Volume 9, Number 3
1009-1963/2000/09(03)/0199-04
CHINESE PHYSICS
March, 2000
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DYNAMICS IN A DRIVEN DISORDERED VORTEX LATTICE: A NUMERICAL STUDY*

Cao Yi-gang(曹义刚) and Jiao Zheng-kuan(焦正宽)

Department of Physics, Zhejiang University, Hangzhou 310027, *China*

(Received 17 September 1999; revised manuscript received 25 October 1999)

Using a repulsive vortex-vortex interaction model, we numerically investigate the dynamics of vortices in a driven disordered two-dimensional superconductor. At low driving forces and temperatures, we have seen the evidence of plastic and filamentary flow. At high driving forces and very low temperatures, the evidence of an ordering of the moving vortices is seen in the flux flow regime. Our results are in agreement with the previous simulation results and recent experiments.

PACC: 7460G; 0570; 7460J

Much attention has been focused lately on the driven disordered vortex lattice, due to its obvious relevance to typical transport measurements.[1−3] Brant[4], Jensen *et al.*[5] and Cao *et al.*[3] proposed that for all but very weak pinning, the vortex lattice is plastically deformed and the onset of vortex motion takes place through channels. This result goes beyond the applicability of the collective pinning theory for I-V characteristics.[6] An actual dynamic phase transition between the plastically deformed phase and a moving lattice for the vortex lines was first proposed in the two-dimensional (2D) system of pancake vortices by Koshelev and Vinokur.[7] Their idea is that the random pinning noise diminishes with increasing vortex velocity, allowing a high-velocity reordering. Numerical evidences came from the zero-temperature case[2,8] and the finite temperature case.[9]

In experiments on layered superconductor 2H-$NbSe_2$ Bhattacharya *et al.*[10] observed three distinct regimes as a function of increasing current: (i) pinned vortices, no motion; (ii) disordered, plastic motion; and (iii) coherently moving vortex lattice. Structural evidence of these regimes was found in neutron scattering measurements.[11] Experiments[12] on amorphous $Mo_{77}Ge_{23}$ thin films found that the onset of voltage shows abrupt steps in the differential resistance. Furthermore, experiments[13] on $YBa_2Cu_3O_{7-\delta}$ revealed a regime between (i) and (ii) with "steps" in the I-V curve.

In this paper, using a simple repulsive vortex-vortex interaction model, we numerically examine the dynamics of a disordered 2D vortex system subjected to a uniform driving force. We model a transverse 2D slice (in the x-y plane) of a superconducting infinite slab containing rigid vortices that are parallel to the sample edge ($\boldsymbol{H} = \boldsymbol{H} \cdot \hat{\boldsymbol{z}}$). The flow behavior of the driven 2D vortex lattice is numerically studied with varying temperatures. The subject is the evolution in velocity-force dependence (VFD) and its differential, corresponding to the I-V characteristics and the differential resistance $\mathrm{d}V/\mathrm{d}I$ of a type-Ⅱ superconductor in the mixed state, respectively.

The dynamic behavior of the vortex system is described by the Langevin equation,

$$\eta \frac{\mathrm{d}\boldsymbol{R}_i}{\mathrm{d}t} = -\sum_{i\neq j} \nabla_i U_v(\boldsymbol{R}_i - \boldsymbol{R}_j) - \sum_{j'} \nabla_i U_p(\boldsymbol{R}_i - \boldsymbol{r}_{j'}) + \boldsymbol{F}_i^L(t) + \boldsymbol{F}_{\mathrm{d}r}, \tag{1}$$

where $\boldsymbol{R}_{i,j}$ and $\boldsymbol{r}_{j'}$ are the vortex and pin coordinates, respectively; $U_v(\boldsymbol{R})$ is the vortex-vortex interaction potential, $U_p(\boldsymbol{R}-\boldsymbol{r})$ is the vortex-pin interaction potential, $\boldsymbol{F}_{\mathrm{dr}}$ is the external driving force, and η is the viscosity coefficient. The Langevin force $\boldsymbol{F}_i^L(t)$ describes the coupling with a heat bath, and can be expressed by the correlation function[14]:

$$< F_{i\alpha}^L(t) F_{j\beta}^L(t') >= 2\eta T \delta_{ij}\delta_{\alpha\beta}\delta(t-t'). \tag{2}$$

