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PREPARATION OF ENTANGLED STATES OPTIMAL FOR UNIVERSAL QUANTUM CLONING MACHINE*

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Considering a model of one photon in three separated cavities, we propose a method to create a more general three-cavity entangled state with only one photon optimal for universal quantum cloning machine.

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There has been considerable interest in the study of quantum entanglement in recent years. Entangled states of two particles were first proposed by Einstein, Podolsky and Rosen (EPR states),[1] and they have long been demonstrated experimentally.[2] Cirac and Zoller[3] have proposed a scheme for generating an entangled state of two atoms based on resonant atom-cavity-field interaction. Gerry[4] has prepared a two-cavity entanglement containing only one photon. Such entangle states are in the realm of the original derivation of Bell's theorem,[5] which states that local hidden-variable theories of quantum mechanics are not possible, and are the basis of some quantum effects, such as teleportation of quantum state,[6] certain types of quantum cryptography.[7]

Greenberger, Horne, Shimony and Zeilinger[8] have shown that much stronger refutations of local realism can be provided by entangled states involving three or more particles (GHZ states). Cirac and Zoller[3] have prepared a GHZ state of three atoms if the cavity through which the atoms pass is initially prepared in a superposition of a three-photon number state and a vacuum state. Song and Guo[9] have also prepared entangled states of three atoms via atoms interacting with the cavity-field in $SU(1,1)$ coherent state. Gerry[10] has proposed a scheme to produce the four-atom GHZ state in terms of cavity quantum electrodynamics (QED). Bergou and Hillery et al.[11] have created a three-cavity entangled state, in which either each cavity is in a one-photon number state or each is in the vacuum state. Very recently, a three-photon GHZ state has been observed experimentally[12] based on the method proposed by Zeilinger et al.[13] The entangled states of three- or multi-particle are fascinating quantum systems, the realization of such states will open the door to many novel quantum phenomena and applications in quantum computer and quantum communication.

The entangled states mentioned above are the maximal entangled states. It has been shown that such states are of central importance for quantum information theory. Whereas the general (non-maximal) entangled states have also important applications, particularly, in quantum copying and cloning.[14−18] Recently, Bruβ et al.[15] used the more general three-particle entanglement,

$$|\Psi\rangle = \sqrt{\frac{2}{3}}|100\rangle - \sqrt{\frac{1}{6}}|010\rangle - \sqrt{\frac{1}{6}}|001\rangle, \qquad (1)$$

in order to reach the optimal condition of the universal quantum cloning machine.[15−18]

In this paper, we consider how a three-cavity setup with a single photon excited nonlocally in the cavities may be used to produce the more general entangled state of the three-probe atoms. Our scheme to prepare state (1) is shown in Fig.1. The cavities 1, 2 and 3 are three identical micromaser cavities. We consider two-level atoms with upper state $|e\rangle$, lower state $|g\rangle$. In the interaction picture the resonant interaction of an atom with the cavity field is described by Hamiltonian

$$H_{\rm I}^{i} = \hbar g(\sigma_{+}a_{i} + a_{i}^{+}\sigma_{-}), \qquad (2)$$

where g is the atom-field coupling constant, σ_{+} and σ_{-} are the atomic raising and lowering operators, a_i and a_i^{+} $(i = 1, 2, 3)$ are the annihilation and creation operators for the field in cavity $i\,(i = 1, 2, 3)$. This

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Hamiltonian effects the following transitions:

$$|e\rangle|0\rangle_i \to \cos(gt)|e\rangle|0\rangle_i - \mathrm{i}\sin(gt)|g\rangle|1\rangle_i, \quad (3)$$

$$|g\rangle|1\rangle_i \to \cos(gt)|g\rangle|1\rangle_i - \mathrm{i}\sin(gt)|e\rangle|0\rangle_i. \quad (4)$$

