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DISPERSION RELATION OF A MAGNETIZED PLASMA-FILLED BACKWARD WAVE OSCILLATOR*

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A linear theory and a more general dispersion relation of electromagnetic radiation from a magnetized plasma-filled backward wave oscillator with sinusoidally corrugated slow-wave structure driven by a solid intense relativistic electron beam have been given. The comparisons show good agreement with the previous works when $B_0 \to \infty$ and $\omega_b = 0$ from this dispersion relation.

PACC: 5235; 5240M; 5275

I. INTRODUCTION

Recently, much attention has been attracted to the study of a plasma-filled high-power backward wave oscillator (BWO) and other microwave devices for efficiency enhancement.[1] The introduction of plasma into generators of coherent electromagnetic radiation potentially has several beneficial effects, including (a) charge neutralization, allowing operation above the vacuum space-charge limiting current, (b) enhanced generation mechanisms involving the plasma electrons, (c) frequency tunability, and (d) operation without a guiding magnetic field. Up to now, both experimental and theoretical works have been greatly developed in studying of plasma-filled BWO[2−6]. However, in the previous theoretical works, Ref.[4] just considered the case of a solid intense relativistic electron beam (IREB) injected into a plasma-loaded BWO with an infinitely large guiding magnetic field, so there are not the coupling of TM modes with TE modes. As we know, the TM and TE modes are always coupled to form the HE and EH hybrid modes in a magnetized plasma-filled waveguide. Thus the field expressions and dispersion relation are more complicated. Reference [6] presented the propagated theory of electromagnetic wave in a finite magnetic field plasma waveguide, so it is a cold cavity case. If we want to know the interaction of waveguide mode with electron beam, we should consider the case of existing IREB.

In this paper, we present a linear theory of electromagnetic radiation from a backward wave oscillator in an uniform plasma-filled sinusoidally corrugated slow-wave structure and obtain a more general dispersion relation by considering a finite external magnetic field and IREB.

II. BASIC EQUATIONS

The analyses presented here are based on the slow-wave structure shown in Fig.1. The slow-wave structure consists of an axisymmetrical and cylindrical waveguide whose wall radius, $R(z)$, varies sinusoidally according to the relation

$$R(z) = R_0 + h\cos(k_0 z), \tag{1}$$

where h is the corrugation amplitude. $k_0 = 2\pi/z_0$ is the corrugation wavenumber, and z_0 is the length of the corrugation period.

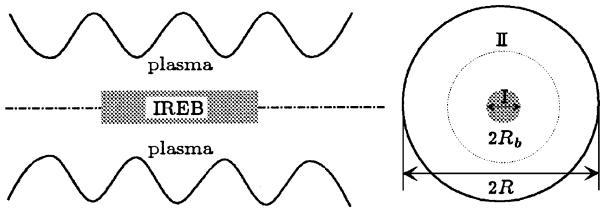


Fig.1. An uniform magnetized plasma-filled sinusoidally corrugated slow-wave structure with perfectly conducting inner wall and beam model.

A solid IREB of density N_b and radius R_b propagates through the waveguide, which is loaded completely with a cold, uniform and magnetized collisionless plasma of density N_p. The dielectric tensor of magnetized plasma can be given by

$$[\varepsilon] = \varepsilon_0 \begin{bmatrix} \varepsilon_1 & j\varepsilon_2 & 0 \\ -j\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix}, \tag{2}$$

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where

$$\begin{aligned}\varepsilon_1 &= 1 = \omega_p^2/(\omega^2-\omega_c^2)\\ \varepsilon_2 &= j\omega_p^2\omega_c/[\omega(\omega^2-\omega_c^2)]\\ \varepsilon_3 &= 1-\omega_p^2/\omega^2.\end{aligned} \tag{3}$$

Here, $\omega_p = (N_p e^2/m_0\varepsilon_0)^{1/2}$ is the electron plasma frequency, $\omega_c = eB_0/m_0$ is the electron cyclotron frequency, and ω is the angular frequency of the electromagnetic wave.

