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FRINGE STRUCTURE IN THE PHASE-SPACE DYNAMICS OF ATOMIC STABILIZATION IN AN INTENSE FIELD*

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An analytical expression of a Floquet operator, which describes the evolution of a wave packet in combined atomic and an intense laser field, is obtained approximately in the stabilization regime. Both the classical and quantum versions of the Floquet operator are used to study the phase-space dynamics of atomic stabilization, and the "fringe structure" in the phase-space is clearly demonstrated. Furthermore, we investigate the dynamical mechanism and characters of this striking structure, and find that this structure is very closely related to the classical invariant manifolds.

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I. INTRODUCTION

Atomic stabilization means that the ionization rate of an atom decreases with increasing intensity of high-frequency intense laser field, which is one of the most interesting nonperturbative phenomena observed in laser-atom interaction and attracts much attention recently. This effect has been predicted by both quantum theories[1] and classical theories[2] and has recently been verified experimentally.[3,4] In particular, Sundaram and Jensen[5] connect successively this exciting phenomenon with the chaotic dynamics.

More recently, by numerically solving time-dependent Schrödinger equation for one-dimensional model, some researchers have investigated the phase-space dynamics of intense-field stabilization and found the existence of a striking structure-fringe structure.[6,7] These findings enrich greatly the discussions on atomic stabilization and helpful to better understand the stabilization mechanism. However, the dynamical mechanism of the fringe structure is far from fully understood.

In this paper, using the approximate analytic expression of the evolution operator of the atom-laser system, we shall demonstrate the fringe structure from classical distribution and quantal Wigner distribution. Furthermore, in light of nonlinear dynamics we shall discuss the dynamical mechanism underlying this striking structure. As we shall see later that, the fringe structure associates closely with the ionization process in the regime of stabilizatin.

The one-dimensional model is widely used in studying the atom stabilization phenomenon[1,2] in the strong-field laser-atom physics, its Hamiltonian is

$$H(x,v,t) = \frac{v^2}{2} + V(x) - xF\cos\omega t,$$
$$V(x) = -1/\sqrt{1+x^2}\,, \qquad (1)$$

where F, ω are the strength and frequency of the oscillating electric field, respectively. For all calculations in this paper, the atomic units ($m_e = e = \hbar = 1$a.u.) is adopted for simplicity.

Usually, the stabilization phenomenon is discussed in the Kramers-Henneberger (KH) frame, i.e., the laser-atom interaction is viewed from the stand point of a free classical electron oscillating in a lasert field. By this way, the Hamiltonian becomes

$$H_{\rm KH}(q,p,t) = \frac{p^2}{2} + V(q - \alpha_0\cos\omega t)\,, \qquad (2)$$

where $\alpha_0 = F/\omega^2$ is the amplitude of an electron's quiver motion. Notice that the wave function in the KH frame is related to that in the length gauge by an unitary transformation.

This is a periodically driven system. With the help of a time-order exponential, the formal evolution operator at $t > 0$ can be written as,

$$\widehat{U}(t) = P\left[\exp\left(-\mathrm{i}\int_0^t \mathrm{d}t'\widehat{H}_{\rm KH}(t')\right)\right], \qquad (3)$$

where P is the time order operator.

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The time evolution operator referring to one period T is the so-called Floquet operator

$$\widehat{F} = \widehat{U}(T)\,. \tag{4}$$

If the frequency of the laser field is assumed to be asymptotically high, we can discard all but the time average of the KH potential, namely,

$$V_0(q) = \frac{1}{T}\int_0^T V(q-\alpha_0\cos\omega t)\mathrm{d}t\,. \tag{5}$$

Then the Hamiltonian becomes time independent and takes the form of

$$\overline{H}_{\mathrm{KH}} = \frac{p^2}{2} + V_0(q)\,. \tag{6}$$

The eigenstates of this system is called KH states. So one is led to the prediction that all KH states will be stable. However, in practical case of finite field frequency rather than an asymptotically high frequency, Sundaram and Jensen pointed out that the high order Fourier terms are also important and cannot be neglected for simplicity. With the assumption that every coefficients of the Fourier expansion of $V(q-\alpha_0\cos\omega t)$ are equally important, the potential can be described by a periodic train of delta kicks as follows,

$$H_{\mathrm{KH}} = \frac{p^2}{2} + V_0(q)\sum_{n=-\infty}^{+\infty}\delta\left(\frac{t}{T}-n\right)\,. \tag{7}$$