Different from Ref.[9], where the Bessel-function form for the vortex-vortex interacting potential was chosen, we choose the vortex-vortex interacting potential as the repulsive logarithmic:

$$U_v(\boldsymbol{R}) = A_v\left(\frac{R^2}{R_v^2} - 1\right)\ln\left(\frac{R}{R_v}\right). \tag{3}$$

The vortex-pin interaction is modeled by an attracting Gaussian potential

$$U_p(\boldsymbol{R}-\boldsymbol{r}) = -A_p r_p/2 \exp[-(R-r)/r_p^2]. \tag{4}$$

* Project supported by the National Center for Research and Development on Superconductivity of China (Grant No. Yu-06-9704).

The parameters η and A_p are fixed to unity. We choose the dimensionless prefactor of the vortex-vortex interaction term $A_v = 0.001$. All the lengths will be measured with respect to the lattice constant a_0 of the ideal triangular lattice, which is related to the vortex density n_v by $(3/4)^{1/2}a_0^2 = n_v^{-1}$, and the scale for temperature is chosen as the "bare" Kosterlitz-Thouless melting temperature $T_{m0} = c_{66}a_0^2/4\pi$.[15] We consider $N_v = 100$ 2D vortices in a thin superconducting film with $N_p = 400$ randomly distributed pinning centers. The cutting length R_v and the size of pinning centers r_p are chosen to be $R_v = 3.6a_0$ and $0.5a_0$, respectively.

We change the temperature over 4 orders from high to very low. For each value of temperature, we initially place the vortices in a perfect lattice, slowly increasing the driving force $F_{\rm dr}$ from zero to 2.0 and measure average velocity $V_x = \dfrac{1}{N_v}\sum_{i=1}^{N_v}\boldsymbol{V}_i \cdot \hat{\boldsymbol{x}}$ and its differential $\mathrm{d}V_x/\mathrm{d}F_x$. Each time $F_{\rm dr}$ is increased by 0.04, and then we halt the increases in $F_{\rm dr}$, checking that the velocity signal V_x is stationary over time, and then collecting detailed velocity information for a long time interval, 2×10^5 molecular dynamics steps, at a single driving force.

Different VFD curves from low to high temperatures are plotted on a double- logarithmic scale in Fig.1 The results at low temperatures (see the curve

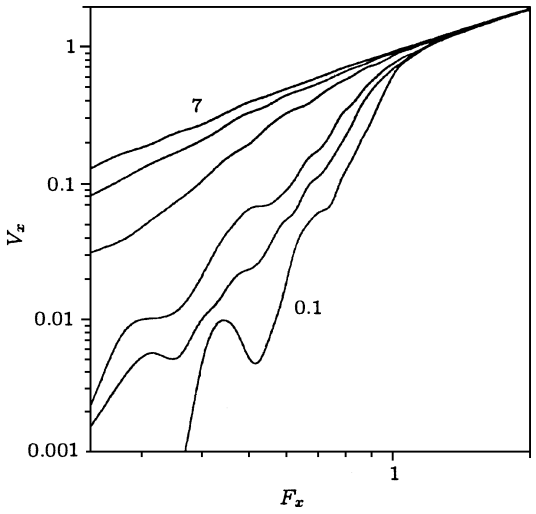


Fig.1. Series of VFD curves at T/T_{m0} (from top to bottom)=7, 5, 3, 1, 0.5, 0.1.

