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ANALYSES FOR LASING THRESHOLD AT 2 AND 1.5 μm WAVELENGTHS IN Tm:YVO_4 CRYSTAL*

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From transition-rate-equations and transmission-equations the formulas of threshold power are deduced for an end-pumped laser, in which the probability of cross-relaxation cannot be ignored and the nonradiative transition-probability of the excited state is the same order as the radiative transition-probability of upper level of the lasing transition. The lasing performance of Tm:YVO_4 at $\lambda = 1.9$ and $1.5\,\mu$m is discussed by dint of these equations of threshold power.

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I. INTRODUCTION

When the Tm^{3+} ions doped in a crystal are excited by pumping light, they will transit from the $^3H_6$ ground state to high level, then relax to $^3F_4$ level and transit from the $^3F_4$ level to the ground state in succession. A lasing light at $\lambda = 1.9\,\mu$m wavelength will be radiated out simultaneously during the transition $^3F_4 \rightarrow {}^3H_6$. This is a three-level lasing system. Since the width of the fluorescent spectrum of transition $^3F_4 \rightarrow {}^3H_6$ is about $0.4\,\mu$m, the tunable laser at near infrared band can be made. Because the waveband is different from that of a Cr: laser and Ti: Sapphire laser, Tm^{3+} laser is favorable.[1,2] Ermeneux *et al.*[3] studied the laser at $\lambda \sim 1.5\,\mu$m wavelength corresponding to the $^3H_4 \rightarrow {}^3F_4$ transition of the Tm^{3+} ions. This is the four-level system laser whose wavelength is in eye-safe band and in the window of atmospheric transmission. Therefore it will be applied to the surgery and communication potentially.

The absorption band of the rare-earth ions in YVO_4 host is broader than that in YAG host, thereby the performance of the YVO_4 laser overmatches the YAG laser. But due to the difficulty of the growth of long crystal rod and the inferior of the thermodynamics-performance, the application of YVO_4 crystal is restricted. The success to exploit laser diodes at near infrared band urges flourish development of completely solidified YVO_4 laser with compact configuration. Saito *et al.*[4] and Ermeneux *et al.*[3] have manufactured small size Tm:YVO_4 lasers with a microchip at $\lambda = 1.94$ and $1.48\,\mu$m, respectively, pumped by the Ti: Sapphire and the diode laser at $\lambda = 798$ nm. In this paper we will deduce the expression of the threshold of laser end-pumped, if the cross-relaxation between active ions exists. The study of the relation between the lasing threshold and parameters, such as the concentration of active ions, the length of the crystal rod, the reflectivity of output-mirror of the resonant cavity, etc., will help us design the laser.

II. THE BASIS OF THEORETICAL ANALYSES

The experiment[1] show that the slope efficiency of the Tm: YVO_4 laser at $\lambda = 1.9\,\mu$m is larger than the ratio of frequencies of lasing and pumping light (ν_s/ν_p). This implies that the cross relaxation between the $^3H_4 \rightarrow {}^3F_4$ and $^3H_6 \rightarrow {}^3F_4$ transitions cannot be neglected if the concentration of Tm^{3+} ions in the crystal is high. Because the energy difference between $^3H_4$ and $^3H_5$ states is larger, the magnitude of the probability of nonradiative transition of the $^3H_4$ level cannot count for much in the comparison with the probability of radiative transition of the $^3F_4$ level. Hence the expressions to estimate the threshold of three- and four-level laser system in Refs.[5,6] ought to be modified. If the wavelength of the pumping light is 800 nm, the kinetic process of excited Tm^{3+} ions is schematically shown in Fig.1. According to the Fig.1, the transition-rate-equations of the three- and four-level laser systems of Tm^{3+} ions corresponding to $^3F_4 \rightarrow {}^3H_6$ and $^3H_4 \rightarrow {}^3F_4$ transitions can be written as Eqs.(1) and (2) respectively.

