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INTERACTION OF A TWO-LEVEL ATOM WITH FINE STRUCTURE MOVING IN AN ELECTROMAGNETIC FIELD

Zhang Jing-tao(张敬涛), Chen Zhao-yang(陈朝阳), and Xu Zhi-zhan(徐至展)
Laboratory for High Intensity Optics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, *China*

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Dynamics of a moving two-level atom with fine structure interacting with an electromagnetic field is studied. The fine structure increases the damping rate induced by the atomic momentum distribution. When the field is in a Fock state, the Rabi oscillation becomes very complicated; when the field is in a superposition state, the collapse-revival phenomenon disappears for the wider atomic wavepackets. In the presence of the fine structure, each atomic level splits into three sublevels when the field is in a Fock state and the atom has a definite momentum.

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I. INTRODUCTION

The subject of atomic motion in an electromagnetic (e.m.) field has attracted much attention because of its important applications, such as the deflection of the atom beams, the selection of the atomic species in an atom beam, the cooling and trapping of atoms.[1−4] When an atom travels through an e.m. field, correlation can build up between its internal and external dynamics and the e.m. field with which the atom interacts. The subsystems may affect one another for two reasons: the exchange of real or virtual photons and the dependence on the atomic position of the matter-field coupling. A possible description of the interaction is Jaynes-Cummings model. This model describing the interaction of a two-level atom with an e.m. field has been extended to include the atomic motion by Schlicher.[5] When the kinetic energy of the atom is comparable to the atom-field interaction energy, effects of the quantum mechanical center-of-mass motion become important, it is necessary to study the atom-field interaction with the consideration of the atomic motion.[6,7] On the other hand, it is known that the atomic level is not singlet. When one examines the spectral lines of an atom with instrument of high resolving power, he will find that many of the lines consist of doublets, which are two lines very closer together. Such lines are said to exhibit fine structure and have significant influence on the atomic and the field dynamics.[8]

In our previous paper[7] we have given a general solution to the problem that a two-level atom moves in an e.m. field. In this paper we focus on this problem in which the atomic fine structure is taken into account. Because of the important influence of the fine structure on the atomic and the field dynamics, this study is necessary. We will show that the atomic fine structure increases the damping rate induced by the atomic momentum distribution.

II. SOLUTION TO THE SCHRÖDINGER EQUATION

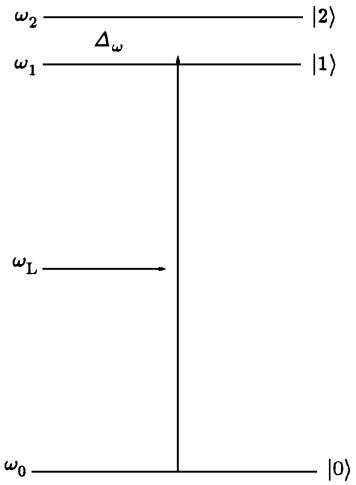


Fig.1. The level diagram of a two-level atom with fine structure. The excited state is a doublet with an energy separation Δ_ω smaller compared to the free energy of each atomic level.

The two-level atom with fine structure is plotted in Fig.1. The energies of the atomic states are, respectively, ω_0, ω_1 and ω_2. We assume that the energy difference between the excited states is much smaller than that of the excited states to the ground state, i.e., $\omega_2 - \omega_0 > \omega_1 - \omega_0 \gg \omega_2 - \omega_1 := \Delta_\omega$. In a reference frame that eliminates the energy of the free field (with frequency $\omega_{\rm L}$), the atom moving in the e.m. field is

described by the Hamiltonian ($\hbar = 1$)

$$H = \frac{P^2}{2M} + \omega_0\sigma_{00} + \omega_1\sigma_{11} + \omega_2\sigma_{22} + g[\mathrm{e}^{\mathrm{i}kz}(a^\dagger\sigma_{01}\mathrm{e}^{\mathrm{i}\omega_\mathrm{L}t} + a^\dagger\sigma_{02}\mathrm{e}^{\mathrm{i}\omega_\mathrm{L}t}) + \mathrm{H.c.}], \quad (1)$$

where P and M are, respectively, the momentum in z direction and the mass of the atom, $\sigma_{ij} = |i\rangle\langle j|$ $(i, j = 0, 1, 2)$ is the atomic operator; $a^\dagger$ and a are field operators; g is the atom-field couple; k is the wave vector of the e.m. wave.

