the coordinate system.

From the symmetry of the problem, it is enough to study the upper half-space $z>0$ or lower half-space $z<0$. In this case, for studying the upper half-space, the boundary conditions of the problem are described by

$$\sqrt{r^2+z^2}\to\infty:\sigma_{zz}=p, H_{zz}=0, \tag{9a}$$

$$z=0, 0\le r\le a:\sigma_{zz}=\sigma_{rz}=0; H_{zz}=H_{rz}=0, \tag{9b}$$

$$z=0, r>a:\sigma_{rz}=0, u_z=0; H_{rz}=0, w_z=0, \tag{9c}$$

Using the principle of linear superposition, the problem is reduced to a superposition of two problems, one of them has the boundary conditions

$$\sqrt{r^2+z^2}\to\infty:\sigma_{ij}=H_{ij}=0; \tag{10a}$$

$$z=0, 0\le r\le a:\sigma_{zz}=-p, \sigma_{rz}=0; H_{zz}=H_{rz}=0; \tag{10b}$$

$$z=0, r>a:\sigma_{rz}=0, u_z=0; H_{rz}=0, w_z=0. \tag{10c}$$

The other is that the quasicrystal material without crack is subjected to a tension p at infinity, the solution of the problem is known, i.e:, $\sigma_{zz}=p$ the rest of the stress components are zero. So, it is enough to solve the boundary-value problems (10) of the partial differential equation (6).

In the Hankel transform domain, Eq.(6) is reduced to an ordinary differential equation (7) with the foll0wing characteristic roots

$$\gamma_{1,2}=\pm\lambda_1\xi, \gamma_{3,4}=\pm\lambda_2\xi, \gamma_{5,6}=\pm\lambda_3\xi, \gamma_{7,8}=\pm\lambda_4\xi,$$

where $\lambda_i (i=1,2,3,4)$ are known constants composed of the elastic constants (given in the Appendix).

We may assume $\lambda_i\neq\lambda_j (i\neq j)$ and $\lambda_i>0$ (if $\lambda_i=\lambda_j$ for some values of i and j, or $\lambda_i\le 0$, or λ_i are complex numbers for some values of i, for these cases the problem can be solved similarly. Only when λ_i are pure imaginary numbers for some values of i, there is no physical meaning for solutions), from the condition (10-1), we obtain

$$\begin{aligned}\overline{F}(\xi,z)=&A_1(\xi)e^{-\lambda_1\xi z}+A_2(\xi)e^{-\lambda_2\xi z}\\&+A_3(\xi)e^{-\lambda_3\xi z}+A_4(\xi)e^{-\lambda_4\xi z},\end{aligned} \tag{11}$$

where $A_i(\xi)(i=1,2,3,4)$ are to be determined.

Substituting Eq.(11) into Eqs.(8), and from the conditions (10-2) and (10-3), we obtain the dual integral equations

$$\int_0^{+\infty}A_i(\xi)\xi^8 J_0(\xi r)\mathrm{d}\xi=M_i p, 0\le r\le a, \tag{12a}$$

$$\int_0^{+\infty}A_i(\xi)\xi^7 J_0(\xi r)\mathrm{d}\xi=0, r>a, \tag{12b}$$

where $M_i (i=1,2,3,4)$ are known constants composed of the elastic constants (listed in the Appendix).

Solving the dual integral equations, we obtain

$$A_i(\xi)=2a^2M_i p(2\pi a\xi)^{-1/2}\xi^{-7}J_{3/2}(a\xi), \tag{13}$$

where $J_{3/2}(a\xi)$ is the Bessel function with order $3/2$ of the first kind.

The problem is then solved. Substituting Eqs.(11) and (13) into Eqs.(8), the analytical expressions of all the stress and displacement components can be evaluated.