For the corrugated waveguide, the field components should be expanded into space harmonics according to Floquet's theorem, e.g.:

$$\begin{aligned}E_z &= \sum_n E_{zn}\exp(j\omega t - jk_{zn}z + jm\theta)\\ H_z &= \sum_n H_{zn}\exp(j\omega t - jk_{zn}z + jm\theta)\end{aligned} \tag{4}$$

where $k_{zn} = k_z + nk_0$, k_z is the axial wavenumber. E_{zn} and H_{zn} must satisfy the following equations which are obtained from Maxwell equation and Eq.(2),

$$\begin{cases}\nabla_\perp^2 E_{zn} + aE_{zn} = bH_{zn} + jw\mu_0 J_{zn} - \dfrac{jk_{zn}}{\varepsilon_0\varepsilon_1}\rho,\\ \nabla_\perp^2 H_{zn} + cH_{zn} = \mathrm{d}E_{zn} - (\nabla\times\vec{J})_z - \dfrac{\omega\varepsilon_2}{\varepsilon_0}\rho,\end{cases} \tag{5}$$

where

$$\begin{aligned}\nabla_\perp^2 &= \nabla^2 - \partial^2/\partial z^2,\\ a &= (k^2\varepsilon_1 - k_{zn}^2)\varepsilon_3/\varepsilon_1, \quad k^2 = w^2/c^2,\\ b &= jk_{zn}\omega\mu_0\varepsilon_2/\varepsilon_1,\\ c &= -k_{zn}^2 + k^2(\varepsilon_1^2-\varepsilon_2^2)/\varepsilon_1,\\ d &= -jk_{zn}\omega\varepsilon_0\varepsilon_2\varepsilon_3/\varepsilon_1.\end{aligned} \tag{6}$$

From Eq.(5), we can see that the TM and TE modes are coupled to each other, they cannot exist independently, while there are HE or EH modes.[6] J_{zn} and ρ are the perturbed electron beam current density and charge density respectively. They can be expressed by E_{zn} according to the equations of motion and continuity as

$$J_{zn} = -\frac{j\omega\varepsilon_0 w_b^2}{\gamma_0^3(\omega - k_{zn}v_0)^2}E_{zn}, \tag{7}$$

$$\rho = k_{zn}J_{zn}/\omega, \tag{8}$$

where $\omega_b = (N_b e^2/m_0\varepsilon_0)^{1/2}$ is the electron beam frequency, v_0 is the velocity of the electron beam along z axial. From Eqs.(5)–(8) we can get

(i) In the beam region ($|r| \le R_b$):

$$\begin{cases}\nabla_\perp^2 E_{zn} + a'E_{zn} = bH_{zn},\\ \nabla_\perp^2 H_{zn} + cH_{zn} = d'E_{zn},\end{cases} \tag{9}$$

where

$$\begin{aligned}a' &= \frac{k^2\varepsilon_1 - k_{zn}^2}{\varepsilon_1}\left[\varepsilon_3 - \frac{\omega_b^2}{\gamma_0^3(\omega - k_{zn}v_0)^2}\right],\\ d' &= -jk_{zn}\omega\varepsilon_2\left[\varepsilon_0\varepsilon_3/\varepsilon_1 - \frac{w_b^2}{\gamma_0^3(w - k_{zn}v_0)^2}\right].\end{aligned} \tag{10}$$

In getting Eq.(9) we neglect the transverse perturbed beam current. So for symmetric modes (m=0), we can get:

$$\begin{cases}E_{zn} = A_{ln}J_0(q_1 r) + A_{2n}J_0(q_2 r),\\ H_{zn} = A_{ln}H_l J_0(q_1 r) + A_{2n}H_2 J_0(q_2 r),\end{cases} \tag{11}$$

where

$$q_{1,2}^2 = \frac{1}{2}\{(a'+c) \pm [(a'+c)^2 - 4(a'c - bd')]^{1/2}\}, \tag{12}$$

$$H_{1,2} = (a' - q_{1,2}^2)/b. \tag{13}$$

(ii) Outside the beam region ($R_b \le |r| \le R$)

$$\begin{cases}\nabla_\perp^2 E_{zn} + aE_{zn} = bH_{zn},\\ \nabla_\perp^2 H_{zn} + cH_{zn} = \mathrm{d}E_{zn}.\end{cases} \tag{14}$$

The solution to Eq.(14) is

$$\begin{cases}E_{zn} = B_{ln}J_0(p_1 r) + C_{ln}N_0(p_1 r) + B_{2n}J_0(p_2 r) + C_{2n}N_0(p_2 r),\\ H_{zn} = [B_{ln}J_0(p_1 r) + C_{ln}N_0(p_1 r)]h_1 + [B_{2n}J_0(p_2 r) + C_{2n}N_0(p_2 r)]h_2,\end{cases} \tag{15}$$

$p_{1,2}$ and $h_{1,2}$ are determined similarly to Eqs.(12) and (13) by replacing a' and d' with a and d, respectively. In Eqs.(11) and (15), J_0 and N_0 are 0-order Bessel and Neumann functions, respectively.

