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CHAOTIC SOLITON IN COUPLED LONG JOSEPHSON JUNCTIONS*

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The interaction between soliton and sinusoidal wave in two weakly coupled long Josephson junctions is studied. Theoretical analysis reveals that the soliton may be embedded in Melnikov chaotic attractors and the Fiske-step-modes are implied in the boundedness condition of the system. Comparison between the chaotic soliton oscillators and synchronized soliton oscillators shows that the former possesses greater maximal velocity and energy.

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I. INTRODUCTION

The perturbed sine-Gordon equations (PSGEs) have attracted much attention as the model equations of long Josephson junctions (LJJs).[1−4] Dynamical chaos in this model was investigated theoretically and numerically.[5−8] Recently, the Fiske-step-modes were found experimentally in the stacked junctions.[9] These current steps were called the devils' staircase, which is a typical chaotic singnature.[10] The recent investigations[11,12] have shown that as a critical state chaos makes the system parameters the extreme values. Maximal velocity and energy of the motion of a system should be associated with its chaotic state thereby. For purpose of applicability, the methods for controlling chaos have also been established for various systems.[13,14]

In the present paper, we consider two weakly coupled LJJs governed by the coupled PSGEs.[15] As a perturbation the interaction between soliton and sinusoidal wave leads to chaotic oscillation in the system. We analytically prove that the chaotic solutions are bounded under the perturbation, if and only if a relation among the system parameters and initial conditions is satisfied. The Fiske-steps are implied in the boundedness relation. From this relation we can obtain the maximal velocity of motion associated with maximal energy of the chaotic soliton. The comparison between the chaotic soliton oscillators and synchronized soliton oscillators exhibits that the maximal energy of the former is much larger than the latter. The results present a theoretical basis for designing efficient Josephson effect devices by controlling chaos in the system.

II. CHAOTIC SOLITON IN A SYSTEM

The coupled PSGEs governing the two LJJs are[15]

$$\phi_{i,xx} - \phi_{i,tt} - \sin\phi_i = \alpha\phi_{i,t} - \beta\phi_{i,xxt} - \eta_i + \Delta_1\phi_{j,xx} + \Delta_2\phi_{j,tt}, \quad (1)$$

for i,j=1,2 and $i \neq j$. The field variables ϕ_i denote the quantum mechanical phase differences over each of the junctions. The space-time dimensions x and t are normalized to the characteristic Josephson length λ_J and inverse plasma frequency ω_J^{-1} respectively,

$$\lambda_J = \sqrt{\frac{c\Phi_0}{8\pi^2 j_c d}}, \quad \omega_J = \sqrt{\frac{2\pi c j_c}{C\Phi_0}},$$

where c and C are the light velocity and junction capacitance per unit area, respectively; Φ_0 is the flux quantum, j_c the critical current density of the superconductor and $d = 2\lambda_L + b$ with λ_L being the London penetration length and b the thickness of the dielectric barrier. The parameters α and β are the normalized loss factors and η_i is the normalized bias current density in the junction i. The parameters Δ_1 and Δ_2 represent the inductive and capacitive couplings respectively. For brevity, we study the travelling wave solutions of the forms

$$\phi_i = \psi_i(\xi) + \pi, \ \ \xi = \gamma(u)(x - ut), \ \ \gamma(u) = 1/\sqrt{1-u^2}, \quad (2)$$

with constant velocity u such that Eqs.(1) become two ordinary differential equations[15]

$$\psi_{i,\xi\xi} + \sin\psi_i = -\alpha u\gamma(u)\psi_{i,\xi} + \beta u\gamma^3(u)\psi_{i,\xi\xi\xi} - \eta_i + (\Delta_1 + \Delta_2 u^2)\gamma^2(u)\psi_{j,\xi\xi}. \quad (3)$$

