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THEORY OF INTERBAND COHERENCE AND INVERSIONLESS BISTABILITY IN SEMICONDUCTOR LASERS*

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Coherent transitions of charge carriers between the conduction and valence bands of a semiconductor medium are essential for the operation of a semiconductor laser. In this paper, we study how such interband coherence can be set up by an injection current and a coherent pump-field. In the absence of the pump-field, the injection current is the only source to establish the interband coherence in a semiconductor laser system. A laser threshold is obtained, which shows that a strongly coupled high-Q microcavity has a low threshold value. However, when an external pump-field serving as another mechanism to create the interband coherence is applied, the threshold value of the injection current can be lowered and it vanishes for sufficiently strong field. Besides, if the pump-field exceeds a threshold value, it is even possible to achieve a bistability in the inversionless region. Some fundamental macroscopic properties, including polarization, absorption and dispersion for the semicoductor system, are also obtained analytically.

Keywords: interband coherence, inversionless bistability, semiconductor laser
PACC: 4250, 4265, 4262

I. INTRODUCTION

Over the past years there has been great interest in electromagnetically induced transparency (EIT)[1,2], lasing without inversion (LWI)[3,4], and enhancement of dispersion with reduced absorption[5] in atomic systems. The physical mechanisms behind these effects are atomic coherence and quantum interference, which can drastically change the optical properties of atomic systems. They can lead, for example, to a large dip in the absorption spectrum of a noninverted system. Such a non-absorbing resonance has been observed in the experiments for a Λ scheme proposed by Alzetta *et al.*[6] and also for a V scheme proposed by Harris and co-workers[1]. Lasing without inversion is another important application of quantum coherence and interference, which leads not only to a change of laser theory, but also to a generation of laser light with phase noise quenching[7] or amplitude squeezing[8]. It was understood that the generation of laser coherence stems from atomic coherence. This means that, to obtain laser light, the most important thing one needs to do is to create atomic coherence whether by atom-field interaction or by population inversion. In fact population inversion in conventional laser systems is nothing but the condition of atomic coherence in that case. What we are now concerned with is that, can one apply the principle of atomic coherence to semiconductor lasers to create LWI? The answer should be positive.

It is well known that many optical properties that are first realized in atomic laser systems can also be similarly obtained in semiconductor laser systems. For example, pump noise reduction and amplitude squeezing in semiconductor laser have been observed experimentally[9]; the experiment of Marin *et al.*[10] examined intensity squeezing in three different semiconductor laser systems; most recently, the publication of Kilper *et al.*[11] has shown that squeezed light can be generated from a semiconductor microcavity laser; the Rabi-like splitting [12], modification of spontaneous emission lifetime[13], optical bistability and multistability[14], as well as thresholdless laser and LWI[15] are extensively studied in semiconductor (microcavity) lasers. However, as the main subject in telecommunication and optical information processing, semiconductor lasers cannot be replaced by lasers whose active media are atoms. A concerted effort has begun to create high quality semiconductor lasers. Much experimental and theoretical effort has been devoted to the realization and study of low-dimensional (or mesoscopic) semiconductor structures, such as vertical cavity surface emitting laser (VCSEL), microdisk laser and quantum dot (QD) laser[15−19]. The development of such semiconductor nanostructures opened the possibility to modify the lifetime of electrons and

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holes. The enhanced spontaneous emission into the laser mode leads to a reduction of laser threshold. Björk, Karlsson, and Yamamoto[15] proposed that the threshold condition for a laser can be defined by the mean cavity photon number being unity, instead of the conventional "definition" that the pump power at which the optical gain equals the cavity loss. In a semiconductor microcavity, the coupling between exciton and cavity mode (normal-mode coupling) or cavity-polaritons can be observed[12]. A systematic approach investigating excitonic signatures in semiconductor microcavity luminescence begins with a many-body Hamiltonian that includes the Coulomb interactions of electrons and holes and relevant quantities can be evaluated either with the nonequilibrium Green's function or the density matrix approach. Much progress in this field has been made recently by Koch's group and other groups as well[20].

In this paper, we focus our attension mainly on the laser action, interband coherence and other macroscopic properties induced in a semiconductor laser medium. We will generalize the principle of atomic coherence, which is extensively studied recently in quantum optics and laser systems, to a driven semiconductor laser system. Since a semiconductor laser can be modeled as a "two-level" system [16,17], which can be described by the density matrix formalism. The term interband coherence in this paper is defined as the off-diagonal elements of the density matrix of charge carriers between the conduction and valence bands. Starting from a quantum mechanical description, we show that the interband coherence in a semiconductor laser depends crucially on the injection current. Beisdes, the systematic decay rates and the Rabi frequancy of a laser system could also affect the lasing threshold. Our results show that a strongly coupled high-Q semicoductor microcavity laser may have a lower threshold value due to the enhancement in the spontaneous decay rates. On the other hand, when an external pump-field is directly applied to the two bands of semiconductor, which serves as an extrinsic mechanism to establish the coherence required for lasing action, the threshold value of the injection current decreases. When the external field coupling is further increased, it could even create a bistability in the inversionless region similar to the discussions in the atomic laser systems[4,21]. It has been shown that by controlling atomic coherence, the quantum noise of an atomic system can be reduced to the vacuum level[21]. Therefore, we expect that a suitably controlled semiconductor laser system may have the same property.

It is well known that the macroscopic behavior of a semiconductor laser is related to its microscopic properties. In order to describe the macroscopic behaviors of a semiconductor laser, such as the porlarization, absorption and dispersion, one has to match both the microscopic and macroscopic descriptions. To do this we introduce the macroscopic Maxwell-Bloch equations. Two parameters κ and ζ, as proposed in the paper of Yao *et al.*[16], are employed to describe the macroscopic Maxwell-Bloch equations. The parameter κ governs changes of the susceptibility in the medium, resulting from integrating over the band states in semiconductors, and affects both the gain and the refractive index. On the other hand, ζ governs modification of the interband relaxation time for the macroscopic polarization P. Exact determination of these two parameters has been considered not easy, in fact, even impossible. However, by comparing both the macroscopic and microscopic descriptions, we obtain an analytic formula for the parameter κ in terms of the laser detuning.

