Volume 9, Number 12
1009-1963/2000/9(12)/0881-04
CHINESE PHYSICS
Number, 2000
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A POSSIBLE EXPERIMENTAL TEST ON THE GRAVITATION THEORY OF COMPLEX FIELD

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(Received 15 May 2000; revised manuscript received 28 September 2000)

By appropriately choosing additional dimensions of space-time, the mass spectrum of mesons was obtained, and the calculated results agree with the experimental data.

Keywords: gravitation, complex field, experiment inspection
PACC: 0314

I. INTRODUCTION

Since the 1980s, in the field of grand unification theory (GUT) and supergravity theory, researchers have made extensive research work, using the idea of gauge field, the idea of decomposing space geometry, and the idea of super symmetry and super string, etc., Kaluza[1] put forward the 5- dimensional unification idea of gravitation and electromagnetic field. A kind of complex field equation by using the method of decomposing 5-dimensional space into (4+1)-dimensional one has been established by the present author and her coworker.[2] They have given a static spherically symmetric solution. From the asymptotic form of the solution one can see that the theory is reasonable.

The best testing method for the theory of complex field is to compare the theoretical predictions with the experimental results. In this paper, the mass spectrum of meson has been obtained through appropriately choosing additional dimensions of space-time. The predictions of the theory are in agreement with the experimental results.

II. WAVE FUNCTION OF MESON

We interpret the charged and neutral scalar fields introduced in the theory[3] of 5- dimensional complex fields as the wave functions of meson. To make neutral scalar field still have mass and the electric charge still equal to zero, one must introduce the seventh dimension. We consider the 7-dimensional Riemann manifold, which is of the topological form of $V^7 = V^4 \times S^1 \times S^1 \times S^1$. We take its metric tensor as

$$\mathrm{d}\overset{(7)}{S}{}^2=(e^{8/5})^{\sigma}\left\{\left(g_{\mu\nu}-\frac{4k}{c^4}A_\mu A_\nu\right)\mathrm{d}x^\mu \mathrm{d}x^\nu + \frac{4k}{c^2}A_\mu \mathrm{d}x^\mu \mathrm{d}x^5 - \mathrm{d}x^{5^2}+\mathrm{d}x^{6^2}+\mathrm{d}x^{7^2}\right\}, \quad (1)$$

where $g_{\mu\nu}$ is the metric tensor for the 4-dimensional Riemann space, A_μ is the 4-dimensional potential of the electromagnetic field, $e^{2\sigma}$ is conformal factor, and k is Newton gravitation constant, with μ, ν =0,1,2,3.

Taking Lagrange density as

$$\overset{(7)}{L}=(-\overset{(7)}{g})^{1/2}\overset{(7)}{R}, \quad (2)$$

making conformal transformation and decomposition of (4+1+1+1), we get

$$\begin{aligned}\overset{(7)}{L}=(-g)^{1/2}\Bigg\{&e^{4\sigma}\left[R+\frac{k}{c^4}F_{\mu\nu}F^{\mu\nu}\right]\\&-\frac{24}{5}g^{\mu\nu}e^{2\sigma}\left(\nabla_\mu+\frac{2\sqrt{k}}{c^2}A_\mu\frac{\partial}{\partial x^5}\right)\\&\cdot\left(\nabla_\nu+\frac{2\sqrt{k}}{c^2}A_\nu+\frac{\partial}{\partial x^5}\right)e^{2\sigma}\\&+\frac{24}{5}[(e^{2\sigma})_{,55}-(e^{2\sigma})_{,66}+(e^{2\sigma})_{,77}]e^{2\sigma}, \quad (3)\end{aligned}$$

where R and g are formed by $g_{\mu\nu}$, ∇_μ is the usual covariant derivative for $g_{\mu\nu}$.