Thus stress intensity factor K_I, strain energy W_I and strain energy release rate G_I can be defined as follows[15]

$$K_I=\lim_{r\to a^+}\sqrt{2\pi(r-a)}\sigma_{zz}(r,0), \tag{14}$$

$$W_I=\int_0^a 2\pi r\sigma_{zz}(r,0)u_z(r,0)\mathrm{d}r, \tag{15}$$

$$G_I=\frac{1}{2\pi a}\frac{\partial W_I}{\partial a}. \tag{16}$$

Calculation shown that

$$K_I = \frac{2p\sqrt{a}}{\sqrt{\pi}}, \quad W_I = Mp^2a^3, \quad G_I = \frac{3Mp^2a}{2\pi}, \quad (17)$$

where M is known constant composed of the elastic constants (shown in the Appendix)

V. CONCLUSION AND DISCUSSION

It has been shown that the basic equation system of elasticity for the quasicrystal is much more complicated than conventional crystals. The solution in straightforward manner is not available. In the work of our group, including the present paper, we have developed a procedure, which is named the eliminating variable procedure, by introducung the displacement function or stress function to simplify the original basic equations to a single higher-order equation, In this way, the complicated boundary-value problem of elasticity of cubic quasicrystal is solved, and the exact analytic solution for a penny-shaped crack is achived.

The experiment[16] indicated that quasicrystal is quite brittle, and in Ref.[16] was reported the measurment of fracture toughness of quasicrystal $Al_{65}Cu_{20}Co_{15}$, It is well-known that the failure of brittle solid is always connected with crack that was observed by Griffith[17]. The present work attempts to extend the classical Griffith work to quasicrystal. However, at present, due to lack of experimental results for penny-shaped crack in cubic quasicrystal, the exact analytic solution of this paper provides a complete theoretical predication for experiment investigation and an assessment for numerical analysis. The formulation and mathematical method developed in this study can also be used for reference for other problems of elasticity of the quasicrystal.

APPENDIX

The expressions of the constants in the text are as follows:

$$\begin{aligned}
A_1 =& \frac{R_{11}b_1 + K_{11}b_2}{C_{11}K_{11} - R_{11}^2}, \quad A_2 = \frac{R_{11}b_3 + K_{11}b_4}{C_{11}K_{11} - R_{11}^2}, \quad A_3 = \frac{R_{11}b_5 + K_{11}b_6}{C_{11}K_{11} - R_{11}^2},\\
B_1 =& \frac{(R_{12} + R_{44})b_1 + (K_{12} + K_{44})b_2}{(C_{12} + C_{44})(K_{12} + K_{44}) - (R_{12} + R_{44})^2},\\
B_2 =& -\left\{ \frac{(R_{12} + R_{44})b_3 + (K_{12} + K_{44})b_4}{(C_{12} + C_{44})(K_{12} + K_{44}) - (R_{12} + R_{44})^2}, \right.\\
& \left. + \frac{c_2b_1 - c_1b_2}{(C_{11}K_{11} - R_{11})^2[(C_{12} + C_{44})(K_{12} + K_{44}) - (R_{12} + R_{44})^2]} \right\},\\
B_3 =& \frac{(R_{12} + R_{44})b_5 + (K_{12} + K_{44})b_6}{(C_{12} + C_{44})(K_{12} + K_{44}) - (R_{12} + R_{44})^2},\\
& + \frac{c_2b_3 - c_1b_4}{(C_{11}K_{11} - R_{11})^2[(C_{12} + C_{44})(K_{12} + K_{44}) - (R_{12} + R_{44})^2]},\\
B_4 =& - \frac{c_2b_5 - c_1b_6}{(C_{11}K_{11} - R_{11})^2[(C_{12} + C_{44})(K_{12} + K_{44}) - (R_{12} + R_{44})^2]},\\
C_1 =& \frac{C_{11}b_1 + R_{11}b_2}{C_{11}K_{11} - R_{11}^2}, \quad C_2 = \frac{C_{11}b_3 + R_{11}b_4}{C_{11}K_{11} - R_{11}^2}, \quad C_3 = \frac{C_{11}b_5 + R_{11}b_6}{C_{11}K_{11} - R_{11}^2},\\
D_1 =& \frac{(C_{12} + C_{44})b_1 + (R_{12} + R_{44})b_2}{(C_{12} + C_{44})(K_{12} + K_{44}) - (R_{12} + R_{44})^2},\\
D_2 =& \frac{(C_{12} + C_{44})b_3 + (R_{12} + R_{44})b_4}{(C_{12} + C_{44})(K_{12} + K_{44}) - (R_{12} + R_{44})^2},\\
& + \frac{c_3b_2 - c_4b_1}{(C_{11}K_{11} - R_{11})^2[(C_{12} + C_{44})(K_{12} + K_{44}) - (R_{12} + R_{44})^2]},\\
D_3 =& \frac{(C_{12} + C_{44})b_5 + (R_{12} + R_{44})b_6}{(C_{12} + C_{44})(K_{12} + K_{44}) - (R_{12} + R_{44})^2},\\
& - \frac{c_3b_4 - c_4b_3}{(C_{11}K_{11} - R_{11})^2[(C_{12} + C_{44})(K_{12} + K_{44}) - (R_{12} + R_{44})^2]},\\
D_4 =& \frac{c_3b_6 - c_4b_5}{(C_{11}K_{11} - R_{11})^2[(C_{12} + C_{44})(K_{12} + K_{44}) - (R_{12} + R_{44})^2]},
\end{aligned}$$