The paper is organized as follows. In Sec. II, we will discuss the kinetic description in terms of the electron representation and the density matrix of the system. The system is described by "two-level"-like form, i.e., conduction band and valence band. In Sec.III, we will discuss the atomic coherence and the condition of noninversion bistability in the semiconductor laser. It is shown that the interband coherence in the semiconductor laser medium is crucially dependent on the injection current. This implies that one can choose the current as the controlling parameter in semiconductor lasers. In Sec. IV, the macroscopic Maxwell-Bloch equations and two parameters κ and ζ are phenomenologically introduced. In Sec.V, by comparing the macroscopic description here with the microscopic description in Sec. II, an analytic formula expressing the parameter κ in terms of the laser detuning, and other properties of the laser field will be obtained. One can see in this section that both the microscopic and macroscopic descriptions are consistent. Lastly, we discuss the physical significance of our results in Sec. VI.

II. KINETIC DESCRIPTION IN ELECTRON REPRESENTATION

The model is similar to that described in Ref.[4]. On the one hand, an external pump-field is directly

applied to the conduction band and valence band of a semiconductor laser, and on the other, carriers of two bands can be modulated by an injection DC-current. For this semiconductor laser system, a quantum mechanical description can be written in terms of the following Hamiltonian:

$$H = H_{\rm F} + H_{\rm S} + H_{\rm SF} + H_{\rm SL}, \tag{1}$$

where

$$H_{\rm F} = \hbar\omega_c b^\dagger b, \tag{2a}$$

$$H_{\rm S} = \sum_k \hbar\omega_{p_k} p_k^\dagger p_k + \sum_{k'} \hbar\omega_{q_{k'}} q_{k'}^\dagger q_{k'}, \tag{2b}$$

$$H_{\rm SF} = \hbar \sum_{k,k'} \{g_{_{kk'}} b c_{kk'}^\dagger + g_{_{kk'}}^* b^\dagger c_{kk'}\}, \tag{2c}$$

$$H_{\rm SL} = \hbar \sum_{k,k'} \{g_{_{Lkk'}} {\rm e}^{-i\omega_L t} c_{kk'}^\dagger + g_{_{Lkk'}}^* {\rm e}^{i\omega_L t} c_{kk'}\}. \tag{2d}$$

The symbols in these terms have the following meanings. ω_c is the laser cavity frequency, b ($b^\dagger$) is the boson annihilation (creation) operator of the laser field; p_k ($q_{k'}$) denotes the fermion annihilation operator for electrons in the conduction (valence) band, ω_{p_k} ($\omega_{q_{k'}}$) is the corresponding frequency associated with the energy of a free electron in state k (k') of the conduction (valence) band; $c_{kk'}^\dagger = p_k^\dagger q_{k'}$ and $c_{kk'} = q_{k'}^\dagger p_k$ represent transition processes from one band to another band or interband polarization, the coupling strength between cavity field and two bands is $g_{_{kk'}}$; ω_L is the external laser field frequency and $g_{_{Lkk'}}$ is the coupling constant of the external field with two bands. Eq.(1) does not include the injection current and the loss terms, which will be introduced in the equations of motion below. In the followinging discussion, we also assume that the number of excited carriers is large enough to establish a steady state of laser field so that we can factorize $< bc_{kk'}^\dagger >$ by $< b >< c_{kk'}^\dagger >$, and so on. It is well known that in an atomic laser system, when the number of excited atoms is large compared to the number of cavity photons the quantum treatment of the system by means of master equation becomes equivalent to the semiclassical treatment of Maxwell-Bloch equations.

Therefore the equations of motion of the system observed in a frame rotating with the frequency ω_L can be written in the rotating wave approximation as

$$\frac{{\rm d} < b >}{{\rm d}t} = -\left(\frac{\gamma_c}{2} + {\rm i}\Delta_c\right) < b > -{\rm i}\sum_{k,k'} g_{_{kk'}}^* < c_{kk'} >, \tag{3}$$

$$\begin{aligned}\frac{{\rm d}n_{ck}}{{\rm d}t} =& -{\rm i}\sum_{k'} g_{_{kk'}} < b >< c_{kk'}^\dagger > \\ & -{\rm i}\sum_{k'} g_{_{Lkk'}} < c_{kk'}^\dagger > \\ & +{\rm i}\sum_{k'} g_{_{kk'}}^* < b^\dagger >< c_{kk'} > \\ & +{\rm i}\sum_{k'} g_{_{Lkk'}}^* < c_{kk'} > -\gamma_k n_{ck} + \Lambda_{ck},\end{aligned} \tag{4}$$

$$\begin{aligned}\frac{{\rm d}n_{vk'}}{{\rm d}t} =& {\rm i}\sum_{k} g_{_{kk'}} < b >< c_{kk'}^\dagger > \\ & -{\rm i}\sum_{k} g_{_{Lkk'}}^* < b^\dagger >< c_{kk'} > \\ & +{\rm i}\sum_{k} g_{_{Lkk'}} < c_{kk'}^\dagger > \\ & -{\rm i}\sum_{k} g_{_{Lkk'}}^* < c_{kk'} > -\gamma_{k'} n_{vk'} + \Lambda_{vk'},\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{{\rm d} < c_{kk'}^\dagger >}{{\rm d}t} =& -[\gamma_{kk'} - {\rm i}(\omega_{kk'} - \omega_L)] < c_{kk'}^\dagger > \\ & -{\rm i}g_{_{kk'}}^* < b^\dagger > (n_{ck} - n_{vk'}) \\ & -{\rm i}g_{_{Lkk'}}^* (n_{ck} - n_{vk'}),\end{aligned} \tag{6}$$

where $\Delta_c = \omega_c - \omega_L$ is the detuning of the cavity field frequency with the external field frequency; $n_{ck} =< p_k^\dagger p_k >$ and $n_{vk'} =< q_{k'}^\dagger q_{k'} >$ express the mean electron numbers in the conduction and valence bands with the wave k and k', respectively; γ_k and Λ_{ck} are the decay rate and pumping rate of electrons in the conduction band; $\gamma_{k'}$ and $\Lambda_{vk'}$ are the decay rate and pumping rate of electrons in the valence band. For a detailed expression, $\Lambda_{\alpha k}, (\alpha = c, v)$ is related to the effective injection current including Fermi-Dirac distributions of carriers[17]. $\gamma_{kk'}$ is the decay rate of the interband relaxation. In general, there are several relaxation processes in a semiconductor, which are related to the dephasing effect and occur on different time scales[16,17]. The interaction of the electrons with the cavity field mode resulting in an optical transition occurs on a relatively slow time scale (of the order of nanoseconds). This process is followed by a thermal redistribution of the electrons within the bands, so that quasi-equilibrium, or local equilibrium within the bands is obtained. The Coulomb interactions (which are not included in the Hamiltonian formalism) that cause scatterings are very strong, much stronger than the interaction between the semiconductor and the cavity field mode. The relaxation time of the polarization is phenomenologically described by