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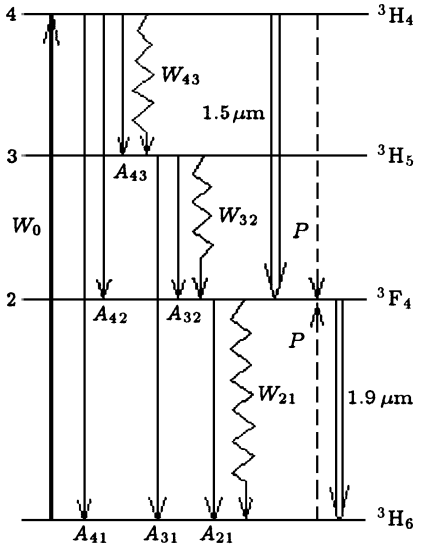


Fig. 1. Schematic of transitions of Tm^{3+} ions. ↓—— radiative, ⇂—— nonradiative, ⇓——lasing, ⇣—— cross-relaxation.

$$\begin{aligned}
\frac{\mathrm{d}N_4}{\mathrm{d}t} =& v\sigma_{\mathrm{ap}}\rho_{\mathrm{p}}(z)N_1 - (P+Q_4)N_4\,,\\
\frac{\mathrm{d}N_3}{\mathrm{d}t} =& Q_{43}N_4 - Q_3N_3\,,\\
\frac{\mathrm{d}N_2}{\mathrm{d}t} =& (2P+Q_{42})N_4 + Q_{32}N_3 - Q_2N_2\\
&+ v[\sigma_{\mathrm{as}}N_1 - \sigma_{\mathrm{es}}N_2][\rho_{\mathrm{s}}^+(z) + \rho_{\mathrm{s}}^-(z)]\,,\\
N_0 =& N_1 + N_2 + N_3 + N_4\,,
\end{aligned} \tag{1}$$

and

$$\begin{aligned}
\frac{\mathrm{d}N_4}{\mathrm{d}t} =& v\sigma_{\mathrm{ap}}\rho_{\mathrm{p}}(z)N_1 - (Q_4+p)N_4\\
&+ v[\sigma_{\mathrm{as}}N_2 - \sigma_{\mathrm{es}}N_4][\rho_{\mathrm{s}}^+(z) + \rho_{\mathrm{s}}^-(z)]\,,\\
\frac{\mathrm{d}N_3}{\mathrm{d}t} =& Q_{43}N_4 - Q_3N_3\,,\\
\frac{\mathrm{d}N_2}{\mathrm{d}t} =& Q_{42}N_4 + Q_{32}N_3 - Q_2N_2 + 2PN_4\,,\\
N_0 =& N_1 + N_2 + N_3 + N_4\,.
\end{aligned} \tag{2}$$

When the laser operates stably, above Eqs. can be rewritten as

$$\begin{aligned}
N_4 =& \frac{v\sigma_{\mathrm{ap}}\rho_{\mathrm{p}}}{Q_4'}N_1\,,\\
N_3 =& \frac{Q_{43}}{Q_3}N_4 = \frac{Q_{43}}{Q_3Q_4'}v\sigma_{\mathrm{ap}}\rho_{\mathrm{p}}N_1\,,\\
N_2 =& N_0 - N_1 - N_3 - N_4\\
=& N_0 - N_1 - \frac{Q_{43}+Q_3}{Q_3Q_4'}v\sigma_{\mathrm{ap}}\rho_{\mathrm{p}}N_1\,,\\
Q_4' =& Q_4 + P\,,
\end{aligned} \tag{3}$$

and

$$\begin{aligned}
N_3 &= \frac{Q_{43}}{Q_3}N_4\,,\\
N_2 &= \frac{(Q_{42}+2P)N_4 + Q_{32}N_3}{Q_2}\\
&= \frac{Q_{42}'Q_3 + Q_{32}Q_{43}}{Q_2Q_3}N_4\,,\\
N_1 &= N_0 - (N_3+N_4) - N_2\\
&= N_0 - \frac{(Q_{43}+Q_3)Q_2 + (Q_{42}'Q_3 + Q_{32}Q_{43})}{Q_2Q_3}N_4\\
&= N_0 - \frac{\alpha}{Q_2Q_3}N_4\,,\\
Q_4' &= Q_4 + P\,,\\
Q_{42}' &= Q_{42} + 2P\,,\\
\alpha &= (Q_{43}+Q_3)Q_2 + (Q_{42}'Q_3 + Q_{32}Q_{43})\,,
\end{aligned} \tag{4}$$