In Ref.[5] the validity of the above assumption has been tested for a wide range of parameters. It is found that in the regime of atomic stabilization the approximation can provide a good qualitative description. Provided that the zero of time is chosen halfway between two consecutive kicks, in terms of Eqs.(3) and (7), the Floquet operator is

$$\widehat{F} = \widehat{R}\circ\widehat{K}\circ\widehat{R}\,, \tag{8}$$

where

$$\widehat{R} = \exp(-\mathrm{i}p^2T/4) \tag{9}$$

represents the half-periodic time evolution of a free electron, and

$$\widehat{K} = \exp(-\mathrm{i}V_0T) \tag{10}$$

represents an impulse on momentum.

The above evolution operator describes the interaction between field and atoms. It is easy to show that the above operator is time-reversal invariant.

Ⅱ. CLASSICAL DESCRIPTION OF IONIZATION PROCESS

The classical version of the above quantum Floquet operator (Eq.8) takes the form

$$M = R\circ K\circ R\,, \tag{11}$$

where $R : (q,p) \to (q + pT/2, p)$, propagates the trajectory for a half-period ahead along the constant momentum; $K : (q,p) \to \left(q, p - T\dfrac{\partial V_0}{\partial q}\right)$, describes an impulse on the electron. Obviously, M is area-preserving in phase plane.

By iterating M for a variety of initial conditions and plotting the trajectories in the phase plane (q,p), we can construct a Poincare surface of section which provides an overview of the classical dynamics. The phase plots of M demonstrate many different structures for different parameters F and ω. A detailed analysis show that, for the case of stabilization the phase structures can be classified into three main cases as we show in Figs.1,2 and 3. Case one, as shown in Fig.1, the P_1(0,0) is the unique 1-periodic elliptic fixed point. Outside the regular island around P_1, the

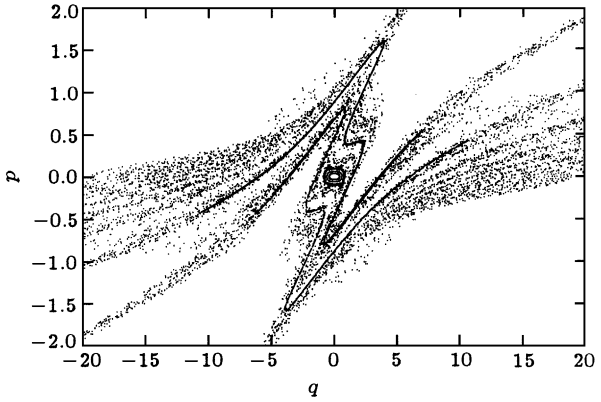


Fig.1. Poincare surface of section for $F = 1.28$ and $\omega = 0.8$ (case 1). Solid line gives the unstable manifolds.

phase plane is filled with chaotic trajectories. In case two (Fig.2), with changing the parameters, this elliptic point P_1 undergoes a pitch bifurcation to an unstable hyperbolic fixed point and gives birth to a pair of 1-periodic elliptic fixed points (P_2, P_3) located at ($\pm\alpha_0$, 0) approximately. Outside these two regular islands the phase plane is full of stochastic orbits. Case three (Fig.3), the P_2 and P_3 becomes unstable, and the phase plane is mainly filled by chaotic trajectories and very small fraction of regular islands. In terms of classical description of ionization, the electrons initiated in the islands will never be ionized. The regular islands around high-periodic fixed points are too small compared with that of 1-periodic fixed points and contribute little to the confinement of the electrons. Therefore, the above analyses provide three

typical parameters (F, ω) which are used in following discussions on ionization suppression.