where

$$
\begin{aligned}
c_1 =&(R_{12}+R_{44})(R_{44}K_{11}-R_{11}K_{44})-(K_{12}+K_{44})(C_{44}K_{11}-R_{11}R_{44}),\\
c_2 =&(R_{12}+R_{44})(C_{11}K_{44}-R_{11}R_{44})-(K_{12}+K_{44})(C_{11}R_{44}-C_{44}R_{11}),\\
c_3 =&(R_{12}+R_{44})(C_{44}K_{11}-R_{11}R_{44})-(C_{12}+C_{44})(R_{44}K_{11}-R_{11}K_{44}),\\
c_4 =&(R_{12}+R_{44})(C_{11}R_{44}-C_{44}R_{11})-(C_{12}+C_{44})(C_{11}K_{44}-R_{11}R_{44}),\\
b_1 =&\frac{R_{12}K_{44}-R_{44}K_{12}}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})},\\
b_2 =&\frac{R_{44}(R_{12}+R_{44})-K_{44}(C_{12}+C_{44})}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})},\\
b_3 =&\frac{K_{11}(R_{12}+R_{44})-R_{11}(K_{12}+K_{44})}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2}+\frac{K_{11}(R_{12}+R_{44})-R_{11}(K_{12}+K_{44})}{C_{11}K_{11}-R_{11}^2}\\
&+\frac{R_{44}c_1+K_{44}c_3}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
b_4 =&\frac{R_{11}(R_{12}+R_{44})-K_{11}(C_{12}+C_{44})}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2}+\frac{C_{11}(K_{12}+K_{44})-R_{11}(R_{12}+R_{44})}{C_{11}K_{11}-R_{11}^2}\\
&+\frac{R_{44}c_2+K_{44}c_4}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
b_5 =&\frac{R_{11}c_1+K_{11}c_3}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
b_6 =&\frac{R_{11}c_2+K_{11}c_4}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
b =&\frac{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2}{C_{44}K_{44}-R_{44}^2}(a_4b_5-a_5b_4+a_6b_3-a_3b_6),\\
c =&\frac{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2}{C_{44}K_{44}-R_{44}^2}(a_1b_6-a_6b_1+a_3b_4-a_4b_3+a_5b_2-a_2b_5),\\
d =&\frac{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2}{C_{44}K_{44}-R_{44}^2}(a_2b_3-a_3b_2+a_4b_1-a_1b_4),\\
e =&\frac{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2}{C_{44}K_{44}-R_{44}^2}(a_1b_2-a_2b_1),
\end{aligned}
$$