Our starting point is the Schrödinger equation. Expanding the state vector $|\phi(t)\rangle$ in terms of the atomic internal state $|I\rangle$ $(I = 0, 1, 2)$, the eigenstates of momentum $|p\rangle$, and the Fock state $|n\rangle$ of field as follows

$$|\phi(t)\rangle = \sum_n \int \mathrm{d}p[\phi_n^0(p,t)|p,n,0\rangle + \phi_n^1(p,t)|p,n,1\rangle + \phi_n^2(p,t)|p,n,2\rangle], \quad (2)$$

then projecting $|p, n, I\rangle$ onto the Schrödinger equation, using their orthogonal relation, we get the following coupled equations

$$\begin{aligned}
\mathrm{i}\dot{\phi}_n^0(p,t) &= \left(\frac{p^2}{2M} + \omega_0\right)\phi_n^0(p,t) \\
&\quad + g\sqrt{n}\mathrm{e}^{\mathrm{i}\omega_\mathrm{L}t}[\phi_{n-1}^1(p-k,t) + \phi_{n-1}^2(p-k,t)], \\
\mathrm{i}\dot{\phi}_{n-1}^1(p-k,t) &= \left[\frac{(p-k)^2}{2M} + \omega_1\right]\phi_{n-1}^1(p-k,t) \\
&\quad + g\sqrt{n}\mathrm{e}^{-\mathrm{i}\omega_\mathrm{L}t}\phi_n^0(p,t), \\
\mathrm{i}\dot{\phi}_{n-1}^2(p-k,t) &= \left[\frac{(p-k)^2}{2M} + \omega_2\right]\phi_{n-1}^2(p-k,t) \\
&\quad + g\sqrt{n}\mathrm{e}^{-\mathrm{i}\omega_\mathrm{L}t}\phi_n^0(p,t).
\end{aligned} \quad (3)$$

Set $\phi_n^I(p,t) = \tilde{\phi}_n^I(p,t)\exp(-\mathrm{i}[p^2/2M + \omega_I]t)$, the above equations have the following simpler form

$$\begin{aligned}
\mathrm{i}\dot{\tilde{\phi}}_n^0(p,t) &= \Omega_n[\mathrm{e}^{\mathrm{i}\beta(p)t}\tilde{\phi}_{n-1}^1(p-k,t) \\
&\quad + \mathrm{e}^{\mathrm{i}\alpha(p)t}\tilde{\phi}_{n-1}^2(p-k,t)], \\
\mathrm{i}\dot{\tilde{\phi}}_{n-1}^1(p-k,t) &= \Omega_n\mathrm{e}^{\mathrm{i}\beta(p)t}\tilde{\phi}_n^0(p,t), \\
\mathrm{i}\dot{\tilde{\phi}}_{n-1}^2(p-k,t) &= \Omega_n\mathrm{e}^{\mathrm{i}\alpha(p)t}\tilde{\phi}_n^0(p,t),
\end{aligned} \quad (4)$$

where $\beta(p) = [p^2 - (p-k)^2]/2M + \Delta_1$, $\alpha(p) = [p^2 - (p-k)^2]/2M + \Delta_2$, $\Delta_I = \omega_\mathrm{L} - \omega_I + \omega_0$ and $\Omega_n = g\sqrt{n}$. These equations can be solved directly. For instance, put $\tilde{\phi}_{n-1}^1(p-k,t) = \exp(-\mathrm{i}\lambda_n(p)t)$, we find $\lambda_n(p)$ satisfies the following equation

$$\lambda_n(p)^3 + A_1\lambda_n(p)^2 + A_2\lambda_n(p) + A_3 = 0 \quad (5)$$

with $A_1 = -\alpha(p)$, $A_2 = \beta(p)\Delta_\omega - 2\Omega_n^2$, $A_3 = \Omega_n^2\Delta_\omega$. Its solution is given by[9]

