Volume 9, Number 12
1009-1963/2000/09(12)/0922-05
CHINESE PHYSICS
December, 2000
© 2000 Chin. Phys. Soc.

DUST-ACOUSTIC SHOCK WAVES IN THE SHEATH OF DUSTY PLASMAS*

Xie Bai-song(谢柏松)[a)b)c)], He Kai-fen(贺凯芬)[a)b)c)], and Chen Yan-ping(陈雁萍)[d)]
[a)] *Key Laboratory in University for Beam Technology and Materials Modification, Beijing Normal University, Beijing* 100875, *China*
[b)] *Institute of Low Energy Nuclear Physics, Beijing Normal University, Beijing* 100875, *China*
[c)] *Beijing Radiation Center, Beijing* 100875, *China*
[d)] *Institute of Physics, Chinese Academy of Sciences, Beijing* 100080, *China*

(Received 18 April 2000; revised manuscript received 23 July 2000)

The effect of dust charging and influence of its fluctuation on dust-acoustic shock waves in the sheath region of the dusty plasma is investigated. By employing the reductive perturbation technique, the Burgers' equation is derived. The weak shock wave solution is obtained and discussed in this dust-charge-fluctuation system.

Keywords: Dusty plasma, shock wave, sheath
PACC: 5235

I. INTRODUCTION

Dusty plasmas have attracted more and more attention because they are of crucial importance for the investigation in space plasma, material processing plasmas and laboratory plasmas[1−7] as well as in thermonuclear fusion devices.[8,9] A new dust-acoustic mode of low-frequency is found both theoretically[11] and experimentally[10] in a dusty plasma and extensively studied by many authors. Shock waves resulting from a balance between wave-breaking nonlinearity and wave-damping dissipation are ubiquitous in nature[12], so recently much work involving the formation of shock wave in a collisionless plasma with dusts or negative ions has been reported.[13−15]

As is well known, in most dusty plasmas the sheath is unavoidable. It is also important in dusty plasma crystals, plasma etching, as well as in the problem of plasma-wall interaction in fusion devices. Some fluid theories of the sheath boundary of a dusty plasma have been proposed.[16,17] Up to now the research work on shock waves is mostly in the region far from the sheath[13−15], and the ions are assumed inertialess with Boltzmann distribution at an equilibrium temperature. However, the situation is different for the sheath region where the ion streaming velocity should be important. In this paper we focus our attention on the possible formation of dust-acoustic shock waves in the sheath region, taking into account the ions streaming velocity.

Ⅱ. BASIC EQUATIONS

We consider a three-component dusty plasma consisting of extremely massive, high negatively charged inertial dust grains, inertialess Boltzmann-distributed free electrons with a finite fraction of number density, and the cold ions but with streaming velocity. Charge neutrality at equilibrium requires

$$n_{i0} = Z_{d0} n_{d0} + n_{e0}, \tag{1}$$

where n_{i0},n_{e0} and n_{d0} are the unperturbed ion, electron and dust number densities respectively, and Z_{d0} is the unperturbed number of charges residing on the dust grain.

In the sheath region the ion is assumed to possess a steady motion. It is valid since the characteristic time for dust-acoustic mode is very long[17] and $kv_d \ll \omega_{pd} \ll kv_i$, where ω_{pd} is the dust plasma frequency and $v_{d,i}$ is the dust (or ion) thermal velocity. Now we consider the cold ions with a streaming velocity that is important in sheath region. Replacing the ion thermal velocity by ion sound velocity, we have $kv_d \ll \omega_{pd} \ll kc_{is}$, so that we obtain

$$\frac{\partial}{\partial x}(n_i u_i) = 0,$$

$$u_i \frac{\partial u_i}{\partial x} = -\frac{e}{m_i}\frac{\partial \phi}{\partial x}.$$

Under the boundary conditions $\phi = 0$, $n_i = n_{i0}$ and $u_{i0} = c_{is} \equiv \sqrt{T_e/m_i}$ with c_{is} being the ions sound velocity. Here the Bohm criterion in the edge of sheath