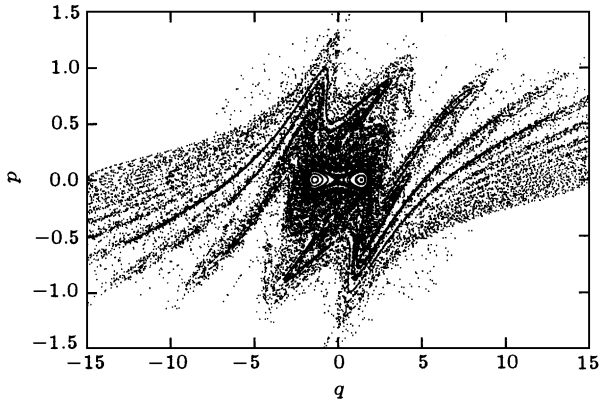


Fig.2. Poincare surface of section for $F = 5$ and $\omega = 1.34$ (case 2). Solid line gives the unstable manifolds.

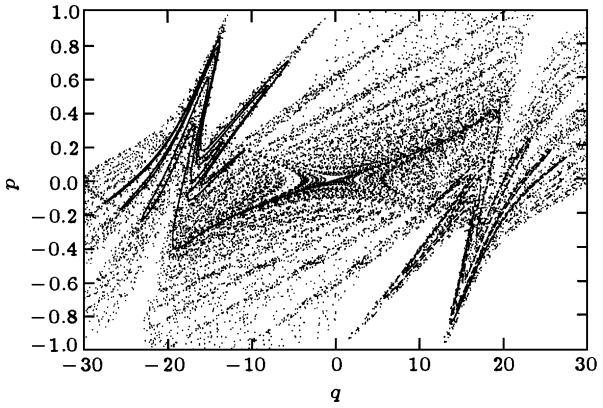


Fig.3. Poincare surface of section for $F = 5$ and $\omega = 0.52$ (case 3). Solid line gives the unstable manifolds.

In addition to those fixed points, KAM tori and Cantori, another striking structures commonly existing in Figs.1,2 and 3 are the fringe pattern in the lower and upper momentum plane. This is so called "fringe structure", first observed in classical simulations of atomic behavior in the stabilization regime by Grobe and Law[8], and recently confirmed by Watson *et al.* in their quantum Wigner functions of quantum description.[6,7]

In view of nonlinear dynamics, the unstable manifold of a steady state is a set of points X such that the orbit going backward in time starting from X approaches this steady state. The stable manifold can be similarly defined for the forward orbit. A stable manifold cannot intersect other stable manifolds, and neither do the unstable manifolds. However, as chaos occur, stable and unstable manifolds can intersect with each other. These invariant manifolds characterize the dynamical properties of the whole system.[9] Therefore, we conjecture that these fringe structures are relics of the unstable invariant manifolds. To identify it, we employ a numerical algorithm (see e.g. Refs.[10,11]) to simulate those unstable manifolds for a rather long time. The results are plotted in the same figure so as to be compared with the fringe structures. The correspondence between the fringe structures and unstable manifolds is obviuos.

In case three, the invariant manifolds are obtained according to the 1-periodic hyperbolic point (0,0). However, in case one and case two, things are much more complicated. Through careful investigations, we find that, in case one, unstable manifolds of 4-periodic saddles ($P(0.4449,\ -0.61567)$, $MP, \cdots, M^3P$) surrounding the stable island associated with the 1-periodic orbit organize the "fringe structure"; in case two, the unstable manifolds of period-6 hyperbolic points ($P(-1.1761,\ -0.6032)$, $MP, \cdots, M^5P$) around the fixed point (0,0) construct the fringe pattern. Because of the symmetry of the map M, the stable manifolds and unstable manifolds are symmetric about x axis. This fact means that the stable and unstable manifolds intersect with each other for infinite times and form a hyperbolic invariant set. Therefore, in the regime of stabilization the classical ionization process is characterized by chaos.

To deeply understand the phase dynamics of ionizaton process in the classical description, we trace 10000 trajectories which initiate on a piece-line of the phase plane described by $x = 0$ and $p \in (-0.5, 0.5)$. The phase plane representation of the time-evolution of this piece-line is shown in Fig.4. It is clear that a classical particle is attracted by the unstable manifolds and comes close to them along the stable manifolds, meanwhile it moves in the unstable direction determined by the unstable manifolds. Therefore, the whole piece-line is folded and stretched along the unstable manifolds and demonstrates a complex geometric configuration. Thus, a particle has more probability to stay near the unstable manifolds, this is the reason for the existence of fringes and those gaps between them.

From the above analyses, we conclude that unstable manifolds play an important role in ionization process and is the dynamical source of the striking fringe structure in the classical phase plane.

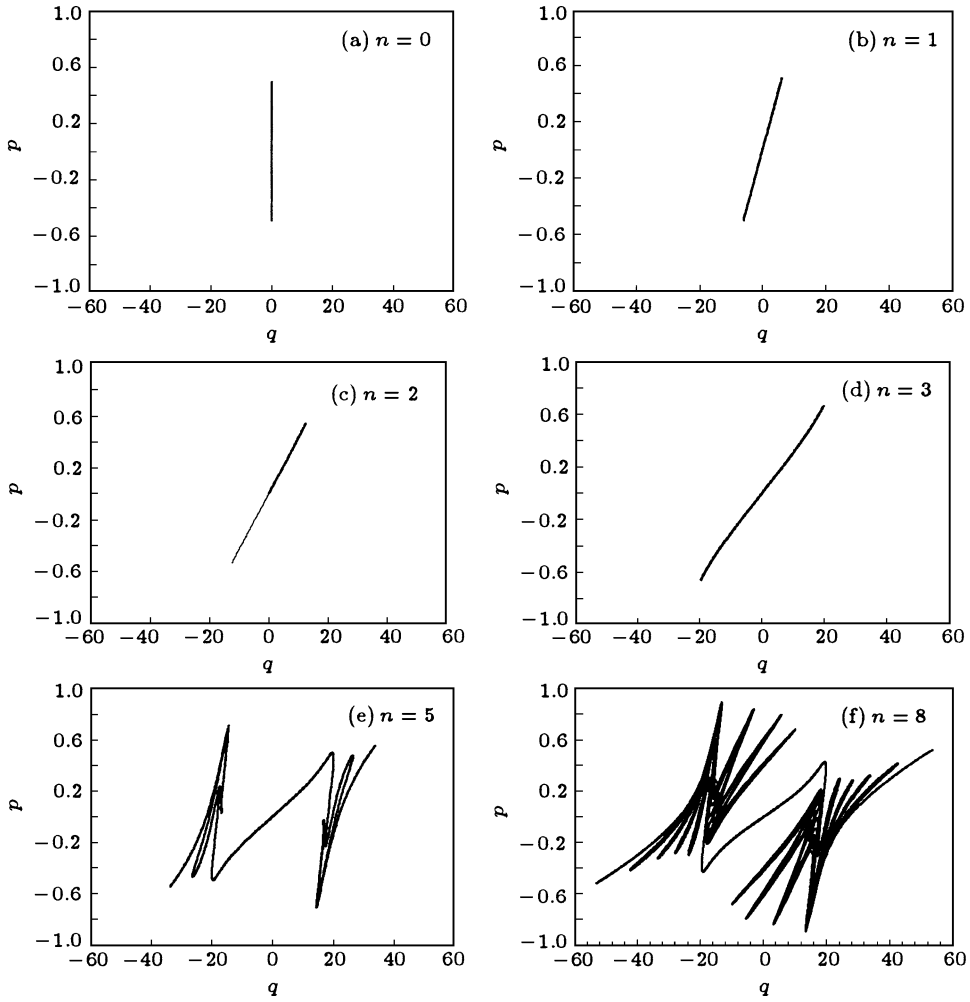


Fig.4. Phase plots of the n-cycle dynamical evolution of 10000 trajectories initiated in a piece-line for $F = 5$, $\omega = 0.52$.

Ⅲ. QUANTUM SIGNATURE

A stable state commonly refers to a quantal state of atom irradiated by laser with long lifetime. As the stabilization phenomenon is first discovered in the case of asymptotically high-frequency field, the eigenstates of the time-average Hamiltonian (KH states) are assumed to be stable states.

However, in order to expect such phenomenon to persist in the face of various experimental realities and can be realistically pursued in the laboratory, the field frequency ω should be a concrete and finite rather than asymptotically high. In this case, as was pointed out by Sundaram and Jensen, the high-order fourier coefficients of the expansion of time-dependence Halmiltonian is also important. Therefore, the quasienergy eigenstates of the time evolution operator $\widehat{U}(T)$ will act as the candidates of the stable states which result in the ionization suppression.