of $T/T_{m0} = 0.1$) clearly show that there are three distinct regimes. (i) At low driving forces ($F_x < 0.36$), the lattice is pinned and the velocity is generated mainly by rearrangements of topological defects, which produce a very slow and unsteady advance of the pinned lattice. (ii) At moderate F_x ($0.52<F_x<1.0$), the VFD curve is strongly nonlinear. (iii) At high F_x, the linear VFD respondence restores. These are in qualitative agreement with the previous simulation results.[9] In addition, an obvious behavior is that a step occurs just above the depinning. "Steps" behavior can be seen most clearly in the curves of $T/T_{m0} = 0.5$ and $T/T_{m0} = 1$. As pointed out in Ref.[16], steps correspond to switches between different patterns of channel flow. At a moment, the flow is filamentary, i.e., the flow is in a one-dimensional path, the effect of increasing F_x is only to increase the speed of the vortices inside the channel, while the structure of the channel and pinned vortices remains stable in a region of F_x, resulting in a linear VFD with a small slope. At another moment, with increasing F_x, the channel becomes unstable and the system switches to a different attractor characterized by a new spatiotemporal channel structure, which is again stable in a range of F_x. This results in sudden jumps in the VFD curve between different linear regimes. With increasing temperature, the influence of disorder decreases, leading to a decrease of the critical pinning force $F_{\rm c}$ and the restoration of the linear VFD at small forces. This can be explained as follows:[17] the thermal fluctuations of the individual vortex lines lead to a dynamical sampling and hence averaging of the disorder potential over the spatial extent of the thermal displacement $< u^2 >_{\rm th}^{1/2}$. Thermal disorder hence opposes quenched disorder via a smoothing of the disorder potential and thereby reduces the critical pinning force in the system. At high temperatures, the interaction between vortices becomes important. Therefore, when the temperature becomes high enough, the linear behavior of VFD is restored even at small forces.

To see the above features clearly, we present three differential curves of VFD at T/T_{m0}=7, 1, 0.1 in Fig.2. One can see that there is no peak in the high temperature curves, and $\mathrm{d}V_x/\mathrm{d}F_x$ vs F_x curves rises monotonically with increasing F_x and asymptotically approaches the flux flow value. The corresponding VFD curve in Fig.1 also exhibits a smooth monotonic rise and asymptotic approach to its linear flux flow behavior. This corresponds to the classical flux flow picture.[14] The behavior of the low temperature curve is different. There is a peak in the $\mathrm{d}V_x/\mathrm{d}F_x$ vs F_x curve, which exceeds the flux flow value. The peak arises since $\mathrm{d}V_x/\mathrm{d}F_x$ at first increases as a driving force is applied and the vortices depin and flow defectively, and then drops as the vortices order and the

dynamic friction they experience decreases. The peak in $\mathrm{d}V_x/\mathrm{d}F_x$ vs F_x curve is associated with a drop in the defect density of the moving vortex configuration[12], and becomes more pronounced in the curve measured at lower temperatures. "Fingerprints" are also observed crossing over the peak in the low temperature curves, as pointed out in Ref.[12]. An interesting behavior is that there are most numbers of steps in the moderately low temperature curves. When the temperature is further decreases, the number of steps shrinks. It suggests some increase in the order of the system of vortices at very low temperatures.

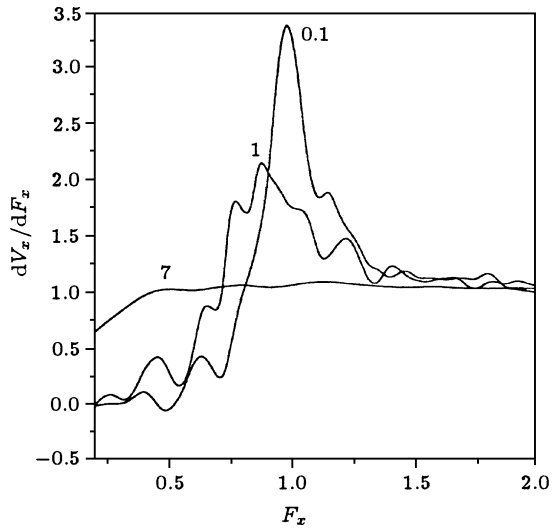


Fig.2. $\mathrm{d}V_x/\mathrm{d}F_x$ vs F_x at T/T_{m0} (from left to right)=7, 1, 0.1.