where N_0 is the concentration of active ions doped; N_i is the density of the populations at i level (i=1, 2, 3, 4); $Q_{i,j}$ is the probability of the transition from i to j level, $Q_i = \sum_j Q_{i,j}$; P is the cross-relaxation probability; v is the light speed in the crystal; $\rho(z)$ is the density of photon number at z; σ_{a} and σ_{e} represent the absorption and the efficient emission cross-section respectively. The superscripts of $\rho(z)$, "+" and "−", indicate the directions of lasing light as the same or contrary with the pumping light respectively.

The subscripts of $\rho(z)$, σ_{a}, σ_{e}, p and s, indicate the photons of the pumping light and the lasing-light respectively.

During the transmission process both the pump-light and lasing light are absorbed, but a stimulated radiation is emitted. For the three- and four-level laser systems of Tm^{3+} ions, transmission Eqs. of the photons can be written, respectively, as

$$\begin{aligned}
\frac{\mathrm{d}\rho_{\mathrm{p}}(z)}{\mathrm{d}z} &= -\sigma_{\mathrm{ap}}\rho_{\mathrm{p}}(z)N_1\,,\\
\frac{\mathrm{d}\rho_{\mathrm{s}}^+(z)}{\mathrm{d}z} &= (\sigma_{\mathrm{es}}N_2 - \sigma_{\mathrm{as}}N_1)\rho_{\mathrm{s}}^+(z)\,,\\
\frac{\mathrm{d}\rho_{\mathrm{s}}^-(z)}{\mathrm{d}z} &= -(\sigma_{\mathrm{es}}N_2 - \sigma_{\mathrm{as}}N_1)\rho_{\mathrm{s}}^-(z)\,,
\end{aligned} \tag{5}$$

and

$$\begin{aligned}
\frac{\mathrm{d}\rho_{\mathrm{p}}(z)}{\mathrm{d}z} &= -\sigma_{\mathrm{ap}}\rho_{\mathrm{p}}(z)N_1\,,\\
\frac{\mathrm{d}\rho_{\mathrm{s}}^+(z)}{\mathrm{d}z} &= (\sigma_{\mathrm{es}}N_4 - \sigma_{\mathrm{as}}N_2)\rho_{\mathrm{s}}^+(z)\,,\\
\frac{\mathrm{d}\rho_{\mathrm{s}}^-(z)}{\mathrm{d}z} &= -(\sigma_{\mathrm{es}}N_4 - \sigma_{\mathrm{as}}N)\rho_{\mathrm{s}}^-(z)\,.
\end{aligned} \tag{6}$$

The effective reflectivities of the input and output mirrors are expressed by R_1 and R_2, respectively (the one-way transmission loss of the cavity is included in R_2), then the boundary condition of lasing field in the resonant cavity can be written as

$$\begin{aligned} \rho_{\rm s}^{+}(0) &= R_1\rho_{\rm s}^{-}(0)\,, \\ \rho_{\rm s}^{-}(L) &= R_2\rho_{\rm s}^{+}(L)\,, \end{aligned} \tag{7}$$

where the input end $z=0$, output end $z=L$. L represents the length of the crystal rod. By Eqs.(5)—(7), the boundary condition can be rewritten as following

$$\begin{aligned} \rho_{\rm s}^{+}(0) &= DR_1^{1/2}, \quad \rho_{\rm s}^{+}(L) = DR_2^{-1/2}\,, \\ \rho_{\rm s}^{-}(0) &= DR_1^{-1/2}, \quad \rho_{\rm s}^{-}(L) = DR_2^{1/2}\,, \end{aligned} \tag{8}$$

where D is a constant, $D^2=\rho_{\rm s}^{+}(z)\rho_{\rm s}^{-}(z)$.