where

$$
\begin{aligned}
a_1 =&\frac{R_{44}(R_{12}+R_{44})-C_{44}(K_{12}+K_{44})}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
a_2 =&\frac{C_{44}R_{12}-C_{12}R_{44}}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
a_3 =&\frac{R_{11}(R_{12}+R_{44})-C_{11}(K_{12}+K_{44})}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2}+\frac{K_{11}(C_{12}+C_{44})-R_{11}(R_{12}+R_{44})}{C_{11}K_{11}-R_{11}^2}\\
&+\frac{C_{44}c_1+R_{44}c_3}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
a_4 =&\frac{C_{11}(R_{12}+R_{44})-R_{11}(C_{12}+C_{44})}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2}+\frac{C_{11}(R_{12}+R_{44})-R_{11}(C_{12}+C_{44})}{C_{11}K_{11}-R_{11}^2}\\
&+\frac{C_{44}c_2+R_{44}c_4}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
a_5 =&\frac{C_{11}c_1+R_{11}c_3}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
a_6 =&\frac{C_{11}c_2+R_{11}c_4}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
E_1 =&\frac{C_{44}K_{44}-R_{44}^2}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},
\end{aligned}
$$

$$
\begin{aligned}
E_2 =& b_4 + \frac{(C_{44}R_{12} - C_{12}R_{44})b_3 + [R_{12}(R_{12}+R_{44}) - C_{12}(K_{12}+K_{44})]b_4}{(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2} \\
&+ \frac{(C_{12}c_1 + R_{12}c_3)b_2 - (C_{12}c_2 + R_{12}c_4)b_1}{(C_{11}K_{11} - R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2]}, \\
E_3 =& -b_6 - \frac{(C_{44}R_{12} - C_{12}R_{44})b_5 + [R_{12}(R_{12}+R_{44}) - C_{12}(K_{12}+K_{44})]b_6}{(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2} \\
&- \frac{(C_{12}c_1 + R_{12}c_3)b_4 - (C_{12}c_2 + R_{12}c_4)b_3}{(C_{11}K_{11} - R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2]}, \\
E_4 =& \frac{(C_{12}c_1 + R_{12}c_3)b_6 - (C_{12}c_2 + R_{12}c_4)b_5}{(C_{11}K_{11} - R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2]}, \\
E_5 =& \frac{(C_{11}R_{12} - C_{12}R_{11})b_1 + (R_{11}R_{12} - R_{11}^2 + C_{11}K_{11} - C_{12}K_{11})b_2}{C_{11}K_{11} - R_{11}^2}, \\
E_6 =& \frac{(C_{11}R_{12} - C_{12}R_{11})b_3 + (R_{11}R_{12} - R_{11}^2 + C_{11}K_{11} - C_{12}K_{11})b_4}{C_{11}K_{11} - R_{11}^2}, \\
E_7 =& \frac{(C_{11}R_{12} - C_{12}R_{11})b_5 + (R_{11}R_{12} - R_{11}^2 + C_{11}K_{11} - C_{12}K_{11})b_6}{C_{11}K_{11} - R_{11}^2}, \\
