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EXCITON STATES IN QUANTUM DISKS*

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The method of few-body physics is applied to the calculation of the low-lying states of the exciton of a GaAs disk-like quantum dot with a parabolic potential. The binding energy of the exciton in quantum disks is calculated for two different thicknesses as a function of the disk size. The four lowest exciton states dependent on the disk thickness are investigated.

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Quantum dots (QD's) are artificial structures, which have attracted intense theoretical and experimental interest in recent years.[1] The QD's have been fabricated in different shapes, such as, boxes,[2] square flat plates,[3] disk-like (cylindrical)[4] and spherical[5,6] shapes. In QD's excitons may be confined in all spatial dimensions, and if the exciton Bohr radius is comparable to the QD size, quantum confinement effects (QCE's) can be observed.[7] Many theoretical studies have been devoted to excitonic ground states in spherical dots[8−19] by use of variational technique. A full numerical analysis of this problem is done by Hu and co-workers,[20,21] expanding the excitonic wave functions in terms of solutions of the single-particle Schödinger equations. There are also works with regard to nonspherical dots,[22,23] and the effects such as dielectric confinement and electron-hole exchange interaction on excitonic states in semiconductor QD's are also studied.[24]

In contrast to the various studies on the exciton states in a spherical QD, theoretical studies related to the QCE's on exciton states for a disk-like QD are still rare. However, in experimentally realized QD's, the motion in the z direction is always frozen out into the lowest subband. Since the corresponding extent of the wave function is much less than that in the xy-plane, we can treat the QD's in the two-dimensional limit of thin disks. What about the QCE and the character of exciton binding energy in such QD's? To our knowledge, there have been no theoretical reports on this problem. Hence, in the present work we will use the method of few-body physics to calculate the low-lying states of the exciton of a GaAs disk-like quantum dot with a parabolic potential. The binding energy of the exciton in quantum disks is calculated for two different thicknesses as a function of the disk size. The four lowest exciton states dependent on the disk thickness are invesgated.

For an exciton confined in a quantum disk of thickness d and subjected to a lateral parabolic confinement, the Hamiltonian in the effective mass approximation is given by

$$H = \sum_{i=\mathrm{e,h}} H_i(\boldsymbol{r}_i) - \frac{e^2}{\epsilon|\boldsymbol{r}_\mathrm{e} - \boldsymbol{r}_\mathrm{h}|}\,, \tag{1}$$

where the single-particle Hamiltonians for the electron ($i = \mathrm{e}$) and hole ($i = \mathrm{h}$) are

$$H_i(\boldsymbol{r}_i) = -\frac{\hbar^2}{2m_i^*}\nabla_i^2 + \frac{1}{2}m_i^*\omega_i^2(x_i^2 + y_i^2) + V(z_i)\,. \tag{2}$$

Here $V(z)$ represents the confining potential associated with the disk thickness.[25] The dielectric constant is denoted by ϵ, the projection of the relative electron-hole distance r in the xy-plane is ρ, and other notation is self-evident.

Since $d \ll R$ (disk radius) in most practical quantum disks, the strong confinement potential $V(z)$ allows us, as a good approximation, to separate the motion in the z-direction from that in the xy-plane and to neglect their coupling.[25,26] The effective quasi-two-dimensional Hamiltonian then takes the form

$$H_\rho = \sum_{i=\mathrm{e,h}}\left[-\frac{\hbar^2}{2m_i^*}\nabla_{i\rho}^2 + \frac{1}{2}m_i^*\omega_i^2\rho_i^2\right] - \frac{e^2}{\epsilon[\rho_\mathrm{eh}^2 + \lambda^2]^{1/2}}\,. \tag{3}$$

We have adopted the simpler model potential[27] $-\dfrac{e^2}{\epsilon[\rho_\mathrm{eh}^2 + \lambda^2]^{1/2}}$ for the electron-hole interaction in the exciton, and the disk is assumed to be described by an infinite square-well confinement. The parameter λ is determined by a least-square fit,[28] and is given by $\lambda = 0.2d$.

In terms of the in-plane relative coordinate $\boldsymbol{\rho}_2 = \boldsymbol{\rho}_\mathrm{e} - \boldsymbol{\rho}_\mathrm{h}$ and center-of-mass coordinate $\boldsymbol{\rho}_1 = (m_\mathrm{e}^*\boldsymbol{\rho}_\mathrm{e} +$

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$m_{\rm h}^*\boldsymbol{\rho}_{\rm h})/(m_{\rm e}^*+m_{\rm h}^*)$, the quasi-two-dimensional Hamiltonian can be rewritten as

$$H_\rho = H_0 - \frac{m_{\rm e}^* m_{\rm h}^*}{M}(\omega_{\rm e}^2-\omega_{\rm h}^2)\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_2 - \frac{e^2}{\epsilon[\rho_2^2+\lambda^2]^{1/2}}\,, \quad (4)$$

with

$$H_0 = -\frac{\hbar^2}{2M}\nabla_{\rm cm}^2 + \frac{1}{2}M\omega_1^2\rho_1^2 - \frac{\hbar^2}{2\mu}\nabla_{\rm rel}^2 + \frac{1}{2}\mu\omega_2^2\rho_2^2\,, \quad (5)$$

where $\mu = m_{\rm e}^* m_{\rm h}^*/M$ is the electron-hole reduced mass and $M = m_{\rm e}^* + m_{\rm h}^*$ is the total mass. We have introduced the center-of-mass frequency $\omega_1 = \sqrt{m_{\rm e}^*\omega_{\rm e}^2 + m_{\rm h}^*\omega_{\rm h}^2}$ and the relative frequency $\omega_2 = \sqrt{m_{\rm h}^*\omega_{\rm e}^2 + m_{\rm e}^*\omega_{\rm h}^2}$.

Owing to the cylindrical symmetry of the problem, the exciton wave function can be labeled by the total orbital angular momentum L. To obtain the eigenenergies and eigenstates, H_ρ is diagonalized in model space spanned by translational invariant harmonic product states[29]

$$\Phi_{[K]} = \varphi_{n_1 l_1}^{\omega_1}(\boldsymbol{\rho}_1)\varphi_{n_2 l_2}^{\omega_2}(\boldsymbol{\rho}_2)\,, \quad (6)$$

where φ_{nl}^{ω} is a two-dimensional (2D) harmonic oscillator state with a frequency ω and an energy $(2n+|l|+1)\hbar\omega$, $[K]$ denotes the set quantum numbers (n_1, l_1, n_2, l_2) in brevity, and $l_1 + l_2 = L$ is the total orbital angular momentum. The dimension of the model space is constrained by $0 \le 2(n_1+n_2) + |l_1| + |l_2| \le 20$. If N is increased by 2, the radio of the difference in energy is less than 0.001%. The matrix elements of H_ρ are then given by the following expressions:

$$\begin{aligned}\left\langle \Phi_{[K]}|H_0|\Phi_{[K']}\right\rangle =& [(2n_1+|l_1|+1)\hbar\omega_1 \\ &+(2n_2+|l_2|+1)\hbar\omega_2]\delta_{[K][K']}\,, \quad (7)\end{aligned}$$

$$\begin{aligned}&\left\langle \Phi_{[K]}\left|-\frac{e^2}{\epsilon[\rho_2^2+\lambda^2]^{1/2}} - \frac{m_{\rm e}^* m_{\rm h}^*}{M}(\omega_{\rm e}^2-\omega_{\rm h}^2)\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_2\right|\Phi_{[K']}\right\rangle \\ &= -U_{n_2 n_2'}\delta_{l_2 l_2'}\delta_{n_1 n_1'}\delta_{l_1 l_1'} - \frac{m_{\rm e}^* m_{\rm h}^*}{M} \\ &\quad\times(\omega_{\rm e}^2-\omega_{\rm h}^2)\pi U_{n_1 l_1 n_1' l_1'}U_{n_2 l_2 n_2' l_2'} \\ &\quad\times[\delta_{l_1 l_1'-1}\delta_{l_2 l_2'+1} + \delta_{l_1 l_1'+1}\delta_{l_2 l_2'-1}]\,; \quad (8)\end{aligned}$$

with

$$U_{m_2 n_2'} = \int_0^\infty R_{n_2 l_2}(\rho_2)\frac{e^2}{\epsilon[\rho_2^2+\lambda^2]^{1/2}}R_{n_2' l_2}(\rho_2)\rho_2 {\rm d}\rho_2\,, \quad (9)$$

$$U_{n_i l_i n_i' l_i'} = \int_0^\infty R_{n_i l_i}(\rho_i)R_{n_i' l_i'}(\rho_i)\rho_i^2 {\rm d}\rho_i\,, \quad (10)$$

where R_{nl} is the radial part of the 2D harmonic oscillator function.

In the following, the energies are in units of ${\rm Ry}^* = e^2/2\epsilon a_{\rm B}^*$, and the length in units of $a_{\rm B}^* = \epsilon\hbar^2/e^2\mu$. Using $\sigma = 0.74$, i.e., $m_{\rm e}^* = 0.067 m_{\rm e}$ ($m_{\rm e}$ is the bare mass of single electron), $m_{\rm h}^* = 0.091 m_{\rm e}$, $\epsilon = 12.5$, which are relevant to GaAs, we find ${\rm Ry}^* = 3.35\,{\rm meV}$ and $a_{\rm B}^* = 17.17\,{\rm nm}$.

By using the method of Ref.[30], we took $d = 1.5\,{\rm nm}$ and calculated the energy spectrum of the low-lying states of an exciton in quantum disks as a function of the disk radius R (Fig.1). As the radius of the disk increases, the exciton energies decrease, i.e., the stronger the confinement, the higher the exciton energy. From Fig.1, we find that the exciton energies are proportional to the total orbital momentum L. However, the size effect in two-electron-dot case is the crossover of the levels.[31] Hence, the Coulombic interaction is important in determining the level ordering.