Ⅲ. DISPERSION RELATION

The field components must satisfy the following continuity and boundary conditions:

(I) at $|r| = R_b$

$$\begin{aligned}E_{zn}^{\mathrm{I}} &= E_{zn}^{\mathrm{II}}, \quad & E_{\theta n}^{\mathrm{I}} &= E_{\theta n}^{\mathrm{II}},\\ H_{zn}^{\mathrm{I}} &= H_{zn}^{\mathrm{II}}, \quad & H_{\theta n}^{\mathrm{I}} &= H_{\theta n}^{\mathrm{II}}.\end{aligned} \tag{16}$$

(Ⅱ) at $|r|=R$

$$\begin{aligned} E_z + E_r\frac{\mathrm{d}R}{\mathrm{d}z} &= 0, \\ H_r + H_z\frac{\mathrm{d}R}{\mathrm{d}z} &= 0. \end{aligned} \tag{17}$$

For the symmetric modes (m=0), the transverse components $E_{\theta n}$, $H_{\theta n}$, E_{rn} and H_{rn} can be expressed by E_{zn} and H_{zn} as

$$E_{rn} = \frac{1}{D}\left[jk_{zn}K^2\frac{\partial E_{zn}}{\partial r} + \omega\mu_0 k_g^2\frac{\partial H_{zn}}{\partial r}\right], \tag{18}$$

$$E_{\theta n} = \frac{1}{D}\left[k_{zn}k_g^2\frac{\partial E_{zn}}{\partial r} - j\omega\mu_0 k^2\frac{\partial H_{zn}}{\partial r}\right], \tag{19}$$

$$H_{rn} = \frac{1}{D}\left[-\omega\varepsilon_0\varepsilon_2 k_{zn}^2\frac{\partial E_{zn}}{\partial r} + jk_{zn}k^2\frac{\partial H_{zn}}{\partial r}\right], \tag{20}$$

$$H_{\theta n} = \frac{1}{D}\left[-j\omega\varepsilon_0(\varepsilon_1 K^2 + \varepsilon_2 k_g^2)\frac{\partial E_{zn}}{\partial r} + k_{zn}k_g^2\frac{\partial H_{zn}}{\partial r}\right], \tag{21}$$

where $D = k^4 - k_g^4$, $k^2 = k_{zn}^2 - k^2\varepsilon_1$, $k_g^2 = k^2\varepsilon_2$. Substituting Eq.(11), (15) and (18)-(21) into Eqs.(16), (17) and using Eq.(4), we can obtain the following matrix expressions:

$$F\cdot A = \sum_{m,n=-\infty}^{\infty} F_{2m2n}A_n = \sum_{m,n=-\infty}^{\infty}\begin{bmatrix}(F_{mn})_1 & (F_{mn})_2\\ (F_{mn})_3 & (F_{mn})_4\end{bmatrix}\begin{bmatrix}A_{1n}\\ A_{2n}\end{bmatrix} = 0. \tag{22}$$

In the above derivation, we applied the same mathematical process as that in Ref.[4]. $(F_{mn})_s (s = 1-4)$ can be expressed in the following forms:

$$(F_{mn})_1 = \sum_{i=1}^{2}\left[1 + (m-n)\frac{k_0k_{zn}K^2 - j\omega\mu_0k_g^2k_0h_i}{D}\right](P_{ni}C_{mn}^{J_i} - R_{ni}C_{mn}^{N_i}), \tag{23}$$

$$(F_{mn})_2 = \sum_{i=1}^{2}\left[1 + (m-n)\frac{k_0k_{zn}K^2 - j\omega\mu_0k_g^2k_0h_i}{D}\right](S_{ni}C_{mn}^{J_i} - T_{ni}C_{mn}^{N_i}), \tag{24}$$

$$\begin{aligned}(F_{mn})_3 = \sum_{i=1}^{2}\Bigg[&\frac{p_i(\omega\varepsilon_0\varepsilon_2k_{zn}^2 - jk_{zn}K^2h_i)}{D}(p_{ni}\triangle_{mn}^{J_i} - R_{ni}\triangle_{mn}^{N_i}) \\ &+ (m-n)\frac{jk_0h_i}{p_i}(P_{ni}\Gamma_{mn}^{J_i} - R_{ni}\Gamma_{mn}^{N_i})\Bigg],\end{aligned} \tag{25}$$