the transverse relaxation time T_2, and the population decay time is measured by the longitudinal relaxation time T_1. They have the same order of picosecond. In general, semiconductor gain media involve a hierarchy of T_1 relaxation times, besides carrier-carrier scatterings, there are also carrier-phonon scattering that may influence both the carrier intraband relaxation and interband kinetics. However, carrier collisions provide the dominant relaxation channel for situations with very high carrier densities, detailed researches on the carrier-carrier scatterings can be found in Ref.[22]. In particular, in quasi-equilibrium approximation, it can be set that[17] $T_1 = T_2 = T$. This relation is just for theoretical simplicity here, rather than necessary for all experimental realities [14]. It is known that it is impossible to rigorously solve the master equation including all these processes. Therefore, in this paper, the scattering effects are simply included in the decay terms. We may assume that the decay rates γ_k and $\gamma_{k'}$ include both intraband and interband relaxations, their values should be determined by the experiment. Normally, they can be assumed to have the same values, i.e., $\gamma_k = \gamma_{k'} = \gamma$. Furthermore, we assume that $\gamma_{kk'} = \Gamma$ for all k and k'. These approximations were also made in some publications [16,17]. From the above discussion we have $\gamma = \Gamma = 1/T$. For pure semiconductor materials with a direct band gap, the k-selection for band-to-band transition is valid, i.e., $g_{_{kk'}} = g\delta_{kk'}$ and $g_{_{Lkk'}} = g_p\delta_{kk'}$. Later, we will see that, for a laser system, the physical process is not affected by single decay parameter, but by the unified action of all decay rates and coupling parameters. A decrease of one parameter can be compensated by the increase of other parameters. So the above approximations do not affect the fundamental physical results. We will concentrate on discussion of physical results, rather than on the effect of some single parameter. Under these approximations, the equations of motion can be rewritten as

$$\frac{\mathrm{d} < b >}{\mathrm{d}t} = -\left(\frac{\gamma_c}{2} + \mathrm{i}\Delta_c\right) < b > -\mathrm{i}\sum_k g^* < c_k >, \tag{7}$$

$$\frac{\mathrm{d} < c_k^\dagger >}{\mathrm{d}t} = -(\Gamma - \mathrm{i}\Delta) < c_k^\dagger > - \mathrm{i}g^*(< b^\dagger > + g_r^*)(n_{ck} - n_{vk}), \tag{8}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(n_{ck} - n_{vk}) = -\gamma(n_{ck} - n_{vk}) - \mathrm{i}2g(< b > + g_r) < c_k^\dagger > + \mathrm{i}2g^*(< b^\dagger > + g_r^*) < c_k > + \Lambda_k, \tag{9}$$

where $\Delta = \omega_{kk} - \omega_L$, $\omega_{kk} = \frac{1}{\hbar}\left(\frac{\hbar^2k^2}{2m_c} + E_g - \frac{\hbar^2k^2}{2m_v}\right)$, m_c and m_v are effective masses of the conduction band and the valence band, respectively, E_g is the energy gap; $\Lambda_k = \Lambda_{ck} - \Lambda_{vk}$ is the difference of population injection rates in two bands, and $g_r = g_p/g$. Defining

$$< b > + g_r = r\mathrm{e}^{\mathrm{i}\phi}, \tag{10}$$

Eqs.(8) and (9) can be solved analytically without resorting to the adiabatic elimination and the perturbation method that were normally performed in many publications (Refs.[16], [17] and references therein).

Ⅲ. COHERENCE AND NONINVERSION BISTABILITY

To solve Eqs.(8) and (9), the terms corresponding to the laser fields are considered to be independent of time, while we deal only with the situation of the steady state field. Without going into the details of calculation, the result obtained for the polarization is

$$\begin{aligned} < c_k(t) > =& c_1\mathrm{e}^{-\Gamma t} + c_2\mathrm{e}^{-(\Gamma - \mathrm{i}\Omega)t} + c_3\mathrm{e}^{-(\Gamma + \mathrm{i}\Omega)t} \\ &+ \frac{1}{\Gamma}\frac{4\Delta\Omega g\Lambda_k}{D(\Omega^2 - \Delta^2)}r\mathrm{e}^{\mathrm{i}\phi} \\ &+ \frac{1}{\Gamma - \mathrm{i}\Omega}\frac{2\Omega g\Lambda_k}{D(\Omega + \Delta)}r\mathrm{e}^{\mathrm{i}\phi} \\ &- \frac{1}{\Gamma + \mathrm{i}\Omega}\frac{2\Omega g\Lambda_k}{D(\Omega - \Delta)}r\mathrm{e}^{\mathrm{i}\phi}, \end{aligned} \tag{11}$$

where $\Omega = \sqrt{4\left|g\right|^2 r^2 + \Delta^2}$, and $D = 3\Omega + \frac{(\Omega - \Delta)^3 + \Delta(\Omega + \Delta)^2}{\Omega^2 - \Delta^2}$. c_i (i=1,2,3) are the integration constants related to the initial values. In solving Eqs.(8) and (9), Λ_k is considered independent of time. What we are physically interested in is the steady state solution of $< c_k(t) >$ at time $t \to \infty$, that is

$$\begin{aligned} < c_k > =& < c_k(\infty) > = \frac{4\Delta\Omega^3 g\Lambda_k}{D\Gamma(\Omega^2 + \Gamma^2)(\Omega^2 - \Delta^2)}r\mathrm{e}^{\mathrm{i}\phi} \\ &+ \mathrm{i}\frac{4\Omega^3 g\Lambda_k}{D(\Omega^2 + \Gamma^2)(\Omega^2 - \Delta^2)}r\mathrm{e}^{\mathrm{i}\phi}. \end{aligned} \tag{12}$$