According to the definition of gain $G(\lambda)$ for lasing light at $\lambda\sim 1.9\,\mu$m

$$G(\lambda)^{(3)} = \int_0^L (\sigma_{\rm es}N_2 - \sigma_{\rm as}N_1){\rm d}z\,, \tag{9}$$

and for lasing light at $\lambda\sim 1.5\,\mu$m

$$G(\lambda)^{(4)} = \int_0^L (\sigma_{\rm es}N_4 - \sigma_{\rm as}N_2){\rm d}z\,. \tag{10}$$

Combining Eqs.(1)—(6), we obtain, respectively,

$$\begin{aligned} G(\lambda)^{(3)} =& \sigma_{\rm es}N_0L + \frac{\sigma_{\rm es}+\sigma_{\rm as}}{\sigma_{\rm ap}}\ln\left[\frac{\rho_{\rm p}(L)}{\rho_{\rm p}(0)}\right] \\ & - \frac{v(Q_3+Q_{43})\sigma_{\rm es}}{Q_4'Q_3}[\rho_{\rm p}(0)-\rho_{\rm p}(L)]\,, \end{aligned} \tag{11}$$

$$\begin{aligned} G(\lambda)^{(4)} =& \frac{\sigma_{\rm es}Q_2Q_3+\sigma_{\rm as}(Q_{42}'Q_3+Q_{32}Q_{43})}{Q_{43}Q_2+Q_{42}'Q_3+Q_2Q_3+Q_{32}Q_{43}} \\ & \left[N_0L+\frac{1}{\sigma_{\rm ap}}\ln\frac{\rho_{\rm p}(L)}{\rho_{\rm p}(0)}\right]. \end{aligned} \tag{12}$$

When the laser operates stably, $G(\lambda)$ is equal to the one-way power loss-factor δ, namely

$$G(\lambda)=\delta=-\ln R=-\ln\sqrt{R_1R_2}\,. \tag{13}$$

Inserting Eqs.(11) and (12) into the Eq.(13), and let $\ln\left[\dfrac{\rho_{\rm p}(L)}{\rho_{\rm p}(0)}\right]=-\Psi(\lambda)$, then we obtain, respectively,

$$\begin{aligned} \frac{\Psi(\lambda)^{(3)}}{\sigma_{\rm ap}}&\left[\sigma_{\rm es}+\sigma_{\rm as}-\sigma_{\rm es}\frac{(Q_3+Q_{43})Q_2}{Q_4'Q_3\beta}\right] \\ &= \sigma_{\rm es}N_0L\left[1-\frac{(Q_3+Q_{43})Q_2}{Q_4'Q_3\beta}\right]+\ln R \end{aligned} \tag{14}$$

and

$$\begin{aligned} \Psi(\lambda)^{(4)} =& \sigma_{\rm ap}\left[\left(\frac{Q_4'Q_3\beta}{\sigma_{\rm es}Q_2Q_3-\sigma_{\rm as}(Q_{42}'Q_3+Q_{32}Q_{43})}\right)\right. \\ & \left.\ln R+N_0L\right], \\ \beta =& \frac{\alpha}{Q_4'Q_3}=\frac{Q_3Q_{42}'+Q_{32}Q_{43}+Q_2(Q_3+Q_{43})}{Q_4'Q_3}\,. \end{aligned} \tag{15}$$

Inserting Eq.(11) into Eq.(9) and Eq.(12) into Eq.(10), and then utilizing the boundary condition of lasing field (8) we obtain, respectively,

$$\begin{aligned} \rho_{\rm s}^{+(3)}(L) =& \frac{\sqrt{R_1}}{(1-R)(\sqrt{R_1}+\sqrt{R_2})}\left\{\frac{Q_2}{v\sigma_{\rm ap}}\Psi(\lambda)^{(3)}\right. \\ & \left.-\frac{Q_2}{v}N_0L+\rho_{\rm p}(0)[1-\exp(-\Psi(\lambda)^{(3)})]\beta\right\} \end{aligned} \tag{16}$$