Let us recall the major characteristics of the 5-dimensional complex field theory. Its most important point is the closed condition of the 5-dimensional manifold for the 5th coordinate, that is, selecting the topology in the form of $V^4 \times S^1$. As proved in Ref.[3],

if we require the electric charge of the scalar field equal to electron charge ($\pm e$), the period T_5 of the 5th coordinate must be very small ($\sim 10^{-30}$cm). Since the scale of T_5 is much smaller than that for the field equation to hold, we can take the average of the Lagrange in the 5-dimension theory according to the period of the 5th coordinate. Now in the Lagrange, there arises the mass term, depending on the second derivative of $e^{2\sigma} = (\overset{(7)}{g}_{55})^{3/4}$ for x^5 and being of magnitude of Planck mass. This term can be regarded as the contribution to the mass of electromagnetic field. To make mass "standard", we introduce the 6th dimension and select $V^4 \times S^1 \times S^1$ as its topology. This treatment shows that the dependence of metric of 6-dimensional manifolds for x^6 does not concern with the electric charge and contributes only to the mass of scalar field with opposite symbol (because x^6 is time-like while x^5 is space-like). We know that the period T_5 for x^5 is decided by the condition that the electric charge of scalar field is equal to $\pm e$, and the period for x^6 is independent of any condition. Thus we can get any value of mass.

In Ref.[3], the charged scalar field is explained as the field describing the charged matter. Assume

$$\varphi = \rho^{1/2} \exp\left(\frac{iS}{\hbar}\right), \tag{4}$$

where S is the classical action, ρ is the probability for particles to appear in region $\mathrm{d}\overset{(3)}{V}$. In the case of classical limit ($\hbar \to 0$) we get the generalized Klein-Gordon-Fock equation that is satisfied by field φ. When transformed to Hamilton-Jacobi equation, the right hand side of Einstein equation is the usual energy-momentum tensor of electromagnetic field, and the right hand side of Maxwell equation is the usual current density. In this paper, we interpret the above-mentioned charged and neutral scalar fields as the wave functions of meson. Thus, in order to make neutral scalar field still have mass and its electric charge be still equal to zero, one must introduce the 7th dimension. It has been proved in Ref.[3] that T_5 must be very small ($\sim 10^{-30}$cm). And we require the mass term to be of the power of mass of elementary particle, so, T_6 must be also very small. Thus naturally we think that the Lagrange under consideration usually should be averaged over x^5, x^6 and x^7:

$$\overset{(4)}{L} = \int_0^{T_5}\int_0^{T_6}\int_0^{T_7} \overset{(7)}{L}\, \mathrm{d}x^5 \mathrm{d}x^6 \mathrm{d}x^7, \tag{5}$$

where $\overset{(4)}{L}$ is the usual 4-dimensional Lagrange density.

We select conformal factor with a form as

$$\begin{aligned} e^{2\sigma} =& 1 + ia_1\varphi(x^p) + ia_2\,\Psi_0(x^p) \\ &+ a_3\,\Psi(x^p) - a_3\,\Psi^*(x^p) \\ =& 1 + ia_1\varphi(x^\mu) f_1(x^5, x^6, x^7) \\ &+ ia_2\,\Psi_0(x^\mu) f_2(x^5, x^6, x^7) \\ &+ a_3\,\Psi(x^\mu) f_3(x^5, x^6, x^7) \\ &- a_3\,\Psi^*(x^\mu) f_3^*(x^5, x^6, x^7), \end{aligned} \tag{6}$$

where p=0, 1, 2, 3, 5, 6, 7; the asterisk denotes complex conjugate. The variables x^μ and x^5, x^6, x^7 in the last step are written in the separate form (this is allowed, because conformal factor can be selected).

We interpret the introduced function $\varphi(x^\mu)$ as wave function to describe the neutral meson with isotopic spin $I = 0$; $\Psi_0(x^\mu)$, $\Psi(x^\mu)$, and $\Psi^*(x^\mu)$ describe three duplicate mesons (when $a_2 = a_3$) or duplex mesons (when $a_2 = \sqrt{2}a_3$); $\Psi_0(x^\mu)$ is the neutral component of the three duplicate (duplex) mesons; a_1, a_2 and a_3 are constant gauge factors (because the dimension of the wave function is $l^{-3/2}$).