*Project supported by the National Natural Science Foundation of China (Grant No.19905001), the Special Funds for Major State Basic Research Projects and partly by the Association for Plasmas Study of China.
b)Mailing address; E-mail:bsxie@bnu.edu.cn

has been used. Then we have

$$u_i = c_{is}\sqrt{1-2e\phi/T_e},\ \ n_i = \frac{n_{i0}}{\sqrt{1-2e\phi/T_e}}.$$

For one-dimensional low-frequency motions, we have the following equations for the cold dust fluid:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0,$$

$$\frac{\partial u_d}{\partial t} + u_d\frac{\partial u_d}{\partial x} = -\frac{Q_d}{m_d}\frac{\partial \phi}{\partial x},$$

where n_d, u_d and m_d refer to the dust grain density, dust fluid velocity and dust grain mass, respectively. Here the dust charge variable $Q_d = -eZ_d$, where $Z_d > 0$ is the charge number of dust grains measured in the unit of electron charge. The Poisson equation is given as

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e(Z_d n_d + n_e - n_i),$$

where

$$n_e = n_{e0}\exp(\frac{e\phi}{T_e})$$

is the electron number density at temperature T_e.

Now we normalize all the physical quantities as follows. The densities are normalized by n_{e0}, Z_d is normalized by Z_{d0}. The space coordinates x, time t, velocity and electrostatic potential ϕ are normalized by the dust Debye length $\lambda_{Dd} = (T_e/4\pi Z_{d0} n_{d0} e^2)^{1/2}$, the inverse of dust plasma frequency $\omega_{pd}^{-1} = (m_d/4\pi n_{d0} Z_{d0}^2 e^2)^{1/2}$, the dust-acoustic speed $C_d = (Z_{d0}T_e/m_d)^{1/2}$, and T_e/e, respectively. Therefore, we obtain a set of dimensionless basic equations

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0, \tag{2}$$

$$\frac{\partial u_d}{\partial t} + u_d\frac{\partial u_d}{\partial x} = Z_d\frac{\partial \phi}{\partial x}, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{P}(Z_{d0}Z_d n_d + n_e - n_i), \tag{4}$$

where $P = Z_{d0}n_{d0}/n_{e0}$[8] and the normalized electron, ion number densities and ions velocity are

$$n_e = \exp(\phi),$$

$$n_i = (P+1)/\sqrt{1-2\phi},$$

$$u_i = \sqrt{\frac{\mu_{di}}{Z_{d0}}}\sqrt{1-2\phi},$$

with $\mu_{di} = m_d/m_i$. In the expression of normalized ion number density we have used the quasineutral condition Eq.(1).

In general, the dust charge variable Q_d is determined by the charge current balance equation[18]:

$$\left(\frac{\partial}{\partial t} + u_d\frac{\partial}{\partial x}\right)Q_d = I_e + I_i.$$

The charge currents originate from the electrons and ions reaching the grain surface. Assuming that the streaming velocity of electrons is much smaller than its thermal velocity, but the streaming velocity of ions is much larger than its thermal velocity, and, according to the well known orbit-motion-limited (OML) probe model[19], we have the following expressions for the electron and ion currents to the spherical dust grains with radius r[13,20]

$$I_e = -e\pi r^2(8T_e/\pi m_e)^{1/2} n_e \exp\left(\frac{e\Phi}{T_e}\right)$$

and

$$I_i = e\pi r^2 n_i u_i\left(1 - \frac{2e\Phi}{m_i u_i^2}\right),$$

where Φ denotes the dust grain surface potential relative to the plasma potential ϕ. In general, $Q_d = C\Phi$, i.e. $\Phi = -eZ_d/r$ since $C \approx r$, so we obtain

$$I_e = I_{e0}\exp\left(\frac{e\phi}{T_e} - \frac{e^2(Z_d - Z_{d0})}{rT_e}\right)$$

and

$$I_i = I_{i0}\frac{1 + 2e^2 Z_d/rm_i u_i^2}{1 + 2e^2 Z_{d0}/rm_i u_{i0}^2},$$

where

$$I_{e0} = -e\pi r^2(8T_e/\pi m_e)^{1/2} n_{e0}\exp\left(-\frac{e^2 Z_{d0}}{rT_e}\right)$$

and

$$I_{i0} = e\pi r^2 n_{i0} u_{i0}\left(1 + \frac{2e^2 Z_{d0}}{rm_i u_{i0}^2}\right).$$