This equation shows that, in equilibrium, the polarization of the system is independent of its initial conditions and is determined by the system parameters, especially by the injection rate which determines the degree of the coherence of the system. Experimentally, this shows that the injection current can drive the system to coherence. When the current or the degree of the coherence reaches a specific level, say a threshold value, the system will undergo a phase transition,

such as lasing action in the absence of an external light pumping field, or bistability in the presence of an external pumping light field as shown later. Physically speaking, the first term of Eq.(12) is related to the dispersion, whereas the second term is related to gain or absorption. Substituting this equation into Eq.(7), one obtains the equations of motion for the amplitude and the phase of the laser field,

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{\gamma_c}{2}r + g_r\left(\frac{\gamma_c}{2}\cos\phi + \Delta_c\sin\phi\right) + \frac{4\Omega^3 g^2 r}{D(\Omega^2+\Gamma^2)(\Omega^2-\Delta^2)}\sum_k \Lambda_k \qquad (13)$$

$$r\frac{\mathrm{d}\phi}{\mathrm{d}t} = -\Delta_c r + g_r\left(\Delta_c\cos\phi - \frac{\gamma_c}{2}\sin\phi\right) - \frac{4\Delta\Omega^3 g^2 r}{D\Gamma(\Omega^2+\Gamma^2)(\Omega^2-\Delta^2)}\sum_k \Lambda_k \qquad (14)$$

These are the general fundamental equations that we obtained for the injected semiconductor laser, which can be used to discuss the basic properties of the laser, such as the steady state amplitude and phase. To verify these, we discuss the resonant case, that is $\Delta_c = \Delta = 0$. In this case, the phase in the steady state should satisfy the condition

$$\phi_0 = 0, \qquad (15)$$

according to $\mathrm{d}\phi/\mathrm{d}t = 0$. Then the steady state field equation can be obtained to be

$$\frac{E\Lambda}{1+E^2} = \frac{E}{G_s} - 2\frac{G_L}{G_s}, \qquad (16)$$

where we have defined dimensionless field amplitude $E{=}2gr/\Gamma$, coupling parameters $G_s{=}2g^2/\gamma_c\Gamma^2$, $G_L{=}g_p/\Gamma$, and $\Lambda{=}\sum_k \Lambda_k = \sum_k(\Lambda_{ck}-\Lambda_{vk})$ expresses the injected population inversion rate of carriers in conduction and valence bands, which is related to the injection current. In equilibrium, the populations of both conduction band and valence band are controlled by Λ. The equation (16) gives rise to the fundamental properties of the system as shown in Fig.1.

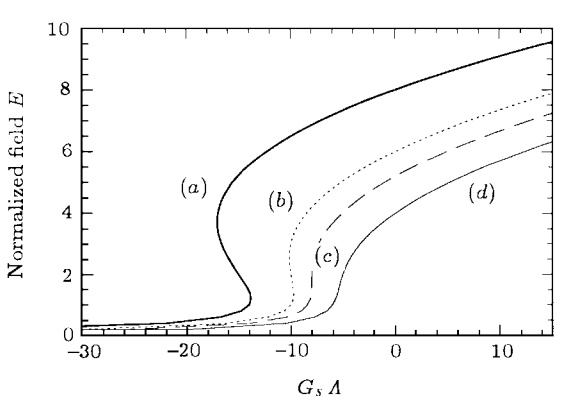


Fig.1. Variation of normalized total laser field with the injected population inversion rate Λ times the parameter G_s. Curves (a, b, c, d) correspond to values of the parameter G_L is given by G_L = 4, 3, 2.598, 2 respectively. It is shown that the bistability occurs in the inversionless region when the parameter G_L is larger than its critical value of $G_{LC} = 2.598$.

First of all, the optical bistability is located in the inversionless region of $\Lambda < 0$, therefore the laser can be operated without the need of population inversion. Second, unlike ordinary semiconductor bistabilities[14], which rely on the change of the input light field, the bistability here relies on the change of the injected population inversion rate, that is the injection current, which is readily controlled experimentally. Third, though a detailed analysis shows that, to achieve this *bistability*, the coupling parameter G_L should be greater than a threshold value $G_{LC} = 2.598$. When the parameter G_L is smaller than G_{LC}, the laser operation still takes place, even in the inversionless region, as curve (d) in Fig.1. It is referred as an inversionless laser (Notice that the population inversion here is corresponding to electrons). This property may be related to the coherent transfer of energy[23]. However, in this paper, our attention is focused on the coherence in the semiconductor interband and its influences on the laser system. It is shown that the external pumping coupling reduces firstly the requirement of coherence degree of the system, such as from lasing with population inversion to without population inversion, and secondly, leads to the generation of nonlinear process, such as bistability (curves a and b). Furthermore, for the bistability, there are switch-on and switch-off values of Λ. As the coupling parameter G_L increases, the switch-on value of Λ decreases, which corresponds to a smaller injection current. We want to point out that the noise property for the system may be modified. For atomic systems, we have shown that noninversion bistable lasers have the phase-noise quenching[21]. The noninversion bistable semiconductor laser discussed here is the counterpart of noninversion lasers with incoherent atomic injection. Therefore, we also expect it to have the same property.

It can be shown that it is the coherent pumping field g_p that leads to the mechanism of inversionless lasing. If $g_p = 0$ (therefore $G_L = 0$), from Eq.(16), we can obtain the laser field equation

$$E = \sqrt{G_s\Lambda - 1}, \qquad (17)$$

then, we obtain immediately the ordinary threshold condition for the semiconductor laser,

$$\Lambda_{\rm th} = \frac{1}{G_s} = \frac{\gamma_c \Gamma^2}{2g^2}. \tag{18}$$

This threshold condition is naturally obtained without any additional definition. From the above equation the parameter G_s should be regarded as the net gain coefficient. It is shown that, the stronger coupling (e.g., small cavity volume), the lower is the threshold value of the system. Actually, in the whole derivation, we have not restricted the dimension of our system. Therefore, the results can, in principle, be suitable to semiconductor laser systems in any dimension. According to the parametric analysis, the microcavity semiconductor laser may be better for achieving the noninversion bistability.