We select functions f_1, f_2 and f_3, so as to make the self-action term of scalar field disappear after averaging for $\overset{(4)}{L}(x^\mu)$ and only the interaction terms of scalar field, gravitation field and electromagnetic field appear, that is, requiring the coupling term, $\varphi\Psi$, $\varphi\partial_\mu\Psi$, $\Psi_0\partial_\mu\Psi$, $\Psi_0\Psi$, $\Psi_0\Psi^*$, $\Psi\partial_\mu\Psi$, $\Psi^*\partial_\mu\Psi^*$, $\varphi\Psi_0$ etc. do not appear. Let

$$\begin{aligned} f_1 &= \frac{2}{T_5\sqrt{T_6}} \cos\frac{2\pi}{T_5}(n+4)x^7 \cdot \cos\frac{2\pi}{T_6}(n+4)x^6, \\ f_2 &= \frac{2}{T_5\sqrt{T_6}} \cos\frac{2\pi}{T_5}(n+1)x^7 \cdot \cos\frac{2\pi}{T_6}(n+1)x^6, \\ f_3 &= \frac{2}{T_5\sqrt{T_6}} \exp\left(-i\frac{2\pi}{T_5}x^5\right) \\ &\quad \times \cos\left(-i\frac{2\pi}{T_6}x^6\right) \cdot \cos\frac{2\pi}{T_6}nx^6, \end{aligned} \tag{7}$$

where $2/T_5\sqrt{T_6}$ is a standardization factor, $n = 0, 2, 3, 4\cdots, (n \neq 1)$, since n=1 will give rise to coupling terms.

Taking the average for x^5, x^6 and x^7, and adding the requirement that the electric charge of the charged scalar field is equal to $\pm e$, we get $T_5 = 4\pi\sqrt{k}\hbar/(ec)$.

Then we get

$$\begin{aligned}\overset{(4)}{L}(x^\mu) =& \sqrt{-g}\Bigg\{\left(1-a_1^2\varphi_n^2-a_2^2\,\Psi_0^2-2a_3^2\,\Psi_n\,\Psi_n^*\right)\\ &\times\left(R+\frac{k}{c^4}F_{\mu\nu}F^{\mu\nu}\right)\\ &-\frac{24}{5}a_1^2\Bigg[g^{\mu\nu}\partial_\mu\varphi_n\partial_\nu\varphi_n\\ &-\left(\frac{4\pi^2}{T_5^2}-\frac{4\pi^2}{T_6^2}\right)(n+4)^2\varphi_n^2\Bigg]\\ &-\frac{24}{5}\Bigg[2a_3^2\Bigg(g^{\mu\nu}\partial_\mu^+\,\Psi_n^*\partial_\nu^-\,\Psi_n\\ &-\left(\frac{4\pi^2}{T_5^2}-\frac{4\pi^2}{T_6^2}\right)\cdot(1+n^2)\,\Psi_n^*\,\Psi_n\Bigg)\\ &+a_2^2\Bigg(g^{\mu\nu}\partial_\mu\,\Psi_{0_n}\partial_\nu\,\Psi_{0_n}\\ &-\left(\frac{4\pi^2}{T_5^2}-\frac{4\pi^2}{T_6^2}\right)(1+n^2)\,\Psi_{n_0}^2\Bigg)\Bigg]\Bigg\},\end{aligned}\tag{8}$$

where $\partial_\mu^\pm \equiv \partial_\mu \pm i(e/\hbar c)A_\mu, \partial_\mu \equiv \partial/\partial x^\mu$. The dimension factor a_3 can be determined by Maxwell equation that is obtained from Lagrange density (8):

$$a_3^2 = \frac{5}{48}e^{2\sigma}\frac{\hbar^2}{m_{\Psi_n}}.\tag{9}$$

Ⅲ. THE MASS SPECTRUM OF MESOMS

From a physical viewpoint, we can consider three reasonable assumptions.