and

$$\begin{aligned} \rho_{\rm s}^{+(4)}(L) =& \frac{\sqrt{R_1}}{(1-R)(\sqrt{R_1}+\sqrt{R_2})}\left\{\frac{Q_2Q_3Q_4'}{\alpha v}\left[N_0L\right.\right. \\ & \left.\left.-\frac{\Psi(\lambda)^{(4)}}{\sigma_{\rm ap}}\right]-\rho_{\rm p}(0)[1-\exp(-\Psi(\lambda)^{(4)})]\right\}. \end{aligned} \tag{17}$$

According to the definition of the output power of the lasing light:

$$P_{\rm out}=Ah\nu_{\rm s}v\rho_{\rm s}^{+}(L)(1-R_2)\,,$$

where A represents the cross-section of the facula; h is Plank constant, $\nu_{\rm s}$ is the lasing frequency. When the output power of the lasing light is zero, the power of the pump light equals to the lasing threshold power. Then the lasing threshold power at $\lambda=1.9\,\mu$m is

$$\begin{aligned} P_{\rm th}^{(3)} =& Avh\nu_{\rm p}\rho_{\rm p}(0)^{(3)} = Ah\nu_{\rm p}\left[N_0L-\frac{\Psi^{(3)}(\lambda)}{\sigma_{\rm ap}}\right]\Big/ \\ & \left\{\left[1-\exp(-\Psi^{(3)}(\lambda))\right]\beta\right\}. \end{aligned} \tag{18}$$

By Eqs.(12) and (17) the lasing threshold power of Tm^{3+} ions at $\lambda=1.5\,\mu$m can be written as

$$\begin{aligned} P_{\rm th}^{(4)} =& \frac{Ah\nu_{\rm p}Q_2Q_3Q_4'(-\ln R)}{Q_2Q_3\sigma_{\rm es}-\sigma_{\rm as}(Q_{42}'Q_3+Q_{32}Q_{43})}\Big/ \\ & [1-\exp(-\Psi(\lambda)^{(4)})]\,. \end{aligned} \tag{19}$$

The superscripts of $G(\lambda)$, $P_{\rm th}$, $\rho_{\rm s}^{+}(L)$ and $\Psi(\lambda)$, (3) and (4), indicate the three- and four-level laser system respectively.

Ⅲ. THE CALCULATION RESULTS AND ANALYSIS

A. The calculation results

The optical parameters in threshold Eqs.(18) and (19) can be obtained from Refs.[3,7,8]. For the sake of convenience these parameters are listed in the Table 1. The superscripts of σ_a and σ_e in the Table 1, (3) and (4), indicate the three- and four-level laser system respectively. Let $R_1 = 0.99$ and $R_2 = 0.94$ to compare with the experimental values in Ref.[2]. Inserting the values of R_1 and R_2 and the parameters in the Table 1 into Eqs.(18) and (19), we calculate out the relations between the laser threshold P_{th} and the length of the crystal rod L (here the relative concentration of the doped ions $x = 5\,\text{at}\%$ is assumed) and between the laser threshold P_{th} and the relative concentration of the doped ions (here the length of the crystal rod $L = 1\,\text{cm}$ is assumed). These are schematically shown in Fig.2. For the sake of the illuminating the effect of the cross relaxation, the relations between P_{th} and x (or L) when the cross relaxation is absence are shown in Fig.2 also.