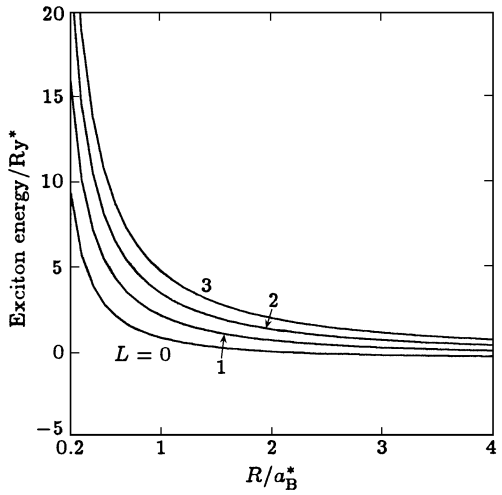


Fig.1. The energy spectra of the four lowest exciton states in a GaAs quantum disk as functions of the disk radius R normalized by the bulk exciton Bohr radius $a_{\rm B}^*$ plotted for $d = 1.5\,{\rm nm}$ and $\sigma = 0.74$. The levels are labeled by the total angular momentum L.

The exciton binding energy $E_{\rm b}$ is defined as

$$E_{\rm b} = E({\rm e}) + E({\rm h}) - E_X\,, \quad (11)$$

where E_X is the energy of an exciton, $E({\rm e})$ and $E({\rm h})$ are the energies of an electron and a hole in the quantum disks, respectively. The dependence of $E_{\rm b}$ on

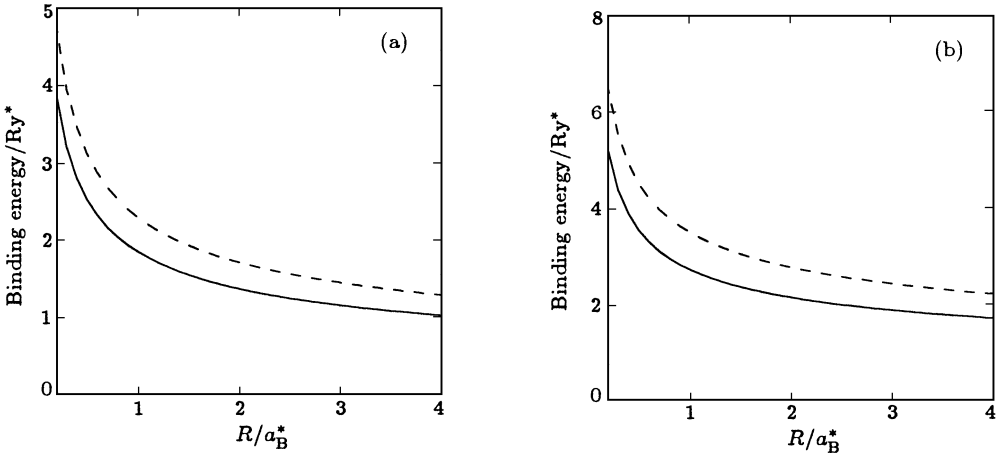


Fig.2. Dependence of the exciton binding energy E_b on the quantum-disk radius R normalized by the bulk exciton Bohr radius a_B^* plotted for $d = 1.5$ nm (a) and $d = 0$ (b) with $\sigma = 0.74$ (solid line) and $\sigma = 0.18$ (dashed line).

the quantum-disk radius R is plotted in Fig.2 for two values of the electron-to-hole mass ratio σ. Two different values of the thickness of the quantum disk, (a) $d = 1.5$ nm and (b) $d = 0$, are considered. The two curves represent the results for a light hole with $\sigma = 0.74$ ($m_h^* = 0.091 m_e$) and a heavy hole with $\sigma = 0.18$ ($m_h^* = 0.377 m_e$). We observe that the heavy hole is the larger the exciton binding energy will be. It is also seen that E_b increases with decreasing R as expected.

In order to see the thickness effect, we plot the energy spectrum as a function of the disk thickness in Fig.3 for the four lowest exciton states with a fixed value of the quantum-disk radius (=32.0 nm). The energy increases with increasing d.

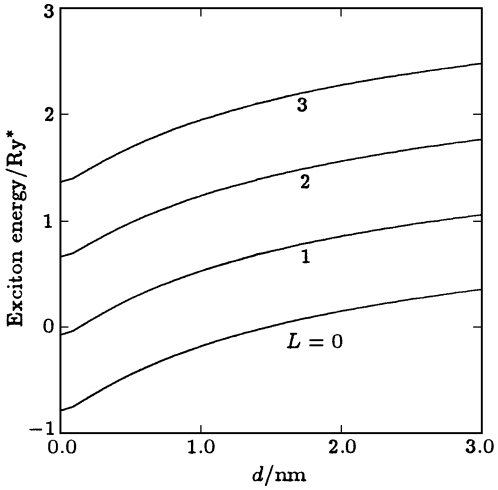


Fig.3. The four lowest exciton levels versus the quantum-disk thickness d with the fixed quantum-disk radius R (=32 nm). The energy is given in units of meV and the thickness is given in unit of nm.

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