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Suppose that the parameters $\alpha, \beta\eta_i$, and Δ_i are so small that the terms proportional to them can be regarded as perturbations added to the unperturbed sine-Gordon equations

$$\psi_{i,\xi\xi}^{(0)} + \sin\psi_i^{(0)} = 0, \quad i = 1, 2. \tag{4}$$

Setting $\psi_i = \psi_i^{(0)} + \psi_i^{(1)}$ for $|\psi_i^{(1)}| \ll |\psi_i^{(0)}|$ and inserting them into Eq.(3) yield the perturbed equations

$$\psi_{i,\xi\xi}^{(1)} + (\cos\psi_i^{(0)})\psi_i^{(1)} = \varepsilon_i, \qquad i = 1, 2. \tag{5a}$$

$$\varepsilon_i = -\alpha u\gamma(u)\psi_{i,\xi}^{(0)} + \beta u\gamma^3(u)\psi_{i,\xi\xi\xi}^{(0)} - \eta_i + (\Delta_i + \Delta_2 u^2)\gamma^2(u)\psi_{j,\xi\xi}^{(0)} \tag{5b}$$

in the first order approximation.

It is clear that Eq.(4) possess the kink solitons and the periodic solutions in terms of the elliptical function. When $\psi_i^{(0)}$ is small, there exists approximate solution in the form of sinusoidal wave. The kink solution corresponds to the well-known heteroclinic one[16−18], and the weak couplings to periodic solution supply periodic perturbations to the heteroclinic orbit in ξ space-time evolution. This is just the mechanism producing Melnikov chaos.[17,18] For convenience of analysis, we only investigate the interaction between the kink soliton and sinusoidal wave

$$\psi_1^{(0)} = 2\arctan\sinh\xi_1, \quad \xi_1 = \xi - \gamma(u)A_1, \tag{6}$$

$$\psi_2^{(0)} = B\sin\xi_2, \quad \xi_2 = \xi - \gamma(u)A_2, \quad B \ll 1 \tag{7}$$

with arbitrary constants A_i and B being determined by the initial and boundary conditions. Conversely, by setting the corresponding initial and boundary conditions we can obtain such soliton and periodic solutions. However, the initial and boundary conditions cannot be set experimentally. Therefore the solutions considered are only theoretically determined and experimentally possible. Knowing this possibility, we may adjust the initial and boundary conditions to produce such solutions which are associated with the greater emitted power experimentally. Applying the direct perturbation technique[11,20] to Eq.(5), we derive the first order solutions, in terms of the solutions (6) and (7), as

$$\psi_i^{(1)} = f_i\int_{B_i}^{\xi} g_i\varepsilon_i\mathrm{d}\xi - g_i\int_{C_i}^{\xi} f_i\varepsilon_i\mathrm{d}\xi, \quad i = 1, 2, \tag{8}$$

where the integration constants B_i and C_i are adjusted by the initial and boundary perturbations $\psi_i^{(1)}(\xi_0)$ and $\psi_{i,\xi}^{(1)}(\xi_0)$ with $\xi_0 = \gamma(u)(x_0 - ut_0)$; f_i and g_i are the functions expressed by $\psi_i^{(0)}$, namely

$$g_i(\xi) = \psi_{i,\xi}^{(0)}, \quad f_i = \psi_{i,\xi}^{(0)}\int(\psi_{i,\xi}^{(0)})^{-2}\mathrm{d}\xi. \tag{9}$$

The substitution of Eqs.(8) and (9) into Eq.(5b) can directly prove the assertion. Applying Eqs.(5b), (6), (7) and (9) to Eq.(8) gives explicit forms of the solutions $\psi_i^{(1)}$. Obviously, $\psi_2^{(1)}$ is a periodic solution, because of the periodicity of the functions f_2, g_2 and ε_2. Since Eq.(6) is a heteroclinic solution[17] in ξ evolution and ε_1 contains periodic perturbation, the solution $\psi_1^{(1)}$ describes chaotic motion of the system.