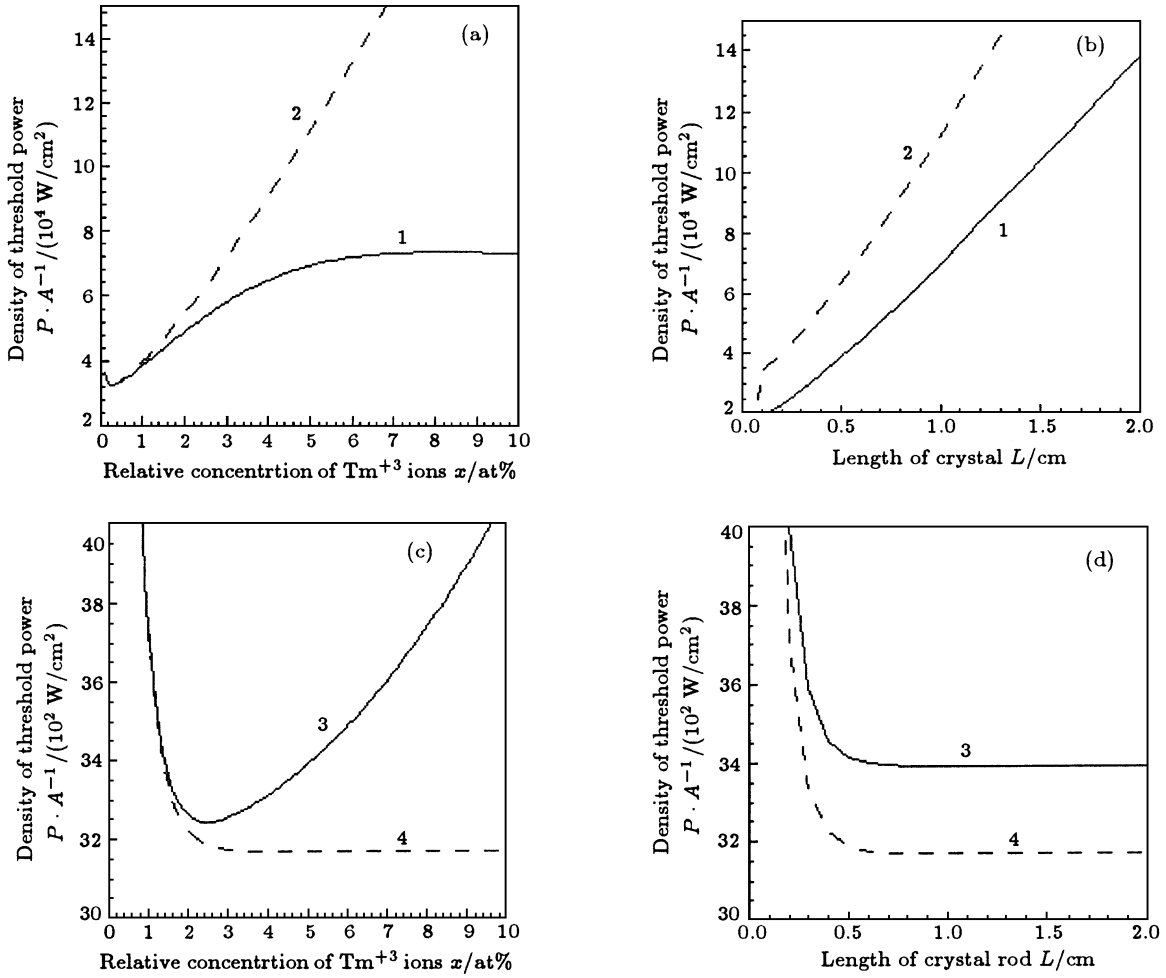


Fig.2. The laser threshold P_{th} versus the length of the crystal rod L (here the relative concentration of the doped ions $x = 5\,\text{at}\%$ is assumed) and the relative concentration of the doped ions x (here the length of the crystal rod L=1 cm is assumed). The lasing wavelength is 1.9 μm in Fig.(a) and (b), λ_s=1.5 μm in Fig.(c) and (d).

Table 1. The optical parameters of Tm^{3+} ions in YVO_4 host.

