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INFLUENCE OF ENTANGLEMENT DEGREE ON SQUEEZING AND PHOTON ANTIBUNCHING IN THE JAYNES-CUMMINGS MODEL*

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We consider the case that one of entangled atoms interacts with a single-mode cavity field. The atom is selectively detected after exiting the cavity, and it is found that the nonclassical properties of field states depend strongly on the entanglement degree between the two atoms. Meanwhile it is proven that squeezing of field and photon antibunching can be greatly enhanced via selective atomic measurements.

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I. INTRODUCTION

In recent years, much interest is devoted to the steering of quantum systems by the postselection of states in the measurement process. A good example of this procedure can be found in the research of Davidovich *et al.*[1], who showed how Schrödinger cat states can be formed in the context of cavity quantum electrody namics (QED). It should be noted that quantum state engineering of this type has a long history within the area of cavity QED. Meystre[2] discussed the effects of repeated quantum measurements on the intercavity field of a lossless micromaser. Krause *et al.*[3] discussed $|n\rangle$-state preparation in a cavity field. More recently Vogel *et al.*[4] showed how to create an arbitrary field state in a single-mode cavity. Guo and Zheng[5] presented a scheme of preparing Schrödinger cat states. Gerry *et al.*[6] showed squeezing enhancement via selective atomic measurements. The basic idea is that the measurement on one of two entangled systems provides information about the other system. The corresponding state reduction can result in a field state with required properties in a well-invented experimented scheme. In our earlier investigation[7], we have shown that nonlinear interaction of an entangled atom with a single-mode field via a one-photon transition can enhance the nonclassical property of sub-Poissonian statistics through conditional measurement on the atom. Here we consider a system of two entangled atoms A and B interacting with a single-mode cavity field and prove the entanglement degree of atoms can strongly influence the nonclassical properties of the cavity field.

II. ENGTANGLEMENT DEGREE BETWEEN TWO ATOMS

If the density operator of the composite system composed of U and V can be written as a tensor product of the reduced density operators assigned to each of the subsystem, i.e., $\hat{\rho} = \hat{\rho}_u \otimes \hat{\rho}_v$, then the subsystem U and V are said to be disentangled; otherwise, they are said to be entangled. Here we use the von Neumann entropy to measure entanglement between two atoms.

The von Neumann entropy S of a system with a density operator $\hat{\rho}$ is given by

$$S = -\mathrm{Tr}(\hat{\rho}\ln\hat{\rho}). \tag{1}$$

Suppose that at the beginning two-level atoms A and B are in the entangled state

$$|\varphi(0)\rangle_{AB} = C_1|e\rangle_a|g\rangle_b - C_2|g\rangle_a|e\rangle_b, \tag{2}$$

where $|e\rangle_a$ is atom A in excited state, $C_1^2 + C_2^2 = 1$.

Thus, the entropy of atoms A and B according to Eq.(1) is given as

$$S = -2C_1^2\ln C_1 - 2C_2^2\ln C_2. \tag{3}$$

It is shown that the greater the von Neumann entropy, the greater the entanglement degree between the two atoms. The entropy reaches the maximum ln2 when $C_1 = C_2 = 1/\sqrt{2} \approx 0.7$, as shown in Fig.1.

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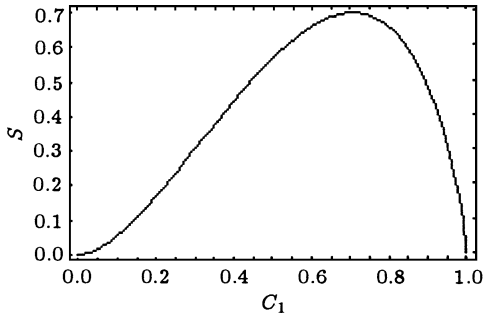


Fig.1. The von Neumann S versus C_2.

Ⅲ. EVOLUTION OF THE FIELD IN JAYNES-CUMMINGS MODEL WITH ENTANGLED ATOMS AFTER SELECTIVE MEASUREMENT ON AN ATOM

A two-level atom B, entangled with atom A, is injected into a high-Q cavity containing a single-mode field, as shown in Fig.2.

Within the rotating-wave approximations, the Hamiltonian of the Jaynes-Cummings model in the

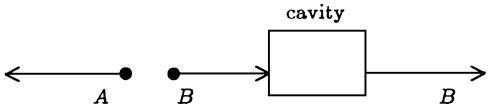


Fig.2. Schematic diagram of the physical model.

interaction picture becomes ($\hbar = 1$)

$$H_{\rm I} = g(a^+ s^- + a s^+),$$

where $s^{\pm}$ are the atomic operators, a and a^+ are the annihilation and creation operators of the cavity field, respectively; and g is the atom-field coupling constant.