F_1 =& b_2 + \frac{C_{44}K_{44} - R_{44}^2}{(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2} + \frac{(C_{11}R_{12} - C_{12}R_{11})b_1 + (R_{11}R_{12} - C_{12}K_{11})b_2}{C_{11}K_{11} - R_{11}^2}, \\
F_2 =& \frac{(C_{12}R_{11} - C_{11}R_{12})b_3 + (C_{12}K_{11} - R_{11}R_{12})b_4}{C_{11}K_{11} - R_{11}^2}, \\
&+ \frac{(C_{44}R_{12} - C_{12}R_{44})b_3 + [R_{12}(R_{12}+R_{44}) - C_{12}(K_{12}+K_{44})]b_4}{(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2} \\
&+ \frac{(C_{12}c_1 + R_{12}c_3)b_2 - (C_{12}c_2 + R_{12}c_4)b_1}{(C_{11}K_{11} - R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2]}, \\
F_3 =& \frac{(C_{12}R_{11} - C_{11}R_{12})b_5 + (C_{12}K_{11} - R_{11}R_{12})b_6}{C_{11}K_{11} - R_{11}^2}, \\
&- \frac{(C_{44}R_{12} - C_{12}R_{44})b_5 + [R_{12}(R_{12}+R_{44}) - C_{12}(K_{12}+K_{44})]b_6}{(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2} \\
&- \frac{(C_{12}c_1 + R_{12}c_3)b_4 - (C_{12}c_2 + R_{12}c_4)b_3}{(C_{11}K_{11} - R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2]}, \\
F_4 =& E_4, \\
G_1 =& \frac{(C_{11}R_{12} - C_{12}R_{11})b_1 + (R_{11}R_{12} - C_{12}K_{11})b_2}{C_{11}K_{11} - R_{11}^2} \\
&+ \frac{[C_{11}(R_{12}+R_{44}) - R_{11}(C_{12}+C_{44})]b_1 + [C_{11}(K_{12}+K_{44}) - R_{12}(R_{12}+R_{44})]b_2}{(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2}, \\
G_2 =& \frac{(C_{12}R_{11} - C_{11}R_{12})b_3 + (C_{12}K_{11} - R_{11}R_{12})b_4}{C_{11}K_{11} - R_{11}^2} \\
&+ \frac{[R_{11}(C_{12}+C_{44}) - C_{11}(R_{12}+R_{44})]b_3 + [R_{11}(R_{12}+R_{44}) - C_{11}(K_{12}+K_{44})]b_4}{(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2}, \\
&+ \frac{(C_{11}c_1 + R_{11}c_3)b_2 - (C_{11}c_2 + R_{11}c_4)b_1}{(C_{11}K_{11} - R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2]}, \\
G_3 =& \frac{(C_{12}R_{11} - C_{11}R_{12})b_5 + (C_{12}K_{11} - R_{11}R_{12})b_6}{C_{11}K_{11} - R_{11}^2} \\
&- \frac{[R_{11}(C_{12}+C_{44}) - C_{11}(R_{12}+R_{44})]b_5 + [R_{11}(R_{12}+R_{44}) - C_{11}(K_{12}+K_{44})]b_6}{(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2}, \\
&- \frac{(C_{11}c_1 + R_{11}c_3)b_4 - (C_{11}c_2 + R_{11}c_4)b_3}{(C_{11}K_{11} - R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2]}, \\
G_4 =& \frac{(C_{11}c_1 + R_{11}c_3)b_6 - (C_{11}c_2 + R_{11}c_4)b_5}{(C_{11}K_{11} - R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44}) - (R_{12}+R_{44})^2]},
\end{aligned}
$$