The absorption and efficiency emission cross-section /10^{-20} cm^2					The probability of transition[7]/s^{-1}						The probability of cross-relaxation /s^{-1}
σ_{ap} [8]	$\sigma_{es}^{(3)}$ [8]	$\sigma_{es}^{(4)}$ [3]	$\sigma_{as}^{(3)}$ [8]	$\sigma_{as}^{(4)}$ [3]	Q_4	Q_{43}	Q_{42}	Q_3	Q_{32}	Q_2	P[7]
1.7	2.4	0.7	2.2	~ 0	2.9×10^3	50	188	3.01×10^3	2.97×10^3	294	$8.24 \times 10^4 x^2$

The one-way transmission loss which is proportionate to the length of the crystal rod is included in the reflectivity of output mirror in Eqs.(19) and (18), so the curves in Figs.2(b) and 2(d) neglect the change of the transmission loss with the length of the crystal rod. This is reasonable if the optical quality of the crystal is good. The population of lower level of lasing transition in four-level system is even less than that in three-level system, so that absorption loss even less also. Nevertheless it is the most important factor depended upon crystal length, so the density of the threshold power tends to be independent of the crystal length in four-level system if crystal length is longer than a certain value.

B. The comparison between calculated and the experimental thresholds of Tm^{3+} laser at $\lambda = 1.9\,\mu$m

For the sake of the comparison with experimental threshold in Ref.[2], the value calculated by Eq.(18) and the experimental results are all reported in Table 2. The parameters in Eq.(18) have been listed in Table 1. The area of a facula A is calculated with $\pi\omega_p^2/4$ (ω_p is the diameter of the beam-waist of the pump-light). The experimental result is smaller than the calculated threshold. Because the beam-waist is at the front of the sample, so the area of a facula A is larger than $\pi\omega_p^2/4$. It is known from Fig.1 in Ref.[2]. From Table 2 it can be seen that the larger the beam-waist, the more approximate to the experimental result the calculated value. Because for larger beam-waist, the focal length of the lens is longer, i.e., the emanative angle is smaller, but the distance between the sample and the beam-waist of the pump-light is not changed approximately. Namely the larger the beam-waist, the smaller the difference between the area of a facula A and $\pi\omega_p^2/4$, thus the calculated value is closer to the experimental results.

Table 2. The comparison between the Tm^{3+} lasing threshold obtained by calculation and experiment at $\lambda = 1.9\,\mu$m.

L/cm	W_p/μm	P_{th}/W (experimental results)	P_{th}/W (calculated value)
0.1	24.6	0.3	0.1
0.1	33.3	0.28	0.18
0.1	47.5	0.42	0.37
0.15	17.6	0.3	0.05
0.15	24.6	0.36	0.11
0.15	47.5	0.44	0.41

C. The comparison with the threshold Eqs. in Refs.[5,6]

In Refs.[5,6] an approximation that the probability of nonradiative transition of Tm^{3+} ions from active state to most near low-level is much larger than the probability of radiative transition of upper state of lasing transition (namely $n_1 + n_2 = n_0$ and $n_1 + n_2 + n_3 = n_0$) was made. Comparing threshold Eqs. in this paper with one in Refs.[5,6], we can find that added β is the primary difference.

In 2 at%Er:YAG Er^{3+} ions are excited to fourth level $^4I_{9/2}$ by Kr^+ laser at $\lambda = 647$ nm and emit the lasing light corresponding to $^4I_{13/2} \rightarrow {}^4I_{15/2}$. The added β is equal to 1.22. In other instance of 1 at%Nd:YAG, that Nd^{3+} ions are excited to $^2G_{9/2}$ level by Ar^+ laser at $\lambda = 514$ nm and emit the lasing light at $\lambda = 1.06\,\mu$m. The probabilities of nonradiative transitions of every level above $^4F_{3/2}$ are all larger, so added $\beta \cong 1$. Therefore the approximations ($n_1 + n_2 = n_0$ and $n_1 + n_2 + n_3 = n_0$) are reasonable in Refs.[5,6]. The calculated threshold is 120 mW in Ref.[5], and the experimental result is 200 mW in Ref.[9]. If the threshold is calculated with Eq.(18) in this paper, it is 150 mW which is more close to the experimental result. In the Tm:YVO_4 the probability of nonradiative transition W_{43} is a little larger than Q_{21}, and there is the cross relaxation between the transitions $^3H_4 \rightarrow {}^3F_4$ and $^3H_6 \rightarrow {}^3F_4$, thus $\beta = 0.3679$ calculated according to Eq.(15) if $x = 5$ at%, and then the approximation ($n_1 + n_2 = n_0$) doesn't evidently adapt to 5 at%Tm:YVO_4 crystal.