The ordering at very low temperatures can be seen explicitly in Fig.3, where we present three VFD

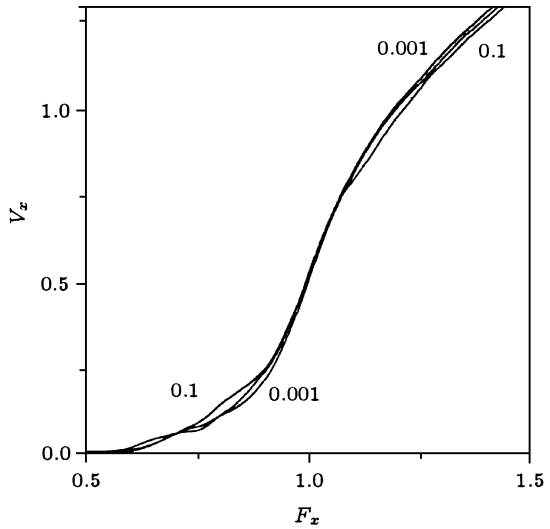


Fig.3. Three VFD curves at very low temperatures T/T_{m0}=0.1, 0.01, 0.001.

curves at very low temperatures T/T_{m0}=0.1, 0.01, 0.001. One can see that a pronounced kink appears in the VFD curves, which signifies a range of the driving force over which the differential exceeds the flux flow value.[9,12]

To further characterize this ordering, we reexamine the VFD curves shown in Fig.1 and Fig.3. At high driving forces, VFD curves all exhibit a well-defined linear region of flux flow. If we extrapolate this linear $V_x = (F_x - F_\mathrm{p})R_{ff}$ back to the driving force axis, as illustrated in the inset of Fig.4, the intercept of driving force axis gives us the average pinning force, F_p, that the vortices feel in the flux flow regime. One can see in Fig.4 that F_p at first increases and then decreases with decreasing temperature. The fact that F_p decreases as the temperature drops below $T^*(T^* \approx 0.5T_{m0})$ suggests that the rigidity of the moving system of vortices increases as the temperature is lowered below T^*, as pointed out in Refs.[9] and [12]. The pinning in 2D films at low temperatures has previously been shown to be collective[18], so the theory of Larkin and Ovchinnikov[6] can give us a qualitative idea of the origin of this increase in the rigidity of the moving vortex configuration. In this theory, the transverse correlation length is given by $R_\mathrm{c} \propto \sqrt{1/I_\mathrm{c}}$ where I_c is the critical current and proportional to the average pinning force F_p. Hence, the drop in F_p at low temperatures implies an increase in the correlation area in the flux flow regime.

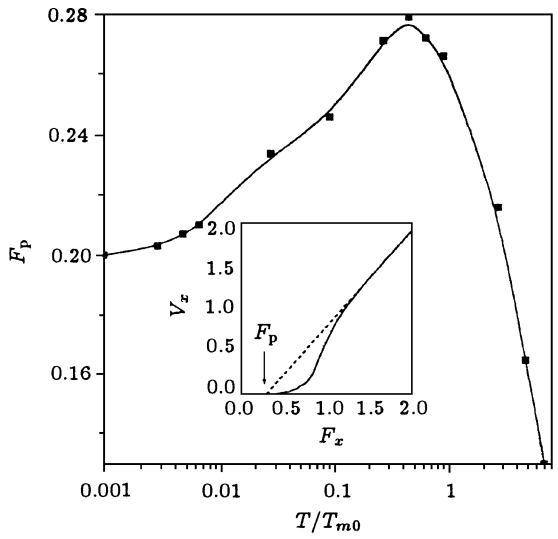


Fig.4. Average pinning force at high driving force vs temperature. Inset: The extrapolation from the flux flow regime to obtain the average pinning force.

The drop in F_p, the pronounced peaks in the $\mathrm{d}V_x/\mathrm{d}F_x$ vs F_x curves and shrinking of the steps in the curves of VFD as well as its differential are all observed at temperatures below T^*, consistent with the simulation results[9] and experiments.[12] These features sug-

gest an ordering of the system at high driving force in this temperature range. The peak in the differential curve characterizes the driving force at which the ordering occurs for a given temperature. The decreasing $F_{\rm p}$ indicates an increasing order in the moving vortex configuration (in the high driving force limit beyond the peak) as the temperature is lowered below T^*.

In conclusion, using a simple repulsive vortex-vortex interaction model, we have numerically studied the dynamics of driven disordered 2D vortex lattices. At low driving forces and temperatures, we have seen the evidence of plastic and filamentary flow. At high driving forces and very low temperatures, the evidence of an ordering of the moving vortices has been seen in the flux flow regime. Our results are in agreement with previous simulation results and recent experiments.

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