$$\begin{aligned}(F_{mn})_4 = \sum_{i=1}^{2}\Bigg[&\frac{p_i(\omega\varepsilon_0\varepsilon_2k_{zn}^2 - jk_{zn}K^2h_i)}{D}(S_{ni}\triangle_{mn}^{J_i} - T_{ni}\triangle_{mn}^{N_i}) \\ &+ (m-n)\frac{jk_0h_i}{p_i}(S_{ni}\Gamma_{mn}^{J_i} - T_{ni}\Gamma_{mn}^{N_i})\Bigg],\end{aligned} \tag{26}$$

where

$$\begin{cases} P_{n1} = \dfrac{h_2 - H_1}{h_2 - h_1}[p_1J_0(q_1R_b)N_1(p_1R_b) - q_1J_1(q_1R_b)N_0(p_1R_b)], \\ P_{n2} = \dfrac{h_1 - H_1}{h_1 - h_2}[p_2J_0(q_1R_b)N_1(p_2R_b) - q_1J_1(q_1R_b)N_0(p_2R_b)], \end{cases} \tag{27}$$

$$C_{mn}^{J_i} = \sum_{s=0}^{\infty}\frac{(p_ih)^{2s+|m-n|}J_0^{(2s+|m-n|)}(p_iR_0)}{2^{2s+|m-n|}s!(s+|m-n|)!}, \tag{28}$$

$$\Delta_{mn}^{J_i} = \sum_{s=0}^{\infty}\frac{(p_ih)^{2s+|m-n|}J_0'^{(2s+|m-n|)}(p_iR_0)}{2^{2s+|m-n|}s!(s+|m-n|)!}, \tag{29}$$

$$\Gamma_{mn}^{J_i} = \sum_{s=1}^{\infty}\frac{(p_ih)^{2s+|m-n|}J_0^{(2s+|m-n|-1)}(p_iR_0)}{2^{2s+|m-n|}s!(s+|m-n|)!}, \tag{30}$$

From Eq.(27), R_{n1} and R_{n2} can be obtained by replacing N_1 with J_1 and N_0 with J_0, S_{n1} and S_{n2} can be obtained by replacing H_1 with H_2 and q_1 with q_2, T_{n1} and T_{n2} can be obtained by replacing H_1 with H_2, q_1 with q_2, N_1 with J_1, and N_0 with J_0, respectively. $C_{mn}^{N_i}$, $\triangle_{mn}^{N_i}$ and $\Gamma_{mn}^{N_i}$ are obtained by replacing J with

N in eqs. (28)–(30). In eq.(29), J_0' (or N_0') denotes the derivative of J_0 (or N_0). So the dispersion relation for the magnetized plasma filled BWO is expressed by the determinant equation

$$F = \det[F] = 0 \tag{31}$$

IV. DISCUSSIONS AND CONCLUSIONS

We can see that the dispersion relation is more complicated when the finite external magnetic field and IREB are considered simultaneously. However, we can learn the radiation electromagnetic wave frequency characteristics and interaction growth rate from this dispersion relation in a magnetized plasma BWO. In the following, we will discuss two degenerated cases, i.e. one with an infinitely large external guiding magnetic field and the other without IREB, with the above dispersion relation as comparisons.

In the first case, the guiding magnetic field $B_0 \to \infty$, then we can get $\varepsilon_2 = 0$, $\varepsilon_1 = 1$ and $\varepsilon_3 = 1 - w_p^2/w^2$ from Eq.(3). So the dispersion equation reduces to two independent equations for TM and TE modes. For TM modes, there are $|(F_{mn})|_1 = 0$. In this case we have $q_1^2 = a'$, $p_1^2 = a$ and $q_2^2 = P_2^2 = c$. Generally, if let $h_1 = H_1 = 0$, we can get

$$p_{n1} = p_1 J_0(q_1 R_b) N_1(p_1 R_b) - q_1 J_1(q_1 R_b) N_0(p_1 R_b), \tag{32}$$

$$R_{n1} = p_1 J_0(q_1 R_b) J_1(p_1 R_b) - q_1 J_1(q_1 R_b) J_0(p_1 R_b). \tag{33}$$

So the dispersion relation can be expressed as

$$|(F_{mn})_1| = \left|\left[1 + (m-n)\frac{k_0 k_{zn}}{K^2}\right](p_{n1} C_{mn}^{J_1} - R_{n1} C_{mn}^{N_1})\right| = 0. \tag{34}$$

That is the results of Ref.[4].