From the current balance equation at equilibrium, we have

$$I_{e0} + I_{i0} = 0.$$

Its normalized expression is

$$\exp(-z_0) = \sqrt{\frac{\pi m_e}{8m_i}}(P+1)(1+2z_0), \tag{5}$$

where $z_0 = e^2 Z_{d0}/rT_e$. In some cases, for example, in plasma etching or fusion devices where the electron temperature is relatively high, $z_0 \ll 1$ is easily satisfied if the dusts have small size and small charge number(see Refs.[8,9]). It is, therefore, valuable to discuss the possible existence of dust-acoustic shock waves in the sheath region.

The normalized equation for the dust charging is

$$\left(\frac{\partial}{\partial t} + u_d\frac{\partial}{\partial x}\right)Z_d$$

$$= |\widetilde{I}_{e0}|\Big\{\exp[\phi - z_0(Z_d-1)] - \frac{1+2z_0Z_d/(1-2\phi)}{1+2z_0}\Big\}, \quad (6)$$

where the normalized current $\widetilde{I}_{e0}$ is

$$\widetilde{I}_{e0} = I_{e0}/(eZ_{d0}\omega_{pd}).$$

Ⅲ. DUST-ACOUSTIC SHOCK WAVES

In order to study the dynamics of small-amplitude dust-acoustic solitary waves in the presence of adiabatic variation of dust charges, we derive the weak shock wave Burgers' equation from Eqs.(2)–(4) by the reductive perturbation technique[21] and the stretched coordinates $\xi = \epsilon(x - v_0t)$, and $\tau = \epsilon^2 t$, where ϵ is a small parameter and v_0 is the solitary wave velocity normalized by C_d. The variables n_d, u_d, ϕ and Z_d are then expanded as

$$n_d = \delta_d + \epsilon n_{d1} + \epsilon^2 n_{d2} + \cdots$$

$$u_d = \epsilon u_{d1} + \epsilon^2 u_{d2} + \cdots$$

$$\phi = \epsilon\phi_1 + \epsilon^2\phi_2 + \cdots$$

$$Z_d = 1 + \epsilon Z_{d1} + \epsilon^2 Z_{d2} + \cdots$$

where $\delta_d \equiv n_{d0}/n_{e0} = P/Z_{d0}$. Substituting these expansions into Eqs.(2)–(4) and Eq.(6), and collecting the terms in different powers of ϵ, in the lowest order, we obtain

$$n_{d1} = -\frac{\delta_d}{v_0^2}\phi_1, u_{d1} = -\frac{\phi_1}{v_0}, Z_{d1} = \alpha\phi_1, \quad (7)$$

where

$$v_0 = 1/\sqrt{\alpha - 1}$$

and

$$\alpha = \frac{1-2z_0}{z_0(3+2z_0)}.$$

The requirement of $\alpha > 1$ gives a range of z_0 as $z_0 < 0.186$. This means that the equilibrium dust charge number is not so high and the electron temperature (or/and dust size) should be relatively high (or/and large). We can express the maximum dust charge number as

$$Z_d < Z_{d\max} \approx 1.2 \times 10^3 [r(\mu\mathrm{m})][T_e(\mathrm{eV})].$$

For instance, for typical experimental values $T_e \approx 3$ eV, we have $Z_d < 36000$, 3600, 360 and 36 for $r \approx 10\mu m$, 1 μm, 0.1 μm and 0.01 μm respectively. From Eq.(5) we can conclude that the limitation of $z_0 < 0.186$ is for $P > 41$. Indeed this condition $P \gg 1$ is easily satisfied in some cases, as in the example described in Ref.[8], it corresponds to some experimental observations, especially the processes of dust agglomeration for small dust charge.