Suppose at the beginning the cavity mode is prepared in the coherent state $|\alpha\rangle$, thus the initial state vector $|\varphi(0)\rangle_{AB\rm f}$ may take the form

$$\begin{aligned}|\varphi(0)\rangle_{AB\rm f} &= |\varphi(0)\rangle_{AB} \otimes |\varphi(0)\rangle_{\rm f} \\ &= \sum_{n=0}^{\infty} Q_n |n\rangle (C_1 |e\rangle_a |g\rangle_b - C_2 |g\rangle_a |e\rangle_b) \quad (4)\end{aligned}$$

where $Q_n = \exp\left(-\frac{\alpha^2}{2}\right)\frac{\alpha^n}{\sqrt{n!}}$ (α is a real number for simplicity).

So in the interaction picture the state of the whole system at any time t under the above initial condition is given by

$$\begin{aligned}|\varphi(t)\rangle_{AB\rm f} =& \sum_{n=0}^{\infty} C_1 Q_n \cos(\sqrt{n}\, gt)|e\rangle_a |g\rangle_b |n\rangle + {\rm i}\sum_{n=0}^{\infty} C_2 Q_n \sin(\sqrt{n+1}\, gt)|g\rangle_a |g\rangle_b |n+1\rangle \\ &- \sum_{n=0}^{\infty} C_2 Q_n \cos(\sqrt{n+1}\, gt)|g\rangle_a |e\rangle_b |n\rangle - {\rm i}\sum_{n=1}^{\infty} C_1 Q_n \sin(\sqrt{n}\, gt)|e\rangle_a |e\rangle_b |n-1\rangle. \quad (5)\end{aligned}$$

After an interaction time t, we perform a state-selective measurement on atom B. If atom B is detected in the excited state $|e\rangle_b$, the system reduces to the state

$$|\varphi_e(t)\rangle_{A\rm f} = \frac{1}{N_e}\left[\sum_{n=0}^{\infty} C_2 Q_n \cos(\sqrt{n+1}\, gt)|g\rangle_a |n\rangle + {\rm i}\sum_{n=1}^{\infty} C_1 Q_n \sin(\sqrt{n}\, gt)|e\rangle_a |n-1\rangle\right], \quad (6)$$

where $N_e = \left[\sum_{n=0}^{\infty} C_2^2 Q_n^2 \cos^2(\sqrt{n+1}\, gt) + \sum_{n=1}^{\infty} C_1^2 Q_n^2 \sin^2(\sqrt{n}\, gt)\right]^{1/2}$.

On the other hand, if atom B is detected in the ground state, atom A-field system is in the state

$$|\varphi_g(t)\rangle_{A\rm f} = \frac{1}{N_g}\left[\sum_{n=0}^{\infty} C_1 Q_n \cos(\sqrt{n}\, gt)|e\rangle_a |n\rangle + {\rm i}\sum_{n=0}^{\infty} C_2 Q_n \sin(\sqrt{n+1}\, gt)|g\rangle_a |n+1\rangle\right], \quad (7)$$

where $N_g = \left[\sum_{n=0}^{\infty} C_1^2 Q_n^2 \cos^2(\sqrt{n}\, gt) + \sum_{n=0}^{\infty} C_2^2 Q_n^2 \sin^2(\sqrt{n+1}\, gt)\right]^{1/2}$.

As shown in Eq.(6) or Eq.(7), the measurement on atom B leaves atom A and the field in an entangled state. Even though atom A does not interact with the cavity field, through atom B, the information of atom A is transferred to cavity field. This means that the entanglement swapping has happened.

Ⅳ. SQUEEZING AND ANTIBUNCHING OF THE FIELD IN THE CAVITY

In order to investigate the squeezing property and

photon antibunching effect of field, we define the functions, $a_1 = \dfrac{a+a^+}{2}$, $a_2 = \dfrac{a-a^+}{2\mathrm{i}}$; $f_i = (\Delta a_i)^2 - \dfrac{1}{4}$, $i = 1, 2$;

$$Q = \frac{< (a^+a)^2 > - < a^+a >^2 - < a^+a >}{< a^+a >}.$$

Then the condition for squeezing is $f_i < 0$ for i=1 or 2 and photon antibunching is defined if $Q < 0$.