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\begin{aligned}
H_1 =&\frac{[R_{44}(C_{12}+C_{44})-C_{44}(R_{12}+R_{44})]b_1+[R_{44}(R_{12}+R_{44})-C_{44}(K_{12}+K_{44})]b_2}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
H_2 =&\frac{(C_{11}R_{44}-C_{44}R_{11})b_1+(R_{11}R_{44}-C_{44}K_{11})b_2}{C_{11}K_{11}-R_{11}^2}\\
&+\frac{(C_{44}R_{12}-C_{12}R_{44})b_3+[C_{44}(K_{12}+K_{44})-R_{44}(R_{12}+R_{44})]b_4}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
&+\frac{(C_{44}c_2+R_{44}c_4)b_1-(C_{44}c_1+R_{44}c_3)b_2}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
H_3 =&\frac{(C_{11}R_{44}-C_{44}R_{11})b_3+(R_{11}R_{44}-C_{44}K_{11})b_4}{C_{11}K_{11}-R_{11}^2}\\
&-\frac{(C_{44}R_{12}-C_{12}R_{44})b_5+[C_{44}(K_{12}+K_{44})-R_{44}(R_{12}+R_{44})]b_6}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
&-\frac{(C_{44}c_2+R_{44}c_4)b_3-(C_{44}c_1+R_{44}c_3)b_4}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
H_4 =&\frac{(C_{11}R_{44}-C_{44}R_{11})b_5+(R_{11}R_{44}-C_{44}K_{11})b_6}{C_{11}K_{11}-R_{11}^2}\\
&+\frac{(C_{44}c_2+R_{44}c_4)b_5-(C_{44}c_1+R_{44}c_3)b_6}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
I_1 =&b_1+\frac{(R_{12}K_{44}-R_{44}K_{12})b_2+[R_{12}(R_{12}+R_{44})-K_{12}(C_{12}+C_{44})]b_1}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
I_2 =&-b_3+\frac{(R_{12}K_{44}-R_{44}K_{12})b_4+[R_{12}(R_{12}+R_{44})-K_{12}(C_{12}+C_{44})]b_3}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
&-\frac{(R_{12}c_2+K_{12}c_4)b_1-(R_{12}c_1+K_{12}c_3)b_2}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
I_3 =&b_5+\frac{(R_{12}K_{44}-R_{44}K_{12})b_6+[R_{12}(R_{12}+R_{44})-K_{12}(C_{12}+C_{44})]b_5}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
&+\frac{(R_{12}c_2+K_{12}c_4)b_3-(R_{12}c_1+K_{12}c_3)b_4}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
I_4 =&-\frac{(R_{12}c_2+K_{12}c_4)b_5-(R_{12}c_1+K_{12}c_3)b_6}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
I_5 =&-b_1-\frac{(R_{11}R_{12}-C_{11}K_{12})b_1+(R_{12}K_{11}-R_{11}K_{12})b_2}{C_{11}K_{11}-R_{11}^2},\\
I_6 =&b_3+\frac{(R_{11}R_{12}-C_{11}K_{12})b_3+(R_{12}K_{11}-R_{11}K_{12})b_4}{C_{11}K_{11}-R_{11}^2},\\
I_7 =&-b_5-\frac{(R_{11}R_{12}-C_{11}K_{12})b_5+(R_{12}K_{11}-R_{11}K_{12})b_6}{C_{11}K_{11}-R_{11}^2},\\
J_1 =&\frac{(C_{11}K_{12}-R_{11}R_{12})b_1+(R_{11}K_{12}-R_{12}K_{11})b_2}{C_{11}K_{11}-R_{11}^2}\\
&+\frac{[R_{12}(R_{12}+R_{44})-K_{12}(C_{12}+C_{44})]b_1+(R_{12}K_{44}-R_{44}K_{12})b_2}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
J_2 =&\frac{(R_{11}R_{12}-C_{11}K_{12})b_3+(R_{12}K_{11}-R_{11}K_{12})b_4}{C_{11}K_{11}-R_{11}^2}\\
&+\frac{[K_{12}(C_{12}+C_{44})-R_{12}(R_{12}+R_{44})]b_3+(R_{44}K_{12}-R_{12}K_{44})b_4}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