We are interested in determining the chaotic solution $\psi_1^{(1)}$. Inserting Eq.(6) into Eqs.(9) and (5b) yields

$$g_1 = 2\mathrm{sech}\,\xi_1, \quad f_1 = \frac{1}{4}(\sinh\xi_1 + \xi_1\mathrm{sech}\,\xi_1), \tag{10}$$

$$\varepsilon_1 = -2\alpha u\gamma\mathrm{sech}\,\xi_1 + 2\beta u\gamma^3\mathrm{sech}\,\xi_1(2\tanh^2\xi_1 - 1) - \eta_1 - B(\Delta_1 + \Delta_2 u^2)\gamma^2\sin\xi_2. \tag{11}$$

Combining Eqs.(10) and (11) with Eq.(8) gives $\psi_1^{(1)}$ of the integral form, in which the integrands $\mathrm{sech}\,\xi_1\sin\xi_2$ and $\xi_1\mathrm{sech}\,\xi_1\sin\xi_2$ are non-integrable. The non-integrability is one of the features of chaos. The solution $\psi_1^{(1)}$ seems to be unbounded, because of the infinite f_1 as $t \to \infty$ or $\xi \to \pm\infty$. However, the unboundedness can be controlled by a necessary and sufficient condition. That is to say, $\psi_1^{(1)}$ is bounded as $t \to \infty$, if and only if the condition

$$\lim_{t\to\infty}\int_{B_1}^{\xi} g_1\varepsilon_1\mathrm{d}\xi = \lim_{\xi\to\pm\infty}\int_{B_1}^{\xi} g_1\varepsilon_1\mathrm{d}\xi = 0 \tag{12}$$

is satisfied. The necessity of Eq.(12) is evident. By using the l'Hospital rule to the $\psi_1^{(1)}$ in Eq.(8), we can derive its finite limit superior and limit inferior for $t \to \infty$ under the condition (12). This is the proof for the sufficiency. Although the travelling wave solutions $\psi_i^{(1)}(\xi)$ are of quite spacial case, they can be directly compared with the known chaotic systems and display many interesting properties of the LJJs. Comparing Eq.(12) with the Melnikov function $M(t_0)$[17−19] of the chaotic systems, we find the former implying

$$M(\xi_0) = \int_{-\infty}^{\infty} g_1\varepsilon_1\mathrm{d}\xi = 0, \tag{13}$$

which is just the Melnikov criterion for the onset of chaos in ξ evolution. In fact, using time t instead of ξ leads $M(\xi_0)$ to the Melnikov function $M(t_0)$. The chaos criterion (13) is contained in the boundedness condition (12). This means the bounded soliton to

be embedded in the chaotic attractor. The condition (12) exhibits sensitivity of the system to the parameters and integration constant B_1[12], which is another feature of chaos.

Ⅲ. MAXIMAL ENERGY OF THE CHAOTIC SOLITON

The necessary and sufficient condition (12) is non-integrable for finite constant B_1. And the initial constant B_1 cannot be set experimentally. Therefore, Eq.(12) is a qualitative result which is determined by the system parameters and initial constants. The Melnikov chaos criterion (13) is only a necessary condition for boundedness, but integrable. Inserting Eqs.(10) and (11) into Eq.(13) and performing the integration yield the relation,

$$\eta_1 = -\frac{4}{\pi}u\gamma(u)\left[\alpha+\frac{1}{3}\beta\gamma^2(u)\right]+\frac{B}{2}(\Delta_1 + \Delta_2 u^2)\gamma^2(u)\mathrm{sech}\frac{\pi}{2}\sin[\gamma(u)A], \quad (14)$$

among the parameters $\eta_1, \alpha, \beta, u, \Delta_i$ and initial constant A. Here $A = A_2 - A_1$ is the distance between the soliton and a crest of the sinusoidal wave. The chaotic region[18] and maximal soliton speed can be derived from Eq.(14) for a set of fixed parameters η_1, α, β and Δ_i since the indeterministic initial constant A only appears in the sinusoidal function. The results restrict the velocity and energy of motion of the system. For example, applying Eqs.(6) and (7) to the energy definitions and omitting the first-order terms, we have

$$H \approx H_0 = \int_{-\infty}^{\infty}\left[\frac{1}{2}(\psi_{1,t}^{(0)})^2+\frac{1}{2}(\psi_{1,x}^{(0)})^2 + 1-\cos\psi_1^{(0)}\right]\mathrm{d}x = 8\gamma(u). \quad (15)$$