We first examine the influence of entanglement degree on the squeezing of the field. We plot f_i versus the scaled interaction time $T = gt$ for various values of entanglement factor C_2 when no selective measurement is made. This is plotted in Fig.3. The result shows that a little squeezing occurs for $C_2 = 0.9$. Suppose now we detect atom B in the excited state. The cavity field is then reduced to the state $|\varphi_{\mathrm{e}}(t)\rangle_{\mathrm{Af}}$ of Eq.(6). The results for f_1 are illustrated in Fig.4. The difference is quite apparent. We find that the degree of squeezing can be made greater than in the case without measurement for the same entanglement factor C_2. Furthermore, we find that the entanglement degree (Fig.1) initially increases with C_2 till $C_2 \approx 0.7$ and then decreases, but the degree of squeezing initially decreases with C_2 till $C_2 \approx 0.7$, beyond this point squeezing starts again to increase. In Fig.4, we show that the squeezing at C_2=0.5 can be made nearly 50%, greater than in the case of C_2=0.7 at short times.

The entanglement degree can be used to measure the degree of correlation between two quantum systems. After B atom, which is in maximum entangled state and can carry the maximum information of A

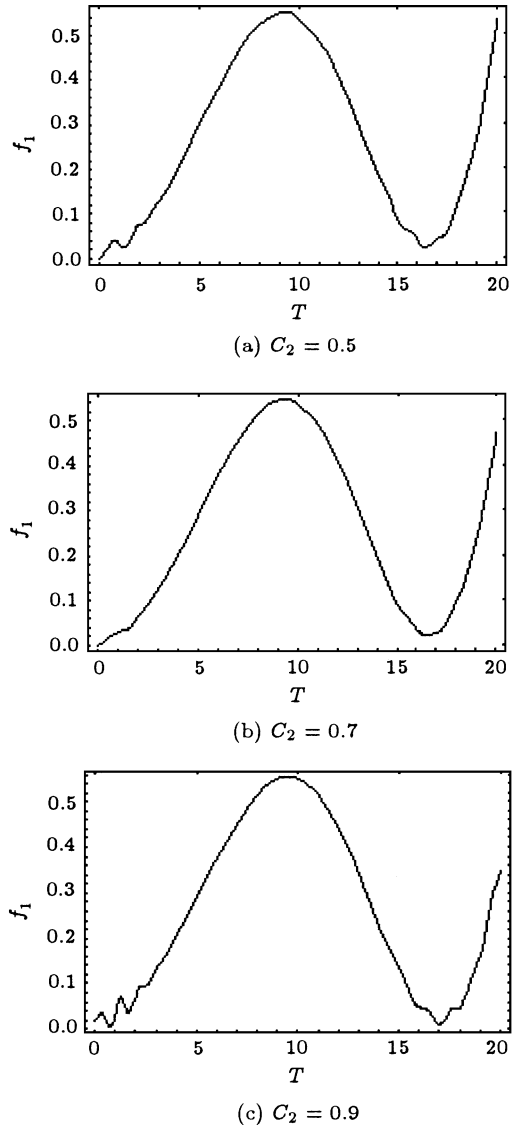


Fig.3. Squeezing for $\bar{n}$=10 when no state selective measurement is made on the atom.

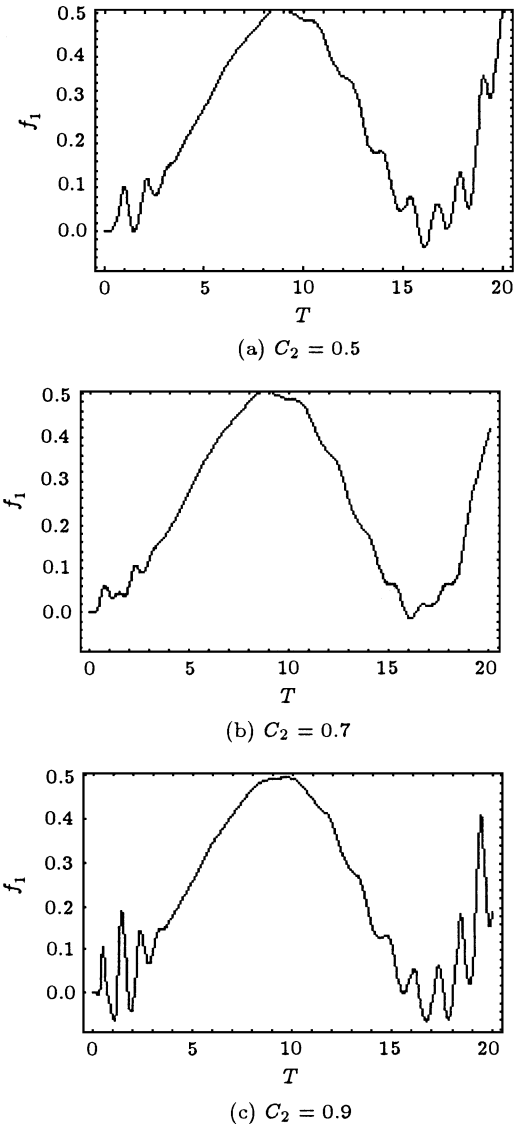


Fig.4. Squeezing for $\bar{n}$=10 but with atom B detected in the excited state.

atom, passes through cavity field, the whole field state is in the maximally entangled mixed-state. The closer the field-state to a pure state, the stronger its nonclassical properties. So we draw the conclusion: with the increase of entanglement degree, the squeezing of the field can be reduced.