We assume that the cavities 1, 2 and 3 are initially prepared in the vacuum states $|0\rangle_1|0\rangle_2|0\rangle_3$ and the atom is prepared in the excited state $|e\rangle$, so the initial cavity-atom state is $|\Psi\rangle_{\mathrm{I}} = |e\rangle|0\rangle_1|0\rangle_2|0\rangle_3$. After the atom is sent through the first cavity in time t_1, through the second cavity in time t_2 and through the third cavity in time t_3, the atom-cavity state then becomes

$$\begin{aligned}|\Psi\rangle_{\mathrm{f}} =& \cos(gt_1)\cos(gt_2)\cos(gt_3)|0\rangle_1|0\rangle_2|0\rangle_3|e\rangle \\ &- \mathrm{i}[\cos(gt_1)\cos(gt_2)\sin(gt_3)|0\rangle_1|0\rangle_2|1\rangle_3 \\ &+ \cos(gt_1)\sin(gt_2)|0\rangle_1|1\rangle_2|0\rangle_3 \\ &+ \sin(gt_1)|1\rangle_1|0\rangle_2|0\rangle_3]|g\rangle.\end{aligned} \quad (5)$$

If the atom is detected to be in the ground state $|g\rangle$, the cavity field is projected into the state

$$\begin{aligned}|\Psi\rangle_{\mathrm{c}} =& R[\sin(gt_1)|1\rangle_1|0\rangle_2|0\rangle_3 \\ &+ \cos(gt_1)\sin(gt_2)|0\rangle_1|1\rangle_2|0\rangle_3 \\ &+ \cos(gt_1)\cos(gt_2)\sin(gt_3)|0\rangle_1|0\rangle_2|1\rangle_3],\end{aligned} \quad (6)$$

where

$$\begin{aligned}R =& [\sin^2(gt_1) + \cos^2(gt_1)\sin^2(gt_2) \\ &+ \cos^2(gt_1)\cos^2(gt_2)\sin^2(gt_3)]^{1/2}\end{aligned} \quad (7)$$

is the normalization factor. Equation (6) is our main result. For arbitary interaction times of atom and fields, the three-cavity entangled state with only one photon can be obtained from Eq.(6) after the atom has been detected to be in the ground state.

If the timings are arranged as $gt_1 = \arctan\frac{1}{2}$, $gt_2 = \pi/4$ and $gt_3 = \pi/2$, we may produce the three-cavity entangled state with only one photon:

$$|\Psi\rangle_{\mathrm{c_1}} = \frac{1}{\sqrt{3}}(|1\rangle_1|0\rangle_2|0\rangle_3 + |0\rangle_1|1\rangle_2|0\rangle_3 + |0\rangle_1|0\rangle_2|1\rangle_3). \quad (8)$$

Now we prepare the more general three-cavity entangled state (1) in terms of Eq. (6). To this end, we set

$$\sin(gt_1) = -2\cos(gt_1)\sin(gt_2), \quad (9)$$

$$\sin(gt_2) = \cos(gt_2)\sin(gt_3). \quad (10)$$

If the timings are arranged so that $gt_1 = \arctan\sqrt{2}$, $gt_2 = \frac{7}{4}\pi$ and $gt_3 = \frac{3}{2}\pi$, then we can easily obtain the more general entangled state suitable for the universal quantum cloning machines[14−16]

$$|\Psi\rangle_{\mathrm{c_2}} = \sqrt{\frac{2}{3}}|100\rangle - \sqrt{\frac{1}{6}}|010\rangle - \sqrt{\frac{1}{6}}|001\rangle. \quad (11)$$

In addition, we can also obtain the more general entangled state of three atoms starting with the general expression of the three-cavity entangled state, i.e., Eq.(6). We suppose that the three atoms initially in their ground states are injected into cavities 1, 2 and 3 (see Fig.1), respectively. Thus the initial atoms-cavity