The broken line in Fig.2 is the case where the cross relaxation doesn't be considered. If the concentration of Tm^{3+} ions doped is lower, the distance between Tm^{3+} ions in the crystal is farther, thus the effect of the cross relaxation between Tm^{3+} ions is faintish. Thereby the solid line where the cross relaxation is considered superposes nearly with the broken line. If the concentration of Tm^{3+} ions doped increases, the cross relaxation also increases. Thereby the error caused from ignoring the cross relaxation becomes larger along with the increasing of Tm^{3+} concentration. The threshold augments along with the Tm^{3+} concentration generally. But the cross relaxation corresponding to $^3H_4 \rightarrow {}^3F_4$ and $^3H_6 \rightarrow {}^3F_4$ transitions brings geminate augment of the population of $^3F_4$ state, which is upper state of lasing light at $\lambda = 1.9\,\mu$m. Consequently as shown in Fig.2(a), if $x > 8\%$ the threshold decreases instead along with

the increasing of Tm^{3+} concentration, although the augment of the concentration of Tm^{3+} ions increases the excited absorption. But the lasing threshold at $\lambda = 1.5\,\mu$m unceasingly increases along with the concentration of Tm^{3+} ions, because the cross relaxation increases the population of the lower level of the lasing transition. It is disadvantageous to the reverse population. If the concentration of Tm^{3+} ions is lower it can be seen from the Eqs.(14) and (15) that $\Psi(\lambda)$ decreases along with the decrease of the concentration N_0 of Tm^{3+} ions doped, thus the threshold increases exponentially along with $[-\Psi(\lambda)]$ as Eqs.(18) and (19). Thereby the optimum concentration x is 2 at% approximately as Fig.2(c) (namely the lasing threshold at $\lambda = 1.5\,\mu$m is smallest if $x = 2\,at\%$ and $L = 1$ cm). It is approach to the concentration of Tm^{3+} ions (1 at%) in the crystal used by Ermeneux.[3]

Because the cross relaxation relates only with the concentration of ions doped in definite crystal, so that the threshold vs. the length of the crystal rod doesn't change by ignoring the cross relaxation as shown in Figs.2(b) and 2(d).

D. The comparison of Tm^{3+} to Er^{3+} lasing lights at $\lambda = 1.5\,\mu$m

The power density of lasing threshold in 2%Tm:YVO_4 crystal at $\lambda = 1.5\,\mu$m (if cavity length is 1 cm) is about 3300 W·cm^{-2} known from Fig.2(c). For 2%Er:YVO_4 crystal with the same length excited by Kr^+ laser at $\lambda = 647\,\mu$m, the power density of lasing threshold of the transition $^4I_{13/2} \rightarrow {}^4I_{15/2}$ is calculated as 2.7×10^4 W·cm^{-2} from Eq.(18). The threshold of Tm^{3+} laser at $\lambda = 1.5\,\mu$m is about 1 order smaller than that of Er^{3+} due to the four-level lasing system. By utilization of codoped Tb^{3+} or Ho^{3+} ions the population of the lower level $^3F_4$ of transition of Tm^{3+} laser at $\lambda = 1.5\,\mu$m can be exhausted through the energy transfer to Tb^{3+}[3] or Ho^{3+}[10]. Thus the threshold is minified further.

The expressions of lasing threshold in this paper deduced from the transition-rate-equations and the transmission equations of pumping and lasing lights in the three- and four-level laser systems can commendably express the lasing performance of crystal. Here the cross relaxation between the doped ions is calculated and the probability of nonradiative transition of excited state cannot count for much as compared with the probability of radiative transition of upper level of the lasing transition. These expressions are valuable for the determination of the parameters to design the laser, such as the concentration of doped ions, the length of the crystal rod and the reflectivity of the output mirror, etc.

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