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NUMERICAL STUDIES ON THE DYNAMICS OF DISORDERED VORTEX LATTICE*

Cao Yi-gang(曹义刚) and Jiao Zheng-kuan(焦正宽)

Department of Physics, Zhejiang University, Hangzhou 310027, *China*

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Flow behavior of the driven two-dimensional vortex lattice is numerically studied with different densities of randomly distributed pointlike pinning centers. Different features in the curves of velocity-force dependence are found between the elastic and plastic regimes. Scaling fit between force and velocity above the critical driving force can be obtained in the elastic regime but fails in the plastic regime. Transition from the elastic to plastic regimes is accompanied by maximum peaks in the differential curves of velocity-force dependence in the disordered vortex lattice.

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The influence of quenched random pinning on the vortex lattice is a subject of longstanding interest. Effects of disorder on the static Abrikosov lattice have been widely discussed.[1,2] Recently the issues of depinning and nonlinear transport in a driven disordered medium, in particular, the dynamics of vortices in disordered type-Ⅱ superconductors driven by an external current has attracted much attention.[3,4]

To solve the summation problem of pinning force, the picture of collective pinning theory was proposed by Larkin and Ovchinnikov.[5] The basic idea is that a vortex lattice breaks up into coherently pinned domains, and these domains are pinned independently. The relative displacements of vortices grow fast with the distance within these domains until they reach the characteristic scale of the pinning potential associated with the vortex core size. In their early work, Brandt and Jensen *et al.*[6] found that for all but very weak pinning, the vortex lattice is plastically deformed and the onset of vortex motion takes place through channels. This result goes beyond the applicability of the collective pinning theory for I-V characteristics. Shi and Berlinsky[7] proposed that even in the case of arbitrarily weak random potential, there are regions of the vortex lattice, where are strain on the vortex lattice is large enough that "phase slips" occur in the vortex lattice and the motion is unstable to plastic flow in the form of channels. Thus the topological nature and the consistency of the elastic description of the vortex lattice has become a subject of extensive debate. Experimenters and theorists are trying to develop tools that are sensitive to the driven motion of vortices, as well as to their equilibrium configurations.

In a theoretical work in the specific context of charge density wave (CDW) systems, Fisher[8] has focused on the nonlinear dynamics above the onset of motion. He proposed, in analogy with static critical phenomenon, that a scaling behavior between force and velocity is expected for the onset of motion,

$$V \sim (F - F_{\rm c})^{\zeta}\,, \tag{1}$$

where $F_{\rm c}$ is the critical driving force and ζ is the critical exponent. Experimentally Bhattacharya and Higgins[9] found, in the layered superconductor 2H-$NbSe_2$, that the scaling fit is valid in the elastic regime and the critical exponent is the same as the measured value for the CDW.[10]

An actual dynamic phase transition between the plastically deformed phase and a moving lattice for the vortex lines was first proposed in the two-dimensional (2D) system of pancake vortices by Koshelev and Vinokur.[11] Their idea is that the random pinning noise diminishes with increasing vortex velocity, allowing a high-velocity reordering. Neutron scattering experiment[12] supported this idea.

Numerical simulations on the behavior of the vortices in an inhomogeneous environment help to establish a correlation between the microscopic defect structure and the macroscopic response of the system. Most simulations have been devoted to the 2D vortex system interacting with point pinning centers. Even this simple system can be used to describe the behavior in superconducting thin films or layered superconductors. In this paper we numerically investigate the flow behavior of the driven 2D vortex lattice with different densities of randomly distributed pointlike pinning centers. The subject is the evolution in velocity-force dependence (VFD), corresponding to the I-V charac-

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teristics of a type-Ⅱ superconductor in the mixed state.