IV.MACROSCOPIC DESCRIPTION OF MAXWELL-BLOCH EQUATIONS

In Secs. II and III, we have used the microscopic kinetic equations to show that it is possible to realize the bistable laser without electronic inversion in the conduction band and valence band of semiconductor systems. To further understand the above results, and parallel other treatments on this topic (see Refs.[16,17,22]), we introduce electron-hole representation and macroscopic polarization description. To do this, we have to introduce some new notations. First, we note that the expectation values of populations of the electrons and holes are $n_{ek} =< p_k^\dagger p_k >$, $n_{hk'} = 1 - n_{vk'} = 1 - < q_{k'}^\dagger q_{k'} >$. To show the role of decay rates γ and Γ, we do not assume that $\gamma = \Gamma$ at first in this section. Similar to the discussion in Sec. II, Eqs.(7–8) can be written in the electron-hole representation as

$$\begin{aligned}\frac{{\rm d}n_{ek}}{{\rm d}t} =& -{\rm i}g(< b > + g_r) < c_k^\dagger > \\ &+ {\rm i}g^*(< b^\dagger > + g_r^*) < c_k > \\ &- \gamma n_{ek} + \Lambda_{ek},\end{aligned} \tag{19a}$$

$$\begin{aligned}\frac{{\rm d}n_{hk'}}{{\rm d}t} =& {\rm i}g(< b > + g_r) < c_{k'}^\dagger > \\ &- {\rm i}g^*(< b^\dagger > + g_r^*) < c_{k'} > \\ &- \gamma_{k'} n_{hk'} + \Lambda_{hk'},\end{aligned} \tag{19b}$$

$$\begin{aligned}\frac{{\rm d} < c_k^\dagger >}{{\rm d}t} =& -(\Gamma - {\rm i}\Delta) < c_k^\dagger > \\ &- {\rm i}g^*(< b^\dagger > + g_r^*)(n_{ek} + n_{hk} - 1).\end{aligned} \tag{19c}$$

The macroscopic variables we want to use are the injected carrier density N related to n_e and n_h as

$$N = \frac{1}{V}\sum_k n_{ek} = \frac{1}{V}\sum_{k'} n_{hk'}, \tag{20}$$

where V is the volume of the semiconductor laser, and the slowly varying polarization P is defined in a manner as in many literatures[15–17],

$$\boldsymbol{P} = \frac{1}{2}\hat{e}[P\exp(-{\rm i}\nu t) + {\rm c.c.}], \tag{21}$$

where P is related to the dipole moment $\wp$,

$$P = 2\wp\frac{1}{V}\sum_k < c_k > . \tag{22}$$

If the sum over k is replaced by an integration over the density of states $D(E)$, N and P can be defined as

$$N = \int_{E_g}^{\infty} n_e(E')D(E'){\rm d}E', \tag{20a}$$

and,

$$P = 2\wp\int_{E_g}^{\infty} c(E')D(E'){\rm d}E'. \tag{21a}$$

Now Eqs. (19) can be changed to the following formation,

$$\begin{aligned}\frac{{\rm d}N}{{\rm d}t} =& -\gamma N - {\rm i}\frac{\wp\varepsilon}{\hbar}\frac{1}{V} \\ &\cdot\left(r{\rm e}^{-{\rm i}\phi}\sum_k c_k - r{\rm e}^{{\rm i}\phi}\sum_k c_k^\dagger\right) + \frac{I}{qV},\end{aligned} \tag{23a}$$

$$\begin{aligned}\frac{{\rm d} < c_k >}{{\rm d}t} =& -\Gamma(1 + {\rm i}\delta) < c_k > \\ &- {\rm i}\frac{\wp\varepsilon}{\hbar}r{\rm e}^{{\rm i}\phi}(n_{ek} + n_{hk} - 1),\end{aligned} \tag{23b}$$

where we have used Eq. (10), and written $g = \wp\varepsilon/\hbar$, $\delta = \Delta/\Gamma$. q is the charge of an electron. To deal with Eqs. (23), we follow a method proposed by Yao *et al.*[16], and define two parameters κ and ζ, so that

$$\int_{E_g}^{\infty}\left(\frac{n_e + n_h - 1}{1 + {\rm i}\delta}\right)D(E'){\rm d}E' \equiv \kappa N, \tag{24}$$

$$\int_{E_g}^{\infty}\left(\frac{2\wp c}{1 + {\rm i}\delta}\right)D(E'){\rm d}E' \equiv \frac{P}{\zeta}. \tag{25}$$

By using these notations, Eqs. (23), describing the motions of population density and the polarization of the system, can be written in terms of Eqs.(21) and (22) as

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\gamma N - \mathrm{i}\frac{\varepsilon}{2\hbar}\left(r\mathrm{e}^{-\mathrm{i}\phi}P - r\mathrm{e}^{\mathrm{i}\phi}P^{\dagger}\right) + \frac{I}{qV}, \quad (26a)$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -\Gamma\zeta P - \mathrm{i}2\frac{\wp^2\varepsilon}{\hbar}\kappa\zeta N r\mathrm{e}^{\mathrm{i}\phi}. \quad (26b)$$

The field equation can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(r\mathrm{e}^{\mathrm{i}\phi}\right) = -\left(\frac{\gamma_c}{2} + \mathrm{i}\Delta_c\right) r\mathrm{e}^{\mathrm{i}\phi} + g_r\left(\frac{\gamma_c}{2} + \mathrm{i}\Delta_c\right) - \mathrm{i}\frac{V\varepsilon}{2\hbar}P. \quad (27)$$

Now the dynamics of semiconductor lasers is governed by a set of macroscopic equations (26a), (26b) and (27), which are the fundamental equations for our model. In Eq.(26a), the equation of motion of population density does not involve terms related to $n_e n_h$ like in some publications[17,22], which is a reasonable approximation for system possessing fast decay rates (see Yamamoto and Björk in Ref.[15] and Yao *et al.* in Ref.[16]). On the other hand, the population decay rate γ here includes the spontaneous emission decay rate and the nonradiative recombination rate, both of them are separately considered in some papers. Actually, in a mathematical sense, this treatment does not change the physical result.

V. DETERMINATION OF LASER PROPERTIES AND PARAMETER κ

In this section, we want to determine the properties of the laser system and those of the parameter κ. Though we have demonstrated the possibility of achieving bistable laser without inversion in this system, we want to further investigate what new properties can be obtained after we have introduced the parameters κ and ζ. Also, it is expected that the property of bistable noninversion lasers matches with the present description.