(1) We do not distinguish the three duplicate mesons or duplex mesons. This is because when taking $a_1 = a_2 = a_3$, we get the Lagrange for singlet state and three duplicate states; and taking $a_1 = a_2 = \sqrt{2}a_3$, we get the Lagrange for singlet state and duplex states.

(2) When n=0, we get $m_{\Psi_0} = m_\Psi = m_{\Psi^*}$. So we do not consider the separation of electric charge in the corresponding state with $n \neq 0$.

(3) If a certain particle does not correspond to m_n, the wave function f for describing it is not characteristic function, but this wave function is the addition of two adjacent characteristic functions.

On the basis of Eq.(2), we consider that Ψ_{0n} and Ψ_n are of equal probability in the three duplicate (or duplex) states, and are described by

$$\tilde{\Psi}_n \equiv \frac{1}{\sqrt{2}}\,\Psi_{0n} + \frac{1}{\sqrt{2}}\,\Psi_n.\tag{10}$$

So, the "average" of three duplicate (or duplex) meson masses is equal to

$$m_{\tilde{\Psi}_n} = m_{\tilde{\Psi}_0}\left(\frac{1+n^2}{2}+\frac{(1+n^2)}{2}\right)^{1/2}.\tag{11}$$

We select the mean square root value of m_{π^0} and m_{π^-} as the initial mass $m_{\tilde{\Psi}_0}$ in Eq.(11) (MeV):

$$m_{\tilde{\Psi}_0}c^2 = \left(\frac{m_{\pi^0}+m_{\pi^-}}{2}\right)^{1/2}c^2 \approx 137.286 \quad \text{MeV}.\tag{12}$$

For

$$m_{\tilde{\Psi}_0}^2\frac{e^2}{\hbar^2} = \left(\frac{e^2c^2}{4k\hbar^2}-\frac{4\pi^2}{T_5^2}\right),\tag{13}$$

we can determine period T_5 from mass m_{Ψ_0} :

$$\frac{4\pi^2}{T_5^2} = \frac{e^2c^2}{4k\hbar^2} - m_{\tilde{\Psi}_0}^2\frac{e^2}{\hbar^2}.\tag{14}$$

The calculation shows that if not taking average for electric charge, the degree of accordance with the experimental result is not as good as Eq.(11).

From Eq.(8) we can get the expression of mass in neutral singlet state (MeV):

$$\begin{aligned}m_{\varphi_n} &= \left(\frac{e^2c^2}{4k\hbar^2}-\frac{4\pi^2}{T_5^2}\right)^{1/2}(n+4)\\ &= 137.268(n+4) \quad \text{MeV}.\end{aligned}\tag{15}$$

On the basis of Eq.(3), if $m_n < m_f < m_{n+1}$, we can write

$$f = f_n\cos\theta + f_{n+1}\sin\theta,\tag{16}$$

or

$$f = f_n\sin\theta + f_{n+1}\cos\theta,\tag{17}$$

From these we can get

$$m_f^2 = (m_n^2\cos^2\theta + m_{n+1}^2\sin^2\theta)^{1/2},\tag{18}$$

or

$$m_f^2 = (m_n^2\sin^2\theta + m_{n+1}^2\cos^2\theta)^{1/2},\tag{19}$$

where θ is the usual shift angle in meson spectrum: $cos\theta = \sqrt{2/3}, \sin\theta = \sqrt{1/3}$.

From Eqs.(11), (12), (15), (18), we can give the mass spectrum containing all known mesons. The theoretic results are all in good agreement with the experimental data in Ref.[4]. These are listed in detail in Tables 2 and 3 of pages 315- 316 in Ref.[5].

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