The dynamic behavior of the vortex system is described by the overdamped equation[4]

$$\frac{\mathrm{d}\boldsymbol{r}_i}{\mathrm{d}t} = -\sum_{i\neq j}\nabla_i U_{\mathrm{v}}(\boldsymbol{r}_i-\boldsymbol{r}_j)-\sum_{j'}\nabla_i U_{\mathrm{p}}(\boldsymbol{r}_i-\boldsymbol{r}_{j'})+\boldsymbol{F}_{\mathrm{dr}}\,, \tag{2}$$

where $\boldsymbol{r}_{i,j}$ and $\boldsymbol{r}_{j'}$ are the vortex and pin coordinates, respectively, $U_{\mathrm{v}}(\boldsymbol{r})$ is the vortex-vortex interaction potential, $U_{\mathrm{p}}(\boldsymbol{r})$ is the vortex-pin interaction potential and $\boldsymbol{F}_{\mathrm{dr}}$ is the external driving force.

The vortex-vortex interaction potential is chosen as the repulsive logarithmic,

$$U_{\mathrm{v}}(\boldsymbol{r}) = \begin{cases} \left[\dfrac{r^2}{(10r_{\mathrm{v}})^2}-0.01\right]\ln\left(\dfrac{r}{r_{\mathrm{v}}}\right) & \text{if } r<r_{\mathrm{v}}, \\ 0 & \text{if } r>r_{\mathrm{v}}, \end{cases} \tag{3}$$

and the vortex-pin interaction is modeled by an attracting Gaussian potential,

$$U_{\mathrm{p}}(\boldsymbol{r}) = -\frac{r_{\mathrm{p}}}{2}\exp\left(-\frac{r^2}{r_{\mathrm{p}}^2}\right). \tag{4}$$

All the lengths will be measured with respect to the lattice constant a_0 of the ideal triangular lattice, which is related to the vortex density n_{v} by $\dfrac{\sqrt{3}}{2}a_0^2 = n_{\mathrm{v}}^{-1}$. The number of vortices is fixed at $N_{\mathrm{v}} = 100$. The cutting length r_{v} and the size of pinning centers r_{p} are chosen to be $r_{\mathrm{v}} = 2.8a_0$ and $r_{\mathrm{p}} = 0.2a_0$, respectively. We start with the ideal triangular lattice subject to periodic triangular boundary conditions. A lattice is gradually shifted over a spatial random set of pinning centers. We increase the driving force along the horizontal symmetry axis (x axis) to compute the average velocity in the x direction, $V_x = \dfrac{1}{N_{\mathrm{v}}}\sum_{i=1}^{N_{\mathrm{v}}}\boldsymbol{V}_i\cdot\hat{\boldsymbol{x}}$, and its differential, $\mathrm{d}V_x/\mathrm{d}F_x$. For each driving force our simulation performs 10^5 sweeps for equilibration and 2×10^5 for measurement.

In order to investigate systematically the influence of density of pinning centers to the VFD, the number of pinning centers N_{p}, is varied from 5 to 500. Results are plotted on logarithmic scales in Fig.1. One can see that, for very small values of N_{p} (curves of $N_{\mathrm{p}} = 5$ and $N_{\mathrm{p}} = 10$) only elastic deformations of the lattice occur and the VFD response is basically linear. With larger value of N_{p} (the curve of $N_p = 20$), there exists a critical driving force F_{c} ($F_{\mathrm{c}} \approx 0.16$ for $N_{\mathrm{p}} = 20$), below which the lattice is pinned. In the pinned regime the velocity is generated mainly by rearrangements of topological defects, producing a very slow and unsteady advance of the pinned lattice, which is characteristic of pinned elastic medium driven by a force.[13] This advance reduces with increasing values of N_{p} (comparing the curve of $N_{\mathrm{p}} = 20$ with the curve of $N_{\mathrm{p}} = 40$, one can see this point). This is in agreement with the viewpoint of Ni *et al.*[2], who investigated the static nonlinear magnetic response of the randomly pinned lattice using variational replica-symmetry-breaking theory[14] and pointed out that the stronger pinning strength makes the response smaller and therefore makes the vortex lines more difficult to move. When N_{p} is larger than certain value ($N_{\mathrm{p}} \approx 50$), the small advance of pinned lattice disappears below F_{c} and the vortices become permanently trapped. F_{c} increases with increasing values of N_{p}. This is consistent with the simulation results of Brass *et al.*[13], in which a Gaussian-like form for the vortex interacting potential was chosen and a molecular-dynamics-annealing method was used. The most prominent behavior of Fig.1 is that below certain value of N_{p} ($\sim$ 50) the velocity rises concave upwards from F_{c} and above which convex upwards. The concave and convex features are the typical depinning characteristics of the elastic and plastic media, respectively. In the elastic regime vortices are bundled together, forming different Larkin domains, and only large enough driving force can drive vortices within the same domain to depin coherently. Therefore, the velocity rises concave upwards from F_{c}. The depinning in the elastic medium can be described by the collective pinning theory. On the contrary, different vortices are trapped in different channels in the plastic regime, the onset of motion for the defective vortex lattice is qualitatively different from that in the elastic regime. Small driving force can cause different vortices to depin independently, therefore the velocity rises rapidly and convex upwards from F_{c}.

Using the methods of CDW systems, we attempt a scaling fit (Eq.(1)) above F_{c} in the elastic regime. For $N_{\mathrm{p}} = 20$ the apparent exponent $\zeta = 1.26\pm0.05$, as shown in Fig.2. This is close to the measured value for the CDW, consistent with the experimental result of Ref.[9]. The scaling fit extends to $F_x \sim 2$—$3F_{\mathrm{c}}$ and then deviates downwards. This is in agreement with the general feature for an elastic medium. However, the scaling fit can not be obtained in the plastic regime.

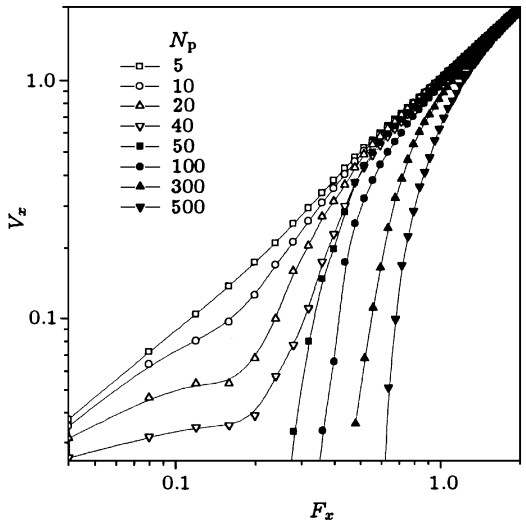


Fig.1. Series of VFD with different numbers of pinning centers.

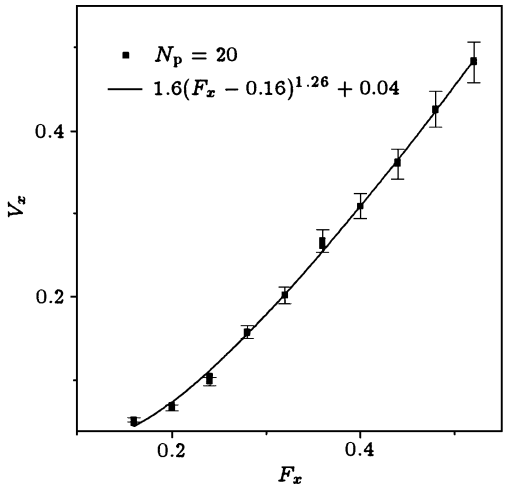


Fig.2. Scaling behavior above F_c in the elastic regime. The line marks the critical exponent $\zeta = 1.26$.

Another obvious behavior in the Fig.1 is, above F_c, a dramatic change in each curve of VFD with a pronounced inflection point from the nonlinear to linear regions. As the value of N_p is increased, the inflection point moves to large value of the driving force. The systematics of the evolution of these curves with increasing values of N_p is better illustrated by the differential curve dV_x/dF_x (corresponding to the differential resistance, $R_d = dV/dI$) in Fig.3. From Fig.3 one can notice that, in the elastic regime ($N_p < 50$), the dV_x/dF_x vs. F_x curve first shows a valley which becomes more deep with increasing values of N_p, and then shows a peak which increases with increasing values of N_p above F_c. The positions of valley and peak corresponds to that of depinning and inflection in the Fig.1, respectively. As pointed out in Ref.[3], the peak indicates occurrence of reordering of vortex motion at high driving forces. This supports the idea of Koshelev and Vinokur.[11] The peak value exceeds the normal resistance and thus does not represent R_f, the asymptotic flux flow value. Crossing over the peak, dV_x/dF_x decreases rapidly and monotonically to the asymptotic flux flow value, indicating that the vortex motion reorders rapidly. However, at entering the plastic regime ($N_p \geq 50$), the valley disappears at F_c and a much larger peak appears slightly above F_c. This is consistent with the recent simulated results.[3] Crossing over the peak, dV_x/dF_x does not decrease monotonically to the asymptotic flux flow value up to very large driving force. This indicates that there exists a defective motion even at very large driving force, consistent with the viewpoint of Olson *et al.*[3] Many "fingerprints" appear when one enters the plastic regime completely (see the curve of $N_p = 300$), in agreement with the recent experimental results.[15] The peak decreases with increasing values of N_p in the plastic regime. Transition from the elastic to plastic regimes accompanies maximum peaks in the curves of dV_x/dF_x vs. F_x, as shown in Fig.4, where we present the peak of dV_x/dF_x vs. N_p curve. One can see that the transition values of N_p from the elastic to plastic regimes are 50—100 for our system.

In conclusion, we have performed numerical simulations on the dynamics of the driven 2D vortex system with different densities of randomly distributed point-like pinning centers. Different features in the curve of velocity-force dependence have been found between the elastic and plastic regimes. Scaling fit between force and velocity above the critical driving force can be obtained in the elastic regime but fails in the plastic regime. Transition from the elastic to plastic regimes is accompanied by maximum peaks in the differential curves of velocity-force dependence in the disordered vortex lattice.

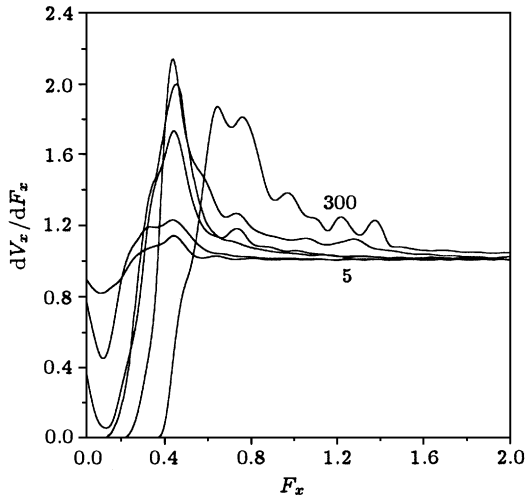


Fig.3. Evolution of the differential curve dV_x/dF_x with increasing values of number of pinning centers (down to up) N_p=5, 10, 40, 50, 100, 300.

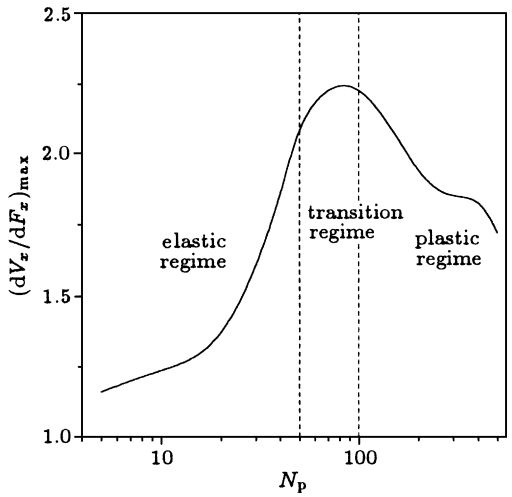


Fig.4. Peak in the differential curve dV_x/dF_x as a function of N_p.

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