As pointed out before, the parameters κ and ζ have different roles. ζ governs modification of the intraband relaxation time for the macroscopic polarization P, which is therefore mostly dependent on the properties of the semiconductor materials that one has chosen. This is not the topic of concern to us in this paper. κ governs changes of the susceptibility in the medium, resulting from integrating over the band states in the semiconductors, and affects both the gain and refractive index. It is determined by the system parameters. Yao *et al.*[16] have numerically shown the relations of κ and ζ with the population density. One of the main fundamental understandings of this paper is the coherence in the two bands of semiconductor, which is controlled by the injection current I. Not only the polarization between the two bands but also the population density and the laser field are related to the current I. Therefore any one of the above macroscopic variables, e.g., the population density, the current and the electric field, can be used as the independent variable. Normally, the susceptibility is determined by the electric field; therefore, we use the electric field as the controlling parameter for κ. By definition, we can see that the parameter κ should vary with respect to the detuning δ.

¿From Eqs.(26a) and (26b), we can obtain the steady state solutions for the population density N and the polarization P, according to $\mathrm{d}N/\mathrm{d}t = 0$, and $\mathrm{d}P/\mathrm{d}t = 0$:

$$N_0 = \frac{I}{qV}\frac{1}{\gamma\left(1 + \dfrac{2\kappa_r\wp^2\varepsilon^2 r^2}{\gamma\Gamma\hbar^2}\right)}, \quad (28)$$

and,

$$P = -\mathrm{i}\frac{2\kappa\wp^2\varepsilon}{\Gamma\hbar}N_0 r\mathrm{e}^{\mathrm{i}\phi} = -\mathrm{i}\frac{2\kappa\wp^2\varepsilon}{\gamma\Gamma\hbar}\frac{\dfrac{I}{qV}}{\left(1 + \dfrac{2\kappa_r\wp^2\varepsilon^2 r^2}{\gamma\Gamma\hbar^2}\right)} r\mathrm{e}^{\mathrm{i}\phi}, \quad (29)$$

where we have let $\kappa = \kappa_r + \mathrm{i}\kappa_i$. Because $P = \epsilon_0\chi E$, where ϵ_0 is the vacuum permittivity, and χ is the susceptibility, we have

$$\chi = -\mathrm{i}\frac{2\kappa\wp^2\varepsilon}{\Gamma\hbar\epsilon_0}N_0 = \chi' + \mathrm{i}\chi'', \quad (30)$$

or,

$$\chi' = \frac{2\wp^2\varepsilon}{\Gamma\hbar\epsilon_0}N_0\kappa_i, \quad (30a)$$

$$\chi'' = -\frac{2\wp^2\varepsilon}{\Gamma\hbar\epsilon_0}N_0\kappa_r. \quad (30b)$$

¿From the above relations we can see that the population density N_0 and the polarization P, as well as the susceptibility χ, can be controlled by the current I. κ_r and κ_i determine the properties of absorption and dispersion functions in semiconductor media.

A. The properties of laser field

We now turn to discuss the properties of laser field. Eq. (27) can be separated into two equations corresponding to the amplitude r and the phase ϕ,

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{\gamma_c}{2}r + \frac{\gamma_c}{2}g_r\cos\phi + \Delta_c g_r\sin\phi - \frac{\kappa_r g^2 r}{\gamma\Gamma}\frac{I/q}{1+\left(\frac{2\kappa_r g^2 r^2}{\gamma\Gamma}\right)}, \tag{31a}$$

$$r\frac{\mathrm{d}\phi}{\mathrm{d}t} = -r\Delta_c - \frac{\gamma_c}{2}g_r\sin\phi + \Delta_c g_r\cos\phi - \frac{\kappa_i g^2 r}{\gamma\Gamma}\frac{I/q}{1+\left(\frac{2\kappa_r g^2 r^2}{\gamma\Gamma}\right)}, \tag{31b}$$

where we have reused $g = \wp\varepsilon/\hbar$. If we choose the detuning of the driving field to be zero, i.e., $\Delta_c = 0$, the steady state solutions of the amplitude r and the phase ϕ, according to $\mathrm{d}r/\mathrm{d}t = 0$ and $\mathrm{d}\phi/\mathrm{d}t = 0$, are

$$\gamma_c g_r\cos\phi = \gamma_c r + \frac{2\kappa_r g^2 r}{\gamma\Gamma}\frac{I/q}{1+\left(\frac{2\kappa_r g^2 r^2}{\gamma\Gamma}\right)},$$

and

$$\gamma_c g_r\sin\phi = -\frac{2\kappa_i g^2 r}{\gamma\Gamma}\frac{I/q}{1+\left(\frac{2\kappa_r g^2 r^2}{\gamma\Gamma}\right)}.$$

¿From these two equations, we can relate the laser field to the electric current I and the system parameters:

$$\frac{E_0}{1+\frac{\kappa_r E_0^2}{2}}\left(\frac{I}{q}\right) = -\frac{E_0\kappa_r}{|\kappa|^2\left(\frac{2g^2}{\gamma_c\gamma\Gamma}\right)} - \frac{1}{|\kappa|^2\left(\frac{2g^2}{\gamma_c\gamma\Gamma}\right)}\cdot\sqrt{|\kappa|^2\left(\frac{4g_p^2}{\gamma\Gamma}\right) - \kappa_i^2 E_0^2}, \tag{32}$$

where we have defined $E_0 = 2gr/\sqrt{\gamma\Gamma}$, which is not equal to E defined in the Sec. III at this stage. However, if $\gamma = \Gamma$, then they are the same. Here the field equation (32) is different from that we obtained in Sec.III, Eq.(16), because we have introduced the parameter κ, which is a function of frequency detuning δ. Whereas in Sec.III the detuning is taken to be zero. So here the field equation (32) is more general. From this equation we can further see that the field depends on the unified action of various decay rates and the coupling coefficients like bracketed factors in the right hand side of the equation. From this point, we can figure out that even one assumes that $\gamma = \Gamma$, the properties of the laser field will remain similar to that of the case $\gamma \neq \Gamma$ as long as the parameters are identical. So for simplicity in the following discussions we may choose $\gamma = \Gamma$ (therefore, $E_0 = E = 2gr/\Gamma$ in the following discussion).

It is particularly worthy to note the square root term. Because the terms in the square root should not be smaller than zero, therefore, it gives a limitation, or a maximum for the laser field of the system for this model,

$$E_{0\max}^2 = \frac{4|\kappa|^2 g_p^2}{\kappa_i^2\gamma\Gamma}. \tag{33}$$

Because the parameter κ is also a function of the field, Eq.(32) is actually very complicated. Theoretically we expect to obtain an explicit expression for κ, and also under the same condition the Eq.(32) should lead to the field equation (16). We will show these in the following subsections.

B. Determination of κ

To determine κ, we need to use Eq.(22) and Eq.(12), they lead to

$$P = \frac{2\wp}{V}\sum_k < c_k >= \frac{2\wp}{V}\frac{4\Omega^3 g}{D(\Omega^2+\Gamma^2)(\Omega^2-\Delta^2)}\cdot\left(\frac{\Delta}{\Gamma}+\mathrm{i}\right)\sum_k \Lambda_k r\mathrm{e}^{\mathrm{i}\phi}. \tag{34}$$

This equation corresponds to $\gamma = \Gamma$ as before. Without loss of generality we will restrict our discussion to this case and compare Eq.(34) with Eq.(29). To do this, we should notice that $\sum_k \Lambda_k = 2(I/q)$, which is easy to be understood from physical consideration. Then we reach the results

$$\kappa_r = \kappa_r(\delta, E) = -\frac{\xi}{1+\xi\frac{E^2}{2}}, \tag{35a}$$

$$\kappa_i = \kappa_i(\delta, E) = \frac{\delta\xi}{1+\xi\frac{E^2}{2}}, \tag{35b}$$

$$|\kappa|^2 = \kappa_r^2 + \kappa_i^2 = \frac{\xi^2(1+\delta^2)}{\left(1+\xi\frac{E^2}{2}\right)^2}, \tag{35c}$$

with

$$\xi = \xi(\delta, E) = \frac{4(\delta^2+E^2)}{\left[2E^2+\delta(3\delta-\sqrt{\delta^2+E^2})\right](E^2+\delta^2+1)}, \tag{36}$$

where $\delta = \Delta/\Gamma$. In particular, if we take the detuning

$\delta = 0$, then we have

$$\kappa_r(0,E) = -\frac{2}{1+2E^2};$$
$$\kappa_i(0,E) = 0;$$
$$\xi(0,E) = \frac{2}{1+E^2}. \qquad (37)$$

We have analytically obtained expressions for κ_r and κ_i. Fig.2(a) shows -κ_r versus Δ, while Fig. 2(b) shows κ_i versus Δ. According to Eq.(30b), one can see that Fig.2(a) relates to the absorption property of a semiconductor medium under different fields. The dotted, dashed, and solid lines correspond to E being 3, 6 and 10, respectively. From Eq.(30a), κ_i is a measure of dispersion property of a semiconductor medium in different fields. The same as in Fig.2(a), E is chosen to be 3, 6 and 10 for the dotted, dashed and solid lines, respectively, in Fig.2(b).

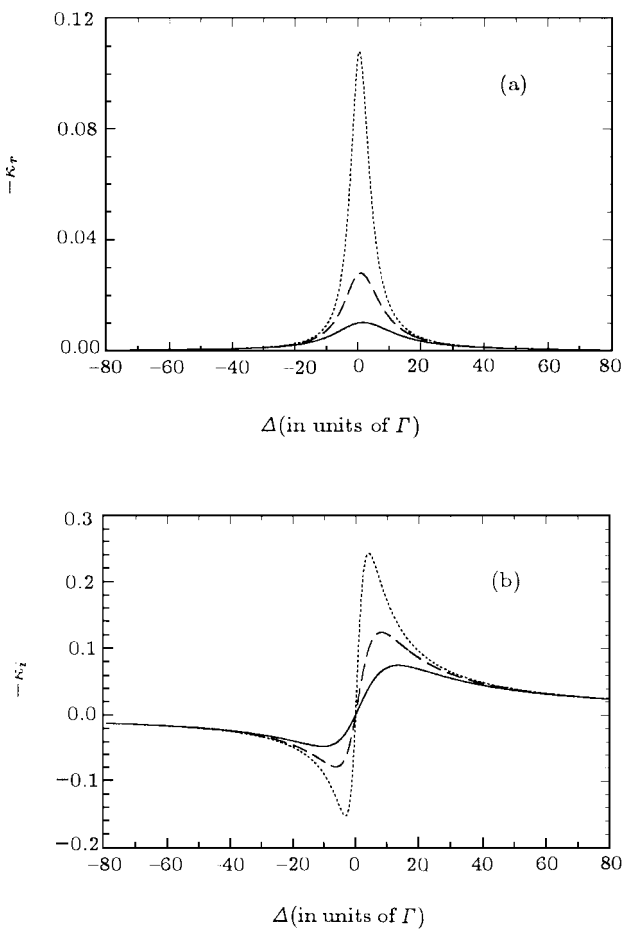


Fig.2. In both figures (a) and (b), E is chosen to be 3, 6 and 10 for the dotted, dashed and solid lines, respectively. (a) κ_r vs laser detuning Δ. (b) κ_i vs laser detuning Δ.

C. More on laser field

Having obtained expressions for κ_r and κ_i, we can rewrite Eq.(32) by inserting Eqs.(35) and (36) into Eq.(32) as

$$\frac{I}{q} = \frac{1-\sqrt{(1+\delta^2)\frac{4G_l^2}{E^2}-\delta^2}}{(1+\delta^2)\xi G_s}. \qquad (38)$$

In this subsection, we are particularly interested in the properties of laser field in the case of the detuning $\delta = 0$. It is expected that when $\delta = 0$, Eq.(38) should be identical to Eq.(16). Actually, if we insert Eq.(37) into Eq.(38), we have

$$\frac{E}{1+E^2}2\left(\frac{I}{q}\right) = \frac{E}{G_s} - 2\frac{G_L}{G_s}, \qquad (39)$$

where $G_s = 2g^2/\gamma_c\Gamma^2$, and $G_L = g_p/\Gamma$ as defined before. So we see that Eq.(39) is exactly what we have expected. The laser property depends on the rate of Rabi frequency g_p of driven field with the interband decay rate Γ. When $g_p > 2.598\Gamma$, the inversionless bistability takes place, as shown in Fig.3(a). Fig.3(a) and Fig.3(b) show the normalized laser field E versus electric current I. In Fig.3(b), the solid, dashed and dotted curves are corresponding to $G_L = 2.5$, 1.5, 0.5, respectively; while the long dashed curve corresponds to $G_L = 0.01$. The former three curves clearly demonstrate the property of an inversionless laser, whereas the curve $G_L = 0.01$ corresponds to a normal laser. What can we understand from Fig.3? We see that, if the Rabi frequency g_p of the driven field is small, the laser actionone is normal. One can realize lasing without inversion for a larger g_p. Further increasing g_p, one can get inversionless bistable laser. The driven field pumps electrons to the conduction band, and increases the coherence degree of the laser system as well.

From Fig.3 one may find that all curves do not show any limitation for the laser field. This is what we want to discuss further in this subsection. Eq.(33) shows a limitation for the laser field of the system. This equation can, if one inserts Eqs.(35) into Eq.(33), be rewritten as

$$E_{\max}^2 = \frac{4g_p^2}{\Gamma^2}\left(1+\frac{1}{\delta^2}\right). \qquad (40)$$

So it is favorable to obtain a high intensity laser field with a small detuning. The field in Eq.(39) is corresponding to the case $\delta = 0$; therefore, according to Eq.(40), $E_{\max} \to \infty$. That means no upper limit for laser field in this case, as it is shown in Fig.3.

It is worth pointing out that if the external field is removed, the laser field will be determined by

$$E = \sqrt{\frac{4g^2}{\gamma_c \gamma \Gamma}\left(\frac{I}{q}\right) - 1}. \tag{41}$$

This equation shows a well known result that a semiconductor laser intensity increases with the electric current I, even for a QD laser this property is also true[19]. In the QD laser, intraband and interband relaxation rates are modified. However, the dependence of the laser field on the current I remains similar to a bulk semiconductor laser. This is because the laser action takes place also between two bands. As E is real-valued, Eq.(41) gives the threshold value for the electric current

$$I_{\rm th} = \frac{\gamma_c \gamma \Gamma}{4g^2} q \tag{42}$$

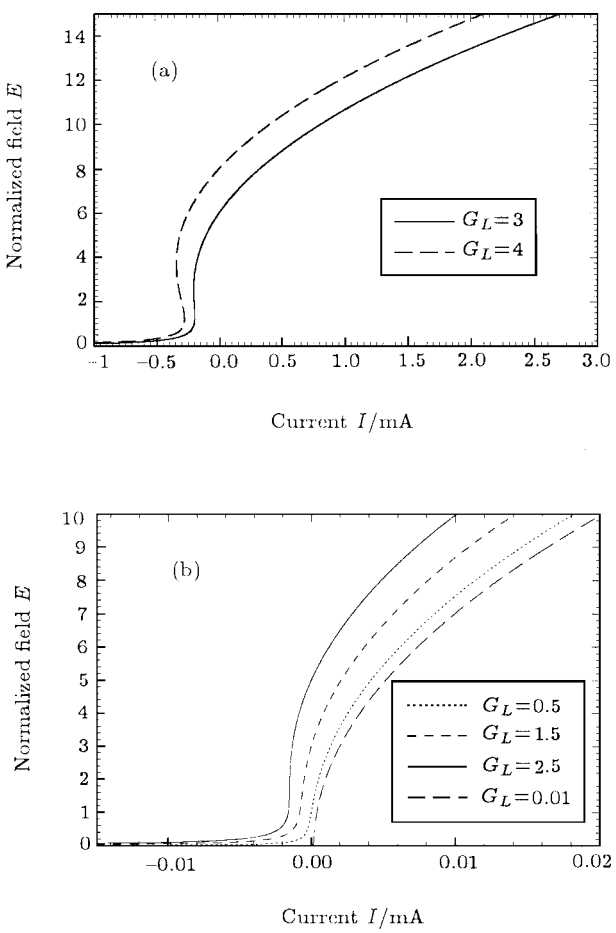


Fig.3. Normalized laser field varys with electric current I under different G_L. (a) Inversionless bistability corresponding to the current. (b) The solid line, dashed line and dotted line correspond to inversionless laser, while the long dashed line corresponds to a ordinary laser.

It is well known that the Rabi frequency g of a laser cavity is inversely proportional to the volume of the laser cavity[16,17], and $\gamma_c = \omega/Q$, where Q is the cavity quality factor and ω is the laser frequency, so this equation can qualitatively explain why a strongly coupled high Q microcavity could have lower laser threshold[18].

VI. DISCUSSION AND CONCLUSION

In summary, a model describing the interband coherence in a semiconductor laser has been discussed in this paper. We have shown that the injection current plays a key role in achieving lasing coherence which could be regarded as the intrinsic (or systematic) mechanism for coherence. If the external field is absent, the injection current is the only mechanism for coherence. Therefore, in order to achieve lasing, it has to exceed a threshold value. Here we emphasize the generality of our results. Yamamoto *et al.*[15] have studied the "definition" of a laser threshold. They pointed out that the threshold of a laser is more appropriately described by the pump power (or current) needed to bring the mean cavity photon number to unity, rather than the conventional "definition" that it is the pump power at which the optical gain equals the cavity loss. In the present paper, we have given an analytic expression Eq.(41) for the laser field, which shows that in order to realize laser action, the threshold condition is the normalized laser intensity being unity. This exactly corresponds to the idea of Yamamoto *et al.* Secondly, from Eq.(42) we can also understand that, for a strongly coulped high-Q microscopic cavity the laser threshold is low. This result also agrees with the recent expremental results[18].

On the other hand, if an external pumping field, serving as an external agent to set up the coherence, is applied, the threshold value of the injection current decreases. When the pump-field is strong enough, the laser field can be established with zero injection current (see Fig.3). When the external pumping field reaches a threshold value, it is even possible to obtain noninversion bistability for a semiconductor laser. For a two-level atomic laser system, we have shown that the noninversion bistability may result in phase noise quenching [21]. Here the semiconductor laser system is the counterpart of an incoherent atomic injection system. Thus, this system is expected to have the same property. Therefore, it is potentially useful in the telecommunication, optical information processing, and optical switching. Besides, we have also ana-

lytically obtained an upper limit for the output laser field of a driven semiconductor laser system and shown that for a small detuning, we may obtain a relatively high laser field. In fact, when the detuning $\delta = 0$, there is no upper limit for the output laser field.

We have studied both the microscopic and macroscopic dynamics of the laser system, and phenomenologically introduced two parameters κ and ζ. ζ governs modifications of interband relaxation time; therefore, it is in a sense determined by the experiment. While κ governs changes of the susceptibility of the semiconductor medium, it is particularly interested theoretically. We have analytically derived the relations of both the real part and imaginary part of κ with respect to the laser detuning δ. This leads to an understanding for the susceptibility. The real part and imaginary part of κ describe the absorption and dispersion properties of semiconductor media, respectively. Under the different fields E, i.e., for different current I, one could achieve different absorption and dispersion properties.

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