The Floquet operator is unitary and satisfies following eigenvalue equation,

$$\widehat{U}(T)|\,\Psi_\lambda\rangle = \mathrm{e}^{-\mathrm{i}\lambda}|\,\Psi_\lambda\rangle\,, \tag{12}$$

where the eigenphase λ is real, λ/T is the quasienergy (QE). The quasienergy wave function Ψ_λ describes QE state. These QE states are obtained by diagonalizing $\widehat{U}(T)$ with a large basis of plane waves, $|n\rangle = |\mathrm{e}^{-\mathrm{i}p_n x}\rangle$ $(n = 1, 2, \cdots, 512)$.

For the operator $\widehat{U}(T)$ is time-reversal invariant, let the time-reversal operator acts on the both sides of eigenvalues Eq.(12), we find that Ψ_λ and Ψ_λ^* are both eigenfunctions with the same quasienergy. In general, the bounded states are not degenerate for one-dimensional quantal system. It is evident that potential $V_0(q)$ is symmetric about zero point. Therefore we conclude that the bound QE states can be described by real functions and parity is a good quantum number.

In following section, we shall take the second case, where the parameters are $F = 5$, $\omega = 1.34$, as an ex-

ample for our discussion. We calculate the lowest two eigenfunctions and find that the ground state (QE0) possesses even parity and mainly concentrats in the zero point; The 1-excited state (QE1) with odd parity demonstrates dichotomous collection at $\pm\alpha_0$.

We use the wave packet propagation method to calculate the evolution of the bound QE states. In this approach, an initial wave packet is propagated by the Fourier spectral method, which is directly applicable to the free propagation step of the Floquet operator, with the impulse delivered once every period. The free time propagation step is divided into many small intervals to reduce the error introduced by boundary conditions. This kinetic propagation is carried out in the momentum space, since time evolution reduces to simple multiplication in that space. The impulsive step is performed in coordinated space for a similar reason. A Fast Fourier Transform (FFT) routine is used to transform the wave function between these two spaces. Since the ionization occurs in our problem, an absorptive filter in the asymptotic region ($> \pm 50\alpha_0$) is used to avoid the unphysical reflection of the wave packet from the grid boundary once it is on its way out. The filter takes the form[12]

$$f(x) = 1/(1 + \mathrm{e}^{\beta(x-a)}), \tag{13}$$

where a is a large distance and β is the width of the filter, the wave function is multiplied by the filter function after each time step.

The time evolution of ground and 1-excited QE states for the case two are plotted in Fig.5, they show

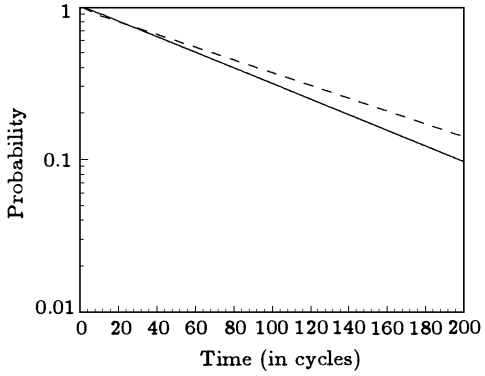


Fig.5. Survival probability on a logarithmic scale as a function of time (in optical cycles) for $F = 5$ and $\omega = 1.34$. The solid and dotted lines represent the decay of norm of initial QE0 and QE1 states, respectively, which are calculated from quantum map.

a good straight line which indicates an exponential decay $\mathrm{e}^{-\gamma t}$. The ionization rate γ of the corresponding QE states is well defined and relates to the extent of the stability of the QE states.

To study the phase-space evolution of a wave packet, we use the Wigner function which is defined by

$$W(q,p,t) = \int_{-\infty}^{+\infty} \phi(q - \tau/2, t) \phi^*(q + \tau/2, t)\mathrm{e}^{\mathrm{i}p\tau}\mathrm{d}\tau \tag{14}$$

for a given wave function $\phi(q,t)$. This Wigner function is formally analog to the classical probability density on phase space and reduces to it in the limit $\hbar \to 0$.