To the next higher order in ϵ, we have the following set of equations,

$$\frac{\partial n_{d1}}{\partial\tau} - v_0\frac{\partial n_{d2}}{\partial\xi} + \frac{\partial(n_{d1}u_{d1})}{\partial\xi} + \delta_d\frac{\partial u_{d2}}{\partial\xi} = 0,$$

$$\frac{\partial u_{d1}}{\partial\tau} - v_0\frac{\partial u_{d2}}{\partial\xi} + u_{d1}\frac{\partial u_{d1}}{\partial\xi} = \frac{\partial\phi_2}{\partial\xi} + Z_{d1}\frac{\partial\phi_1}{\partial\xi},$$

$$Z_{d0}(n_{d2} + Z_{d1}n_{d1} + \delta_d Z_{d2}) + \phi_2 + \frac{\phi_1^2}{2} - (P+1)\left(\phi_2 + \frac{3\phi_1^2}{2}\right) = 0,$$

$$-v_0\frac{\partial Z_{d1}}{\partial\xi} = (\phi_2 - z_0Z_{d2}) + \frac{1}{2}(\phi_1 - z_0Z_{d1})^2 - \frac{2z_0}{1+2z_0}(2\phi_2 + 4\phi_1^2 + 2\phi_1Z_{d1} + Z_{d2}).$$

Using Eq.(7) and eliminating all the second-order terms, we get the standard Burgers' equation

$$\frac{\partial\phi_1}{\partial\tau} + a\phi_1\frac{\partial\phi_1}{\partial\xi} = b\frac{\partial^2\phi_1}{\partial\xi^2}, \quad (8)$$

where

$$a = \frac{v_0^3}{\delta_d}\Big\{\frac{3}{2}\alpha\delta_d + \frac{1}{Z_{d0}} + \alpha \times \frac{1+2z_0}{1-2z_0}\Big[\frac{4z_0(\alpha+2)}{1+2z_0} - \frac{1}{2}(1-\alpha z_0)^2\Big]\Big\},$$

$$b = \frac{\alpha^2}{2\delta_d(\alpha-1)^2}\frac{1+2z_0}{1-2z_0}.$$

For $z_0 < 1/2$, we have $b > 0$. Define the variable $\eta = \xi - v\tau$, then the stationary solution of Eq.(8) can be written as

$$\phi_1 = \frac{\phi_1^b + \phi_1^a\exp[\eta/w]}{1+\exp[\eta/w]},$$

where the shock wave speed v and width w are given as

$$v = \frac{a}{2}(\phi_1^a + \phi_1^b),\ w = \frac{2b}{a}\frac{1}{\phi_1^b - \phi_1^a}.$$

In obtaining the above solution we have used the following boundary conditions:

$$\phi_1 \to \phi_1^a\ (\phi_1^b) \text{ at } \eta \to +\infty\ (-\infty)$$

$$\text{and} \quad \frac{\mathrm{d}\phi_1}{\mathrm{d}\eta} \to 0 \text{ at } \eta \to \pm\infty.$$

Ⅳ. RESULTS AND DISCUSSION

In order to see whether this form of shock waves can be formed in the sheath region of a typical dusty plasma, we need to further analyze the width scale

of shock waves, which is limited not only by the continuity of dust fluid in which $w \gg r$ and $w > a_d$ are required, but also by the sheath scale in which the shock waves can be formed and fully developed. The latter condition requires $w \leq \lambda_{De}$. Here r is the dust grain radius, $a_d \equiv (3/4\pi n_d)^{1/3}$ the average distance between dust grains, and λ_{De} the electron Debye length which is usually the typical distance of sheath region. In general, the condition $w > r$, a_d is easily satisfied for $w \sim O(\lambda_{De})$. For example, when $n_{e0} \sim 10^{10}\text{cm}^{-3}$, $n_{d0} \sim 10^8 - 10^9\text{cm}^{-3}$, $T_e \approx 3\text{eV}$ and the wide range of dust grain radius $r \sim 0.1 - 10\mu\text{m}$, i.e. $\sim 10^{-5} - 10^{-3}\text{cm}$, then we have $a_d \approx [n_d(\text{cm}^{-3})]^{-1/3}\text{cm} \sim 10^{-3}\text{cm}$ and $w \sim \lambda_{De} \approx \sqrt{T_e(\text{eV})/n_e(\text{cm}^{-3})} \times 10^3\text{cm} \sim 10^{-2}\text{cm}$, obviously $w \gg r$ and $w > a_d$ is satisfied. Note that the example given above make the condition of $P \gg 1$ self-consistently satisfied.