Now we consider the case of photon antibunching. Figures 5 and 6 show how the scaled time evolution of Q changes with variations of C_2. It is seen that antibunching is revoked, but recurs periodically with increasing entanglement degree, there is no change in the periodicity of revival. Likewise we find that the entanglement factor C_2 has a strong influence on photon antibunching; with the increase of C_2, antibunching decreases till $C_2 \approx 0.7$ (the maximum value of entanglement degree) and then increases. It is shown that the increase of entanglement degree can enhance the degree of mixed-state, so reduces photon antibunching effect. We also show that in the cases of the selective atomic measurement, the nonclassical property of photon antibunching is much stronger than that in the non-measured cases.

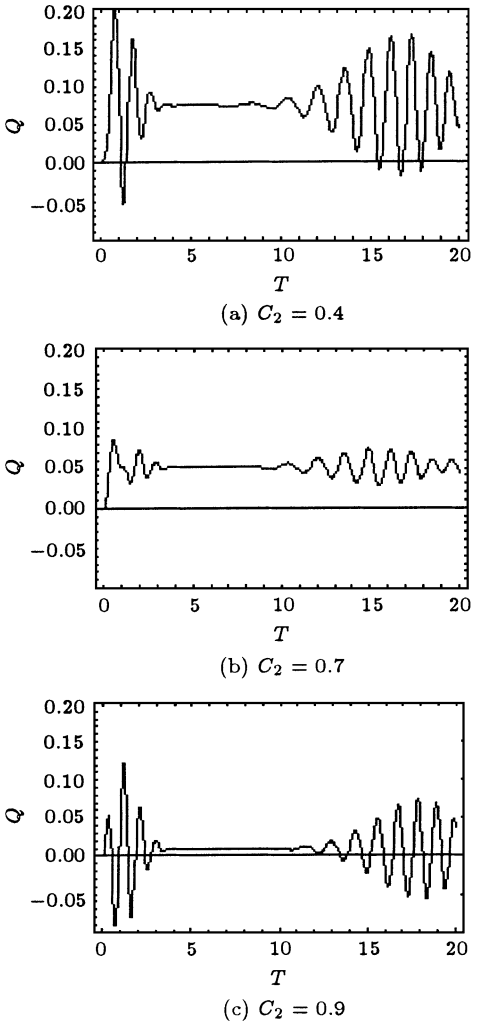


Fig.5. Q parameter for $\bar{n}$=10. No state-selective measurement is made on atom B.

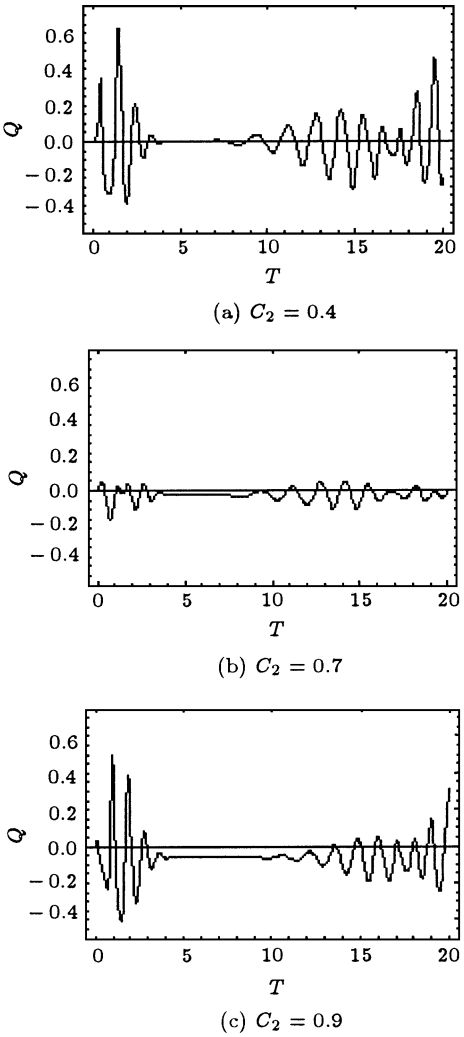


Fig.6. Same as Fig.5 but with atom B detected in the ground state.

V. CONCLUSION

In summary, we have shown that an entangled atom can produce entanglement swapping, but the increase of the entanglement degree of entangled atoms can reduce the squeezing of field and photon antibunching, and also that squeezing and photon antibunching may be enhanced by selective atomic measurements.

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