In the exponential decay regime, the Wigner function is time invariant except for a decay factor. Therefore, we plot the full Wigner function (a) and its contour plot (b) after 20 cycles of evolution in Figs.6 and 7. The most striking feature of this plot is the "fringe structure" observed in the upper and lower region of the figures. We also use the term "fringe" to describe the striking structure observed in the Wigner functions following Watson *et al.*[6]

As to the evolution of the ground QE state, the corresponding Wigner distribution contains a strong positive peak at original point and many weak positive peaks. The strong positive peak at original point results from the even parity character of the wave function. Those weak peaks resulting in the fringe structures relates closely with the ionization process. In the case of 1-excited QE state the strong peak at origin is negative because of the odd parity of the wave function. In our time-reversal invariant model, parity is a good quantum number, therefore the Wigner distributions are symmetric about the original point. This symmetry can be observed clearly in Figs.6 and 7. When we compare the quantum phase space with the classical unstable manifolds, surprisingly we find that the fringe structure in classical phase space also shows up. The fringe structure in both quantum and classical phase space has the same symmetry, and locates at the same regions. Since the size of the fringe structures in Figs.6 and 7 is much larger than the quantum uncertainty, namely, the Planck constant $\hbar(= 1)$, this structure is physically significant. So, we believe that it is quantum signature of the classical unstable manifolds.

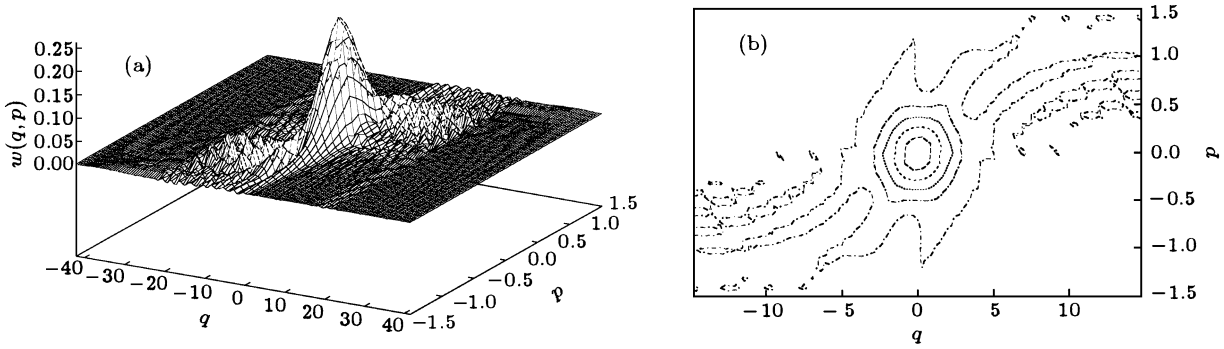


Fig.6. The Wigner function (a) and its contour plot (b) of the wave function after 20 cycles evolution of the QE0 state. $F = 5$ and $\omega = 1.34$.

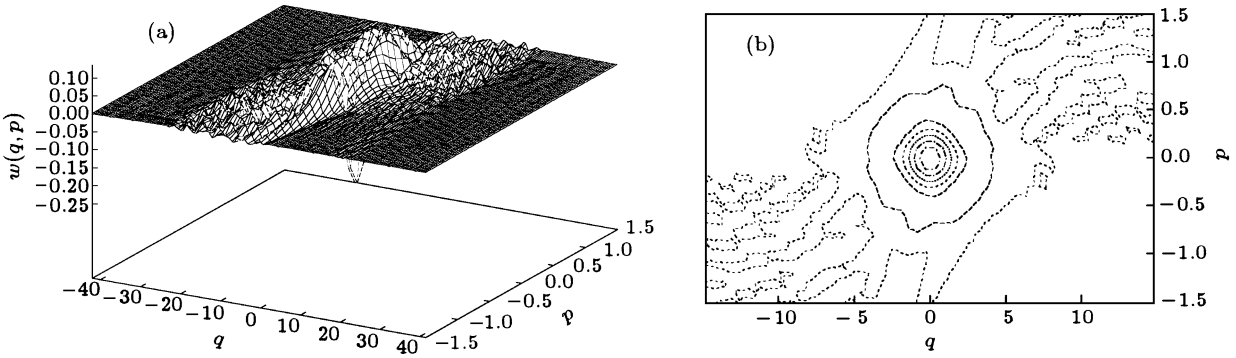


Fig.7. The Wigner function (a) and its contour plot (b) of the wave function after 20 cycles evolution of the QE1 state. $F = 5$ and $\omega = 1.34$.