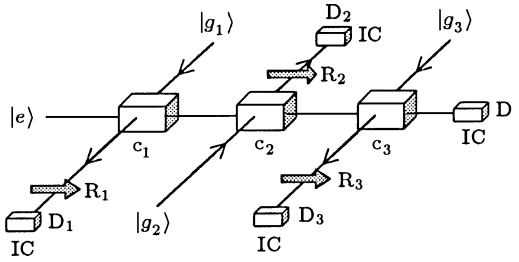


Fig.1. Proposed configuration for the preparation of the entangled states of cavities with only one photon and atoms. The atom in the excited state $|e\rangle$ enters the initially empty cavities c_1, c_2 and c_3, the entangled state of three-cavity with only one photon then produced if the atom is detected in $|g\rangle$. In addition, the three ground-state atoms are injected into the cavities c_1, c_2 and c_3, which results in the preparation of entanglement for the three atoms.

state is $|\Phi\rangle_{\mathrm{i}} = |\Psi\rangle_{\mathrm{c}}|g_1\rangle|g_2\rangle|g_3\rangle$. It is easy to obtain the entangled state in terms of Eqs.(3) and (4) and Eqs.(8) and (11)

$$|\Phi\rangle_{\mathrm{a_1}} = \frac{1}{\sqrt{3}}(|e_1\rangle|g_2\rangle|g_3\rangle + |g_1\rangle|e_2\rangle|g_3\rangle + |g_1\rangle|g_2\rangle|e_3\rangle), \quad (12)$$

$$\begin{aligned}|\Phi\rangle_{\mathrm{a_2}} =& \sqrt{\frac{2}{3}}|e_1\rangle|g_2\rangle|g_3\rangle - \sqrt{\frac{1}{6}}|g_1\rangle|e_2\rangle|g_3\rangle \\ &- \sqrt{\frac{1}{6}}|g_1\rangle|g_2\rangle|e_3\rangle,\end{aligned} \quad (13)$$

and the cavity state is then $|0\rangle_1|0\rangle_2|0\rangle_3$.

In summary, using the cavity QED technique, we have prepared the more general entangled states of three atoms and three fields, respectively, optimal for the universal quantum cloning machines. Our scheme, similar to Refs. [4, 10, 11], to prepare such entangled states is feasible with highly excited circular state Rydberg atoms and microwave cavities.

REFERENCES

[1] A. Einstein, B. Podolsky and N. Rosen, *Phys. Rev.*, **47**(1935), 777.

[2] C. S. Wu, I. Shaknov, *Phys. Rev.*, **77**(1950), 136; S. J. Freedman and J. S. Clauser, *Phys. Rev. Lett.*, **28**(1972), 938; E. Hagley *et al., Phys. Rev. Lett.*, **79**(1997), 1; P. W. Kwiat *et al., Phys. Rev. Lett.*, **75**(1997), 4337.

[3] J. I. Cirac and P. Zoller, *Phys. Rev.*, **A50**(1994), R2799.

[4] C. C. Gerry, *Phys. Rev.*, **A53**(1996), 4583.

[5] J. S. Bell, *Physics*, **1**(1964), 195.

[6] C. H. Bennett *et al., Phys. Rev. Lett.*, **70**(1993), 1895.

[7] A. Ekert, *Phys. Rev. Lett.*, **67**(1991), 661.

[8] D. M. Greenberger, M. A. Horne, A. Shimony and A. Zeilinger, *Am. J. Phys.*, **58**(1990), 1131.

[9] K. H. Song and G. C. Guo, *Chin. Phys. Lett.*, **16**(1999), 160.

[10] C. C. Gerry, *Phys. Rev.*, **A53**(1996), 4591.

[11] J. A. Bergou and M. Hillery, *Phys. Rev.*, **A55**(1997), 4585.

[12] D. Bouwmeester *et al., Phys. Rev. Lett.*, **82**(1999), 1345.

[13] A. Zeilinger *et al., Phys. Rev. Lett.*, **78**(1997), 3031.

[14] W. K. Wootters and W. H. Zurek, *Nature* (*London*), **299**(1982), 802.

[15] D. Bruβ *et al., Phys. Rev.*, **A57**(1998), 2368.

[16] V. Buzěk and M. Hillery, *Phys. Rev.*, **A54**(1996), 1844.

[17] V. Buzěk *et al., Phys. Rev.*, **A55**(1997), 3327.

[18] N. Gisin and S. Massar, *Phys. Rev. Lett.*, **79**(1997), 2513.