Taking $\alpha = 0.05$, $\beta = 10^{-3}$, $\Delta_2 = 0$, $B = 0.2$, $A = 10, u < 0$ from Refs.[4] and [15], we draw the curves of the inverse Lorentz contraction $\gamma(u)$ vs bias η_1 from Eq.(14) for (a) Δ_1=0.1 and (b) Δ_1=0.5 as in Fig.1. In Fig.1(a) we find the maximal value $\gamma_{\max}(u) \approx 12$ for the couplling Δ_1=0.1 and bias $\eta_1 <1$. The $\gamma_{\max}(u)$ increases rapidly with the increase of coupling parameter. From Fig.1(b) we see that $\gamma_{\max}(u)$ can reach 44 for $\sin[\gamma(u)A]$=1, when Δ_1=0.5 and η_1=0.5. Substituting the maximal value $\gamma_{\max}(u)$=44 into Eq.(15) leads to the maximal energy, $H_{\max}$=352. The regions in the shadows of Figs.1(a) and 1(b) are the so-called chaotic regions where the motion of the system sensitively depends on the initial constant A. In Fig.1(c) we show the partial boundary of the chaotic region in Fig.1(b). It is similar to the Fiske-steps[14], since the average voltage is proportional to the inverse Lorentz contraction $\gamma(u)$. With the increase of A, the numbers of steps will increase and the steps will tend to the devils' staircase.[11,12]

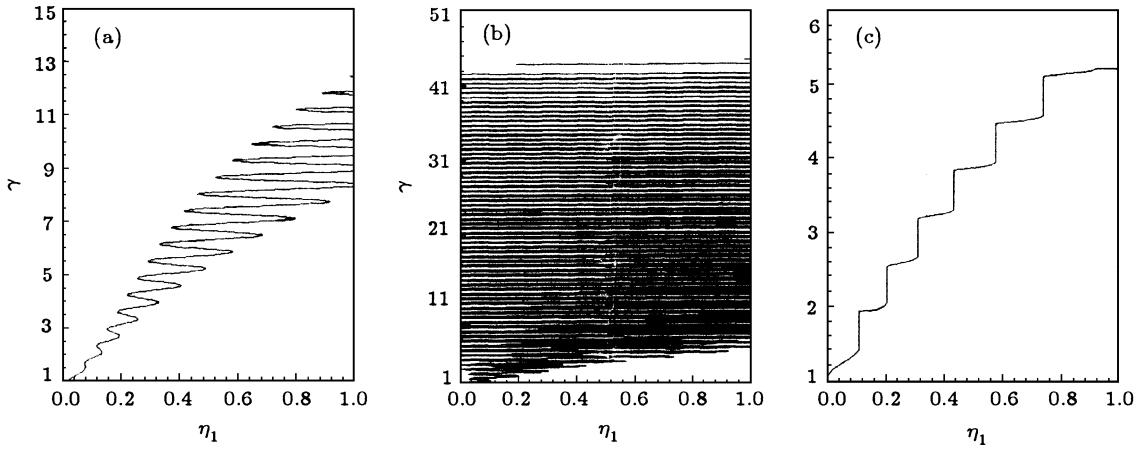


Fig.1. Plot of the inverse Lorentz contraction $\gamma(u)$ vs bias η_1 from Eq.(14) with α=0.05, $\beta = 10^{-3}$, Δ_2=0, B=0.2, A=10, u <0 for (a) Δ_1=0.1 and (b) Δ_1=0.5. (c) is the partial boundary of the chaotic region in (b).