It is well known that due to the different mobilities of electrons and ions, in typical cases the sheath wall has a negative potential with respect to the plasma. Therefore, in the sheath region, i.e. $x > 0$, we have $\phi_1^a < \phi_1^b \approx 0$, which immediately implies that the sheath shock waves can exist for $a > 0$. Indeed for typical dusty parameters (e.g. in Fig.1) $a > 0$ is usually satisfied.

The normalized electron Debye length is $\lambda_e = \lambda_{De}/\lambda_{Dd} = \sqrt{P}$, therefore, the difference between two limits of plasma potential should satisfy the following condition for a fully developed shock wave

$$\Delta\phi \equiv \phi_1^b - \phi_1^a \approx -\phi_1^a > \frac{2b}{a\sqrt{P}}.$$

We have plotted $\Delta\phi$ vs z_0 for a set of rT_e in Fig.1. We can see clearly that the maximum value of $\Delta\phi$ is in the order of $O(10^{-1})$ to $O(1)$, obviously two or three orders of magnitude larger than the corresponding potential jump in the case of plasma region far from the sheath.[14] Without loss of generality, in Fig.1, we have chosen the minimum z_0 corresponding to dust charge number $Z_{d0} = 10$. We can also see that the maximum potential jump occurs in the larger dust charge system for small dust grains and low electron temperature; however, it moves to the smaller dust charge system for larger dust grains and high electron temperature. In our model the ion streaming velocity u_i is important and connected with plasma potential by $u_i = c_{is}\sqrt{1-2\phi}$, where c_{is} is ion sound speed; therefore, we can give the expression of ϕ_1^a near the wall as $\phi_1^a = (1 - M_{\max}^2)/2$, where $M_{\max} = u_{i\max}/c_{is} > 1$ is the maximum Mach number of ion velocity near the wall. The dependence of sheath shock wave width on the ion streaming velocity can be formulated as $w = 4b/a(M_{\max}^2 - 1)$. It is obvious that the narrower the shock wave width, the higher the ion streaming velocity near the wall.

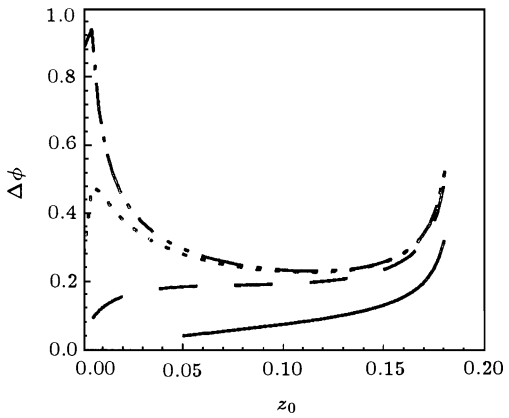


Fig.1. The minimum potential jump $\Delta\phi$ vs the normalized dust charge number $z_0 = Z_{d0}e^2/rT_e$ for a set of dust grains r and electrons temperature T_e. The product of $r(\mu\text{m})$ T_e (eV) are 0.3 (solid line), 3.0 (dashed line), 30.0 (dotted line) and 300.0 (dot-dashed line)