$$\begin{aligned}
\lambda_n^1(p) &= -\frac{1}{3}A_1 + \frac{2}{3}(A_1^2 - 3A_2)^{1/2}\cos\theta, \\
\lambda_n^2(p) &= -\frac{1}{3}A_1 + \frac{2}{3}(A_1^2 - 3A_2)^{1/2}\cos(\theta + 2\pi/3), \\
\lambda_n^3(p) &= -\frac{1}{3}A_1 + \frac{2}{3}(A_1^2 - 3A_2)^{1/2}\cos(\theta + 4\pi/3),
\end{aligned} \quad (6)$$

and θ is determined by $\cos 3\theta = (9A_1A_2 - 2A_1^3 - 27A_3)/[2(A_1^2 - 3A_2)^{3/2}]$. The solution of Eq.(4) works out to be

$$\begin{aligned}
\tilde{\phi}_n^0(p,t) &= \sum_{j=1}^3 C'_{j,n}(p)\exp(-\mathrm{i}[\lambda_n^j(p) - \beta(p)]t), \\
\tilde{\phi}_{n-1}^1(p-k,t) &= \sum_{j=1}^3 C_{j,n}(p)\exp(-\mathrm{i}\lambda_n^j(p)t), \\
\tilde{\phi}_{n-1}^2(p-k,t) &= \sum_{j=1}^3 C''_{j,n}(p)\exp(-\mathrm{i}[\lambda_n^j(p) - \Delta_\omega]t),
\end{aligned} \quad (7)$$

where $C'_{j,n}(p) = C_{j,n}(p)\dfrac{\lambda_n^j(p)}{\Omega_n}$ and $C''_{j,n}(p) = C_{j,n}(p)\dfrac{\lambda_n^j(p)[\lambda_n^j(p) - \beta(p)] - \Omega_n^2}{\Omega_n^2}$. The coefficients are determined by initial conditions.[6] For an initially de-excited atom, these $C_{j,n}(p)$ are given by

$$\begin{aligned}
C_{1,n}(p) &= f(p)c_n\Omega_n\{\lambda_n^3(p)[\lambda_n^3(p) - \beta(p)] - \lambda_n^2(p)[\lambda_n^2(p) - \beta(p)]\}D^{-1}, \\
C_{2,n}(p) &= f(p)c_n\Omega_n\{\lambda_n^1(p)[\lambda_n^1(p) - \beta(p)] - \lambda_n^3(p)[\lambda_n^3(p) - \beta(p)]\}D^{-1}, \\
C_{3,n}(p) &= f(p)c_n\Omega_n\{\lambda_n^2(p)[\lambda_n^2(p) - \beta(p)] - \lambda_n^1(p)[\lambda_n^1(p) - \beta(p)]\}D^{-1}, \\
D &= \lambda_n^1(p)[\lambda_n^1(p) - \beta(p)][\lambda_n^2(p) - \lambda_n^3(p)] \\
&\quad + \lambda_n^2(p)[\lambda_n^2(p) - \beta(p)][\lambda_n^3(p) - \lambda_n^1(p)] \\
&\quad + \lambda_n^3(p)[\lambda_n^3(p) - \beta(p)][\lambda_n^1(p) - \lambda_n^2(p)],
\end{aligned} \quad (8)$$

in which $f(p)$ is initial momentum distribution of the atom, c_n is initial photon distribution of the field.

Solutions in Eq.(7) play the role of wavefunction in momentum space. They denote the occupation probability amplitude of the atom in its internal states for the light field being in a Fock state. A detailed investgation to the solutions shows that when the atom is in its ground state, its momentum is p, the light field has n photons; when the atom is in either of its two excited levels, its momentum is always $p - k$, and the light field has $n - 1$ photons. This fact shows that the

atom jumps to its higher levels by absorbing the same photon, the momental changes are equal.

Ⅲ. SPLITTING OF THE ENERGY

The wave function in coordinate space reads

$$|\psi(t)\rangle = \int \mathrm{d}z \sum_{I=0}^{2} \psi^I(z,t)|z,n,I\rangle,$$
$$|\psi^I(z,t)\rangle = \sum_n \int \mathrm{d}p \phi_n^I(p,t)\exp(\mathrm{i}pz) \quad (I=0,1,2)\,. