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APPROXIMATE MODEL OF INTENSE FIELD STABILIZATION FOR HYDROGEN ATOM*

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An approximate model is proposed to study the stabilization problem of hydrogen atoms under monochromatic intense laser field. The stabilization regime for system parameters such as laser field strength, laser field frequency and atomic magnetic quantum number are obtained by stability analysis of fixed points of the model. The results are consistent with those obtained by other methods.

Keywords: hydrogen atom, intense laser field, stabilization
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I. INTRODUCTION

Recently, very active research interest has attributed to the intense laser field physics ranging from the fast ionization of high Z atoms, high order harmonics generation of soft X-ray to the atomic stabilization dynamics.[1,2] It is well known that the stabilization of atoms in an intense laser field is intrinsically a phenomenon of quantum mechanics.[3–7] However, a great number of theoretical studies in the classical frame have been done,[8–12] which not only help to understand various physical mechanisms for the problem of atomic stabilization, but also give results consistent with quantum mechanics principles to some extents. Therefore, more and more reasonable and effective classical analysis have been proposed and developed. One of the key points for the study of stabilization problem is how to transform the original time-dependent problem to the simple time-independent one. In our earlier paper[11] we made a detailed comparison study on one-dimensional (1D) atomic stabilization for hydrogen atoms starting from both the exact differential equation and the approximate map equation. We pointed out that the stabilization is connected with many aspects of the classical nonlinear dynamics characteristics of the system, especially with the KAM tori structure for (1D) model atom. On the other hand, we developed the two-dimensional map analysis[9] to the full four-dimensional (4D) situation and obtained the improved stabilization border.[12]

In this paper we propose another simple approximate model and derive a map approximation equation to describe the stabilization of three-dimensional real atoms in linearly polarized field. The stabilization border has been obtained by analyze the stability of fixed points, that is

$$\epsilon > \epsilon_{\rm stab} = \beta\frac{\omega}{m},$$

where ϵ and ω are the field strength and frequency respectively, m denotes the z component of the angular momentum, i.e., atomic magnetic quantum number, and $\beta \gg 1$ is a constant which needs to determine by experiment or numerical simulation. This result coincides in the form obtained by Casati *et al.*[10] where $\beta \approx 12$ and our previous work [12] where $\beta \approx 3$.

II. APPROXIMATE MODEL AND MAP EQUATIONS

The classical Hamiltonian in cylindrical coordinates for a hydrogen atom in a sinusoidal driving field along z direction is:(here and throughout this paper we use atomic unit $\hbar = e = m_{\rm e} = 1$)

$$H = \frac{p^2}{2} + \frac{\nu^2}{2} - \frac{1}{\sqrt{\rho^2+z^2}} + \frac{m^2}{2\rho^2} + \epsilon z\sin(\omega t), \quad (1)$$

where p and ν are the momentum in z and ρ directions respectively, m is the z component of the

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angular momentum, i.e., magnetic quantum number. Usually it is convenient for problem to be studied to transform the laboratory frame into the oscillating Kramers-Henneberger frame.[9,12] The corresponding Hamiltonian is changed to[12]

$$H = \frac{p^2}{2} + \frac{\nu^2}{2} + \frac{m^2}{2\rho^2} - \frac{1}{\sqrt{\rho^2 + (z - \alpha\sin(\omega t))^2}}, \quad (2)$$

where $\alpha = \epsilon/\omega^2$ is an amplitude of free electrons quivering in the laser field.

For high frequency $\omega \gg 1$ and large nucleus quiver amplitude $\alpha \gg 1$, it seems that the oscillating nucleus acts like a charge line, which is just the starting point of Casati work.[10] In fact, the density of this charge line should be inversely proportional to the velocity of motion of nucleus. In terms of the normalization condition, i.e., the integral of the charge line density from $-\alpha$ to $+\alpha$ should be $e = 1$. Therefore, the charge line density is written as

$$\sigma(z) = \frac{1}{\pi\sqrt{\alpha^2 - z^2}}.$$

If no further approximation is made, it will be found that the potential determined by $\sigma(z)$ is exactly equivalent to the averaged potential in one optical period determined by the Coulomb potential in the works of Jesen *et al.*[5,9] It is noted that further approximation for potential as $2\sigma(z)\ln(\rho/a)$ (see Eq.(2) of Ref.[10]), is valid only for $z, \rho \ll \alpha$.

For z approaching $\pm\alpha$, the corresponding charge density $\sigma(z)$ becomes infinity, which means that the nucleus is staying most of the time in the neighboring region of $-\alpha$ and $+\alpha$. Thus when $\alpha \gg 1$, we can consider that the two nucleis with charge $+e/2$ are seated approximately at $-\alpha$ and $+\alpha$, respectively. This is the approximate model we proposed. The Hamiltonian of the system has now become

$$H = \frac{p^2}{2} + \frac{\nu^2}{2} + \frac{m^2}{2\rho^2} + V_0(z, \rho), \quad (3)$$

where

$$V_0(z, \rho) = -\frac{1}{2}\Big[\frac{1}{\sqrt{\rho^2 + (z + \alpha)^2}} + \frac{1}{\sqrt{\rho^2 + (z - \alpha)^2}}\Big].$$