&+\frac{(K_{12}c_3+R_{12}c_1)b_2-(K_{12}c_4+R_{12}c_2)b_1}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
J_3 =&\frac{(R_{11}R_{12}-C_{11}K_{12})b_5+(R_{12}K_{11}-R_{11}K_{12})b_6}{C_{11}K_{11}-R_{11}^2}\\
&-\frac{[K_{12}(C_{12}+C_{44})-R_{12}(R_{12}+R_{44})]b_5+(R_{44}K_{12}-R_{12}K_{44})b_6}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{(K_{12}c_3+R_{12}c_1)b_4-(K_{12}c_4+R_{12}c_2)b_3}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
J_4=&\frac{(K_{12}c_3+R_{12}c_1)b_6-(K_{12}c_4+R_{12}c_2)b_5}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
K_1=&\frac{(C_{11}K_{12}-R_{11}R_{12})b_1+(R_{11}K_{12}-R_{12}K_{11})b_2}{C_{11}K_{11}-R_{11}^2}\\
&+\frac{[R_{11}(R_{12}+R_{44})-K_{11}(C_{12}+C_{44})]b_1+[R_{11}(K_{12}+K_{44})-K_{11}(R_{12}+R_{44})]b_2}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
K_2=&\frac{(R_{11}R_{12}-C_{11}K_{12})b_3+(R_{12}K_{11}-R_{11}K_{12})b_4}{C_{11}K_{11}-R_{11}^2}\\
&+\frac{[K_{11}(C_{12}+C_{44})-R_{11}(R_{12}+R_{44})]b_3+[K_{11}(R_{12}+R_{44})-R_{11}(K_{12}+K_{44})]b_4}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
&+\frac{(R_{11}c_1+K_{11}c_3)b_2-(R_{11}c_2+K_{11}c_4)b_1}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
K_3=&\frac{(R_{11}R_{12}-C_{11}K_{12})b_5+(R_{12}K_{11}-R_{11}K_{12})b_6}{C_{11}K_{11}-R_{11}^2}\\
&-\frac{[K_{11}(C_{12}+C_{44})-R_{11}(R_{12}+R_{44})]b_5+[K_{11}(R_{12}+R_{44})-R_{11}(K_{12}+K_{44})]b_6}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
&-\frac{(R_{11}c_1+K_{11}c_3)b_4-(R_{11}c_2+K_{11}c_4)b_3}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
K_4=&\frac{(R_{11}c_1+K_{11}c_3)b_6-(R_{11}c_2+K_{11}c_4)b_5}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
L_1=&-\frac{[K_{44}(C_{12}+C_{44})-R_{44}(R_{12}+R_{44})]b_1+(R_{12}K_{44}-R_{44}K_{12})b_2}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
L_2=&\frac{(C_{11}K_{44}-R_{11}R_{44})b_1+(R_{11}K_{44}-R_{44}K_{11})b_2}{C_{11}K_{11}-R_{11}^2}\\
&+\frac{[R_{44}(R_{12}+R_{44})-K_{44}(C_{12}+C_{44})]b_3+(R_{44}K_{12}-R_{12}K_{44})b_4}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
&+\frac{(R_{44}c_2+K_{44}c_4)b_1-(R_{44}c_1+K_{44}c_3)b_2}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
L_3=&\frac{(C_{11}K_{44}-R_{11}R_{44})b_3+(R_{11}K_{44}-R_{44}K_{11})b_4}{C_{11}K_{11}-R_{11}^2}\\
&-\frac{[R_{44}(R_{12}+R_{44})-K_{44}(C_{12}+C_{44})]b_5+(R_{44}K_{12}-R_{12}K_{44})b_6}{(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2},\\
&-\frac{(R_{44}c_2+K_{44}c_4)b_3-(R_{44}c_1+K_{44}c_3)b_4}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]},\\
L_4=&\frac{(C_{11}K_{44}-R_{11}R_{44})b_5+(R_{11}K_{44}-R_{44}K_{11})b_6}{C_{11}K_{11}-R_{11}^2}\\
&+\frac{(R_{44}c_2+K_{44}c_4)b_5-(R_{44}c_1+K_{44}c_3)b_6}{(C_{11}K_{11}-R_{11}^2)[(C_{12}+C_{44})(K_{12}+K_{44})-(R_{12}+R_{44})^2]}.
\end{aligned}
$$