In order to make comparison between the above results and those from the soliton oscillators, we investigate the motion of two synchronized soliton solutions[15] of Eq.(1) under the perturbation of travelling wave. Setting

$$\phi_1 = \phi_2 = \psi(\xi'),\ \xi' = \gamma(u')(x - u't),$$
$$\Delta_1 = \Delta, \Delta_2 = 0,\ \eta_1 = \eta_2 = \eta, \tag{16}$$
$$x' = \frac{x}{\sqrt{1-\Delta}},\ u' = \frac{v}{\sqrt{1-\Delta}},\ \gamma(u') = \frac{1}{\sqrt{1-u'^2}},$$
$$\beta' = \frac{\beta}{1-\Delta}, \tag{17}$$

and inserting them into Eq.(1) yield

$$\psi_{\xi'\xi'} - \sin\psi = -\alpha u'\gamma(u')\psi_{\xi'} + \beta' u'\gamma^3(u')\psi_{\xi'\xi'\xi'} - \eta. \tag{18}$$

Applying the above perturbation method to Eq.(18), we construct its perturbed solution as

$$\psi = \psi^{(0)} + \psi^{(1)},\ \psi^{(0)} = 4\arctan\exp(\xi'), \tag{19}$$
$$\psi^{(1)} = f\int_{B'}^{\xi'} g\varepsilon \mathrm{d}\xi' - g\int_{C'}^{\xi} f\varepsilon \mathrm{d}\xi'$$
$$B', C' = \text{constants}, \tag{20}$$
$$g = \psi^{(0)}_{\xi'} = 2\mathrm{sech}\,\xi',$$
$$f = g\int g^{-2}\mathrm{d}\xi' = \frac{1}{4}(\sinh\xi' + \xi'\mathrm{sech}\,\xi') \tag{21}$$
$$\varepsilon = -2\alpha u'\gamma(u')\mathrm{sech}\,\xi' + 2\beta' u'\gamma^3(u') \cdot \mathrm{sech}\,\xi'(2\tanh^2\xi' - 1) - \eta. \tag{22}$$

The boundedness condition of the solution and the energy of the soliton oscillators, therefore, are

$$\lim_{\xi'\to\pm\infty}\int_{B'}^{\xi'} g\varepsilon \mathrm{d}\xi' = 0 \to \eta$$
$$= -\frac{4}{\pi}u'\gamma(u')\left[\alpha + \frac{\beta\gamma^2(u')}{3(1-\Delta)}\right], \tag{23}$$
$$H' \approx H'_0 = 2\int_{-\infty}^{\infty}\left[\frac{1}{2}(\psi_t^{(0)})^2 + \frac{1}{2}(\psi_x^{(0)}) + 1 - \cos\psi^{(0)}\right]\mathrm{d}x = 16\gamma(u')/\sqrt{1-\Delta}. \tag{24}$$

Taking $\alpha = 0.05, \beta = 10^{-3}$ and $v < 0$, Eq.(23) gives the values of $\gamma(u')$, which decreases with the increase of Δ. Moreover, it is always less than the maximal $\gamma_{\max}(u)$ for arbitrary values of the coupling parameter and bias current. When $\Delta = 0.5$, and $\eta = 0.5$, the $\gamma(u')$ is equal to 5.6. The corresponding energy associated with Eq.(24) thereby is H'=126.7, which is about one-third of the maximal energy of the chaotic oscillators. In order to raise the energy, we must make the system parameters getting into the chaotic region in Fig.1(b). According to the normal method for controlling chaos[21,22], we can set and adjust some new controllable perturbations to fit the control condition (12) such that the maximal soliton velocity and energy of the system can be obtained experimentally. This is probably useful for enhancing the emitted powers from the system.[15]

IV. CONCLUSIONS

In conclusion, we have investigated the interaction between soliton and sinusoidal wave in two weakly coupled long Josephson junctions. The chaotic soliton solution (8) with Eqs.(9)–(11) has been constructed under the perturbation of travelling waves. The necessary and sufficient condition (12) for boundedness of the solution has also been established, which indicates the existence of chaos and implies the Fiske-steps-modes. The theoretical analysis showed that the maximal energy of the chaotic soliton oscillators could be about three times as large as one of the synchronized soliton oscillators for the fixed bias input. The results show the possibility to design chaotic oscillators with considerably great energy by using the method for controlling chaos.

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