It is noticed that, as pointed out recently by Tsytovich and deAngelis[22], the discreteness of the dust component may create fluctuations in all the components of the dusty plasma (including electrons and ions), the behavior in the framework of kinetic theory of dusty plasmas may be substantially different from that in the framework of fluid theory where dusty plasmas are considered as a multi-component plasma system, as used in the present work. Indeed, the induced effective dust charges will be different from the equilibrium dust charge, the currents can also have fluctuations beyond the description by OML theory, and the screening is not Debye-like due to the plasma particle absorption on the dust grains, ion trapping around the grain and the nonlinearities in screening. However, these effects cannot be serious and are negligible if the characteristic frequency of the system motion is much less than the charging frequency, i.e. $\omega \ll \nu_{ch}$, and, if the dust-wave resonance condition is avoided, i.e. $\omega \ll ku_d$ or $\omega \gg ku_d$. Fortunately, for most parameters regime of z_0 in this study, the above two conditions are both met. Therefore, the fluid dynamics description for dust is valid. It is worth pointing out that the theory presented in this paper would be failure for very small z_0. It is not surprising, because the dust grain charge decreases with decreasing z_0, so that the dust charge fluctuation would be in the order

of equilibrium charge, and the phase velocity of dust-acoustic wave is also approaching the average velocity of dust motion. All this makes the dust-wave resonance occur easier.

In this paper we start from the cold dust fluid equation and dust charge fluctuation, and use the reductive perturbation technique to study the weak shock waves in the sheath region of a dusty plasma with a finite ion streaming velocity. We have obtained the Burgers' equation and weak shock solutions which have larger potential jump. The shock wave problem in the sheath region is very important in plasma processing as well as fusion devices where the dusts are unavoidable.

REFERENCES

[1] R. Bingham, U. de Angelis, V. N. Tsytovich and O. Havnes, *Phys. Fluids* **B3**,(1991),811.

[2] L. Boufendi, A. Plain, J. P. Blondeau, A. Bouchoule, C. Laure and M. Toogood, *Appl. Phys. Lett.,* **60**(1992),169.

[3] T. W. Hartquist, O. Havnes and G. E. Morfill, *Fundam. Cosmic Phys.,* **15**(1992),107.

[4] V. N. Tsytovich and O.Havnes, *Comm. Plasma Phys. Controlled Fusion* **15**(1993),267.

[5] J. H. Chu and Lin I, *Phys. Rev. Lett.,* **72**(1994),4009; Lin I, W. T. Juan, C. H. Chiang and J. H. Chu, *Science* **272**(1996),1626.

[6] B. S. Xie, K. F. He and Z. Q. Huang, *Phys. Lett.,* **A247** (1998),403; **A253**(1999),83.

[7] B. S. Xie, K. F. He, Z. Q.Huang and M. Y. Yu, *Phys. Plasmas* **6**(1999),2997.

[8] V. N. Tsytovich, Y. K. Khodataev, G. E. Morfill, R. Bingham and D. J. winter, *Comm. Plasma Phys. Controlled Fusion* **18**(1998),281.

[9] V. N. Tsytovich and J. Winter, *Physics-Uspekhi* **41** (1998),815.

[10] N. N. Rao, P. K. Shukla and M. Y. Yu, Planet. *Space Sci.,* **38**(1990),543.

[11] A. Barkan, R. L. Merlino and N. D. Angelo, *Phys. Plasmas* **2**(1995),3563.

[12] G. B. Whitham, *Linear and Nonlinear Waves*, (Wiley Inc., New York, 1974).

[13] S. I. Popel, M. Y. Yu and V. N. Tsytovich, *Phys. Plasmas* **3**(1996),4313.

[14] C. B. Dwivedi and B. P. Pandey, *Phys. Plasmas* **2**(1995), 4134.

[15] T. Takeuchi, S. Iizuka and N. Sato, *Phys. Rev. Lett.,* **80** (1998),77.

[16] J. X. Ma, J. Liu and M. Y. Yu, *Phys. Rev,* **E55**(1997), 4627.

[17] X. P. Chen, *Phys. Plasmas* **5**(1998),804.

[18] F. Melandsϕ, F. T. Aslaksen, and O. Havnes, *Planet. Space Sci.,* **41**(1993),312.

[19] M. S. Barnes *et al, Phys. Rev. Lett.,* **68**(1992),313.

[20] Y. N. Nejoh, *Phys. Plasmas* **4**(1997),2813.

[21] H. Washimi and T. Taniuti, *Phys. Rev. Lett.,* **17**(1966), 996.

[22] V. N. Tsytovich and U. de Angelis, *Phys. Plasmas,* **6** (1999),1093.