As a further test, we also calculate the Wigner function of the lowest two KH states and show them in Figs.8 and 9. Since the wave functions are real and has definite parity, they have the reflection symmetry about q axis and p axis. Wigner distributions of the ground and 1-excited states contain strong positive peak and strong negative peak at original point, respectively. This character is the same as that discussed above. However, for these bound KH states, no fringe structure is observed. This also implies that the fringe structure associates closely with the ionization process.

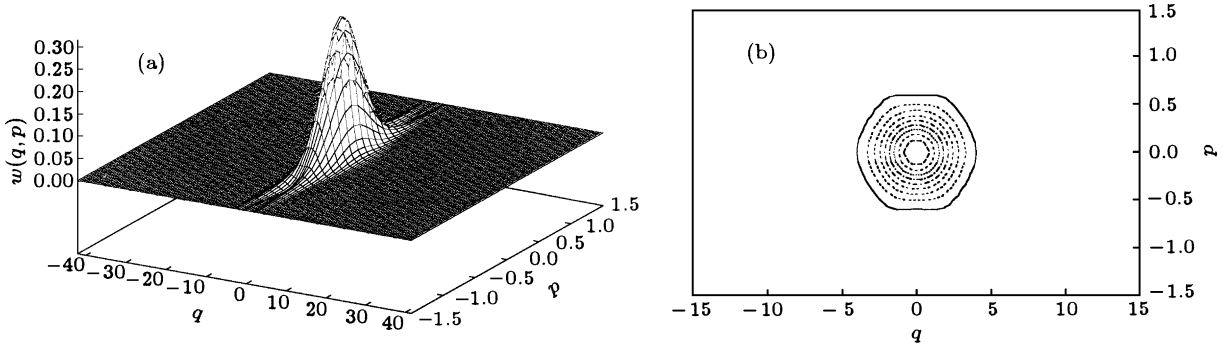


Fig.8. The Wigner function (a) and its contour plot (b) of the ground KH state. $F = 5$ and $\omega = 1.34$.

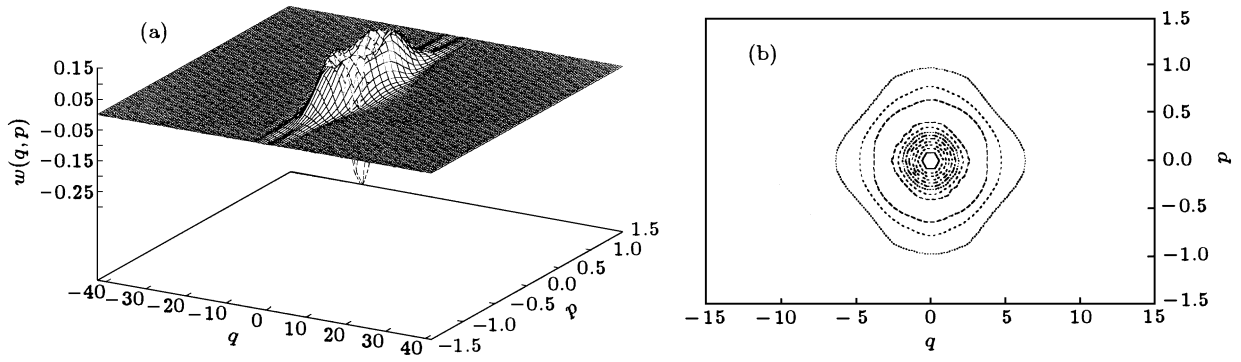


Fig.9. The Wigner function (a) and its contour plot (b) of the 1-excited KH state. $F = 5$ and $\omega = 1.34$.

IV. CONCLUSIONS

In this paper, we have studied the phase-space dynamics of atomic stabilization in the frame of a time-reversal invariant model, both classically and quantum mechanically. Our results show that 1) The dynamical mechanism of the fringe structure in the classical phase space is the unstable manifolds. In this situation the ionization process is characterized by chaos. 2) The fringe structure observed in the quantum Wigner distribution is the quantal signature of the classical unstable manifolds. 3) The fringe structure has geometric symmetry about the original point, and associates closely with the ionization process in the regime of stabilization.

Our results show a strong connection between the unstable manifolds and the ionization process qualitatively. However, it is still an open question that how the ionization rate is related to the fractal structure of the unstable manifolds, quantatitively. The study along this direction is undertaking.

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