$$
\lambda_1=\frac{1}{\sqrt{2}}\Bigg[-\frac{1}{2}\Big(b+\sqrt{b^2-4c+4t}\Big)+\sqrt{\frac{1}{4}\Big(b+\sqrt{b^2-4c+4t}\Big)^2-2\Big(t+\sqrt{t^2-4e}\Big)}\Bigg]^{1/2}\xi,
$$

$$
\lambda_2=\frac{1}{\sqrt{2}}\Bigg[-\frac{1}{2}\Big(b+\sqrt{b^2-4c+4t}\Big)-\sqrt{\frac{1}{4}\Big(b+\sqrt{b^2-4c+4t}\Big)^2-2\Big(t+\sqrt{t^2-4e}\Big)}\Bigg]^{1/2}\xi,
$$

$$
\lambda_3=\frac{1}{\sqrt{2}}\Bigg[-\frac{1}{2}\Big(b-\sqrt{b^2-4c+4t}\Big)+\sqrt{\frac{1}{4}\Big(b-\sqrt{b^2-4c+4t}\Big)^2-2\Big(t-\sqrt{t^2-4e}\Big)}\Bigg]^{1/2}\xi,
$$

$$
\lambda_4=\frac{1}{\sqrt{2}}\Bigg[-\frac{1}{2}\Big(b-\sqrt{b^2-4c+4t}\Big)-\sqrt{\frac{1}{4}\Big(b-\sqrt{b^2-4c+4t}\Big)^2-2\Big(t-\sqrt{t^2-4e}\Big)}\Bigg]^{1/2}\xi,
$$

where, $t=\left(-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}\right)^{1/3}+\left(-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}\right)^{1/3}$,

$$p=bd-4e-\frac{c^2}{3},\quad q=\frac{2}{27}c^3+\frac{c}{3}(bd-4e)-b^2e+4ce-d^2.$$

$M_i=\frac{\Delta_i}{\Delta},(i=1,2,3,4)$, in which

$$\Delta=\begin{vmatrix}a_{11}&a_{12}&a_{13}&a_{14}\\a_{21}&a_{22}&a_{23}&a_{24}\\a_{31}&a_{32}&a_{33}&a_{34}\\a_{41}&a_{42}&a_{43}&a_{44}\end{vmatrix},\quad \Delta_1=\begin{vmatrix}a_{22}&a_{23}&a_{24}\\a_{32}&a_{33}&a_{34}\\a_{42}&a_{43}&a_{44}\end{vmatrix},\quad \Delta_2=-\begin{vmatrix}a_{21}&a_{23}&a_{24}\\a_{31}&a_{33}&a_{34}\\a_{41}&a_{43}&a_{44}\end{vmatrix},$$

$$\Delta_3=\begin{vmatrix}a_{21}&a_{22}&a_{24}\\a_{31}&a_{32}&a_{34}\\a_{41}&a_{42}&a_{44}\end{vmatrix},\quad \Delta_4=-\begin{vmatrix}a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\\a_{41}&a_{42}&a_{43}\end{vmatrix},$$

$$a_{1i}=G_1\lambda_i+G_2\lambda_i^3+G_3\lambda_i^5+G_4\lambda_i^7,\quad a_{2i}=K_1\lambda_i+K_2\lambda_i^3+K_3\lambda_i^5+K_4\lambda_i^7,$$

$$a_{3i}=H_1+H_2\lambda_i^2+H_3\lambda_i^4+H_4\lambda_i^6,\quad a_{4i}=L_1+L_2\lambda_i^2+L_3\lambda_i^4+L_4\lambda_i^6,$$

$M=\frac{4\overline{\Delta}}{3\Delta}$, in which

$$\overline{\Delta}=\begin{vmatrix}a_{21}&a_{22}&a_{23}&a_{24}\\a_{31}&a_{32}&a_{33}&a_{34}\\a_{41}&a_{42}&a_{43}&a_{44}\\a_{51}&a_{52}&a_{53}&a_{54}\end{vmatrix},\quad a_{5i}=B_1+B_2\lambda_i^2+B_3\lambda_i^4+B_4\lambda_i^6.$$

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