On the other hand, the map approximation proposed by Jensen et al. is nevertheless valid for $\alpha \gg 1$,[9,11,12] which are (see, for example, Eqs.(6)–(9) of Ref.[9]):

$$\begin{aligned} z_{n+1} &= z_n + T p_{n+1}, \\ p_{n+1} &= p_n - T\frac{\partial}{\partial z_n} V_{\rm map}, \\ \rho_{n+1} &= \rho_n + T\nu_{n+1}, \\ \nu_{n+1} &= \nu_n - T\frac{\partial}{\partial \rho_n} V_{\rm map}, \end{aligned} \quad (4)$$

where

$$V_{\rm map}(z, \rho) \equiv V_0(z, \rho) + m^2/2\rho^2.$$

For map equation (4) there is one primary fixed point $z_1 = 0, \rho_1 \approx (m^2\alpha^3)^{1/4}$, and two secondary fixed points $z_2 \approx \pm\alpha, \rho_2 \approx 2m^2$. Obviously we can estimate the stability boundary in terms of map equations in z and ρ directions, separately.

Ⅲ. STABILIZATION ANALYSIS

The stability of the fixed point is simply determined by the curvatures of the map potential at the fixed points $\overline{z}, \overline{\rho}$,

$$-4 < S_z = -T^2\frac{\partial^2 V_{\rm map}}{\partial z^2}|_{\overline{z},\overline{\rho}} < 0,$$

$$-4 < S_\rho = -T^2\frac{\partial^2 V_{\rm map}}{\partial \rho^2}|_{\overline{z},\overline{\rho}} < 0.$$

We analyze the stability boundary of the fixed point for two directions of z and ρ respectively and separately.

(1) z direction. We have the following approximate expression for map potential

$$\frac{\partial^2 V_{\rm map}}{\partial z^2}|_{z_1,\rho_1} \approx -\frac{2}{\alpha^3},$$

$$\frac{\partial^2 V_{\rm map}}{\partial z^2}|_{z_2,\rho_2} \approx \frac{1}{8m^6}.$$

Therefore the primary fixed point is unstable in z direction and the secondary fixed points are stable elliptic ones when the following condition

$$m^3\omega > \frac{\pi}{\sqrt{8}} \quad (5)$$

is satisfied.

(2) ρ direction. Map potentials at primary and secondary fixed points are obtained approximately as follows

$$\frac{\partial^2 V_{\rm map}}{\partial z^2}|_{z_1,\rho_1} \approx \frac{4}{\alpha^3},$$

$$\frac{\partial^2 V_{\rm map}}{\partial z^2}|_{z_2,\rho_2} \approx \frac{1}{16m^6}.$$

Thus the stabilization condition, $-4 < S_\rho < 0$ in ρ direction, requires that

$$\alpha^3\omega^2 > 4\pi^2$$

for the primary fixed point, and

$$m^3\omega > \frac{\pi}{4}$$

for the secondary fixed points.

In deriving the stability border mentioned above we have assumed $\rho \ll \alpha$ so that $\alpha \gg m^2$ is also required for the primary fixed point, and $\alpha \gg 2m^2$ for the secondary fixed points. Equivalently we have the following condition

$$\epsilon \gg \sqrt{2\pi m^3\omega}\frac{\omega}{m}$$

for the primary fixed point, and

$$\epsilon \gg \frac{\pi}{2}\frac{\omega}{m}$$

for the secondary fixed points. In addition, combining the stability analyses mentioned above in both z and ρ directions, it can readily be found that the stabilization border of the system could be determined by the following expression

$$\epsilon > \epsilon_{\rm stab} = \beta\frac{\omega}{m}, \qquad (6)$$

where $\beta \gg 1$ is a constant, which can be determined by the fitting of experimental or numerical simulation results, for example, $\beta \approx 12$, in Ref.[10]. It is noted that our results of Eq.(6) are valid not only for the case $z \ll \alpha$ of Ref.[10] but also for the case $z \sim \alpha$. This means that Eq.(6) is applicable to a wide parameter range, and it has an universal characteristic to some extent.

IV.SUMMARY AND DISCUSSION

Because Eq.(4) is a 4D volume-persevered map, there should exist rigorous parameters stabilization regime for fixed points. Once the parameters fall outside the stability boundary, the period-doubling or tangent bifurcations will occur.[13] Thus it is necessary to make complete analysis for 4D map as what we have done in our previous work.[12] However, since both of the crossover derivative terms $\partial^2 V_{\rm map}/\partial z\partial\rho$ at primary and secondary fixed points of our present model equal approximately to zero, the results obtained by considering z direction and ρ direction separately as above are the same as that of global analyzing for the full 4D map in combination of both z and ρ directions as a whole.

In summary, we have proposed a simple approximate model in which the behavior of atoms under an intense laser field seems to experience Coulomb interaction from two equivalent nuclei at positions $\pm\alpha$ with charge $e/2$. By using this model and map approximation, we have again obtained the relation of stabilization border among system parameters, such as field strength ϵ, field frequency ω and z component angular momentum m, which are consistent with those derived in Refs.[10] and [12] in different approaches. This stabilization criterion is applicable to a wide parameter range, and it has played an important role in determining the condition of what and when happens the atomic intense laser field stabilization.

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