The dispersion characteristics of Eq.(34) has been studied carefully in Ref.[4]. Their main conclusions are: the primary effect of the presence of plasma in the slow-wave structure is the increase in the oscillation frequency of the TM_{01} mode. The second effect is the appearance of higher modes. It is due to the growth-rate decrease of the TM_{01} mode when plasma density increases. However, in the meanwhile, they indicated that the linear theory could not explain the enhancement effect. It is possible that a nonlinear process is responsible for the enhancement.

In the second case, there is no IREB, let $R_b = R$ and $\omega_b = 0$, then we have $a' = a$, $d' = d$, $p_1 = q_1$, $p_2 = q_2$, $h_1 = H_1$ and $h_2 = H_2$. So $p_{n1} = S_{n2} = -2/\pi R$, others are zero. The elements of dispersion equation (22) can be simplified as

$$(F_{mn})_1 = \left[1 + (m-n)\frac{k_0 k_{zn} K^2 - j\omega\mu_0 k_g^2 k_0 h_1}{D}\right] C_{mn}^{J_1}, \tag{35}$$

$$(F_{mn})_2 = \left[1 + (m-n)\frac{k_0 k_{zn} K^2 - j\omega\mu_0 k_g^2 k_0 h_2}{D}\right] C_{mn}^{J_2}, \tag{36}$$

$$(F_{mn})_3 = \frac{p_1(\omega\varepsilon_0\varepsilon_2 k_{zn}^2 - j k_{zn} K^2 h_1)}{D}\Delta_{mn}^{J_1} + (m-n)\frac{j k_0 h_1}{p_1}\Gamma_{mn}^{J_1}, \tag{37}$$

$$(F_{mn})_4 = \frac{p_2(\omega\varepsilon_0\varepsilon_2 k_{zn}^2 - j k_{zn} K^2 h_2)}{D}\Delta_{mn}^{J_2} + (m-n)\frac{j k_0 h_2}{p_2}\Gamma_{mn}^{J_2}. \tag{38}$$

The above four equations are the integral results of Eqs.(39)–(42) in the Appendix I of Ref.[6] on the condition of symmetric modes (m=0).

Reference [6] has analyzed the detailed influence of external magnetic field. They concluded that the intensity of magnetic field gives quite strong influences on the dispersion characteristics of the wave propagation in this kind of plasma waveguide. When $B_0 \to \infty$, the dispersion curves of the TM_{01} and TE_{01} modes cross each other at two points in each period and when the B_0 decreases, these two curves are gradually separated and become EH_{01} and HE_{01}, respectively.

So, when we consider the interaction of electron beam with waveguide modes in magnetized plasma-filled BWO, there will be some different results from Ref.[4]. First, it is HE or EH mode that will interact with electron beams, instead of TM or TE mode. Second, the numerical calculation of dispersion relation and the analysis of numerical results will be more complicated than those in Refs.[4] and [6]. Because there should be a finite magnetic field to guide the electron beam in the practical experiments, it is worth to study further.

Recently, Hughes Research Laboratories developed a novel high-power microwave source named PASOTRON—the abbreviation of plasma assisted slow-wave-structure oscillator, whose great uniqueness is that it does not require any external magnetic field for electron beam confinement.[3] In this case, $B_0 = 0$, so we have $\varepsilon_2 = 0$ and $\varepsilon_1 = \varepsilon_3 = 1 - \omega_p^2/\omega^2$. When we neglect the influence of ionchannel effect,[7] the dis-

persion relation (34) still can be used, but K, p_1 and q_1 are different from those for $B_0 \to \infty$, due to the difference of ε_1, so the dispersion characteristics may have some differences.

In conclusion, we have derived the general dispersion relation from a backward wave oscillator driven by a solid intense relativistic electron beam with magnetized plasma filled uniformly in a sinusoidally corrugated slow wave structure. We find the dispersion relation is more complicated due to the coupling of TM and TE modes. The comparisons of our results with the previous works show good agreement. The numerical analyses of this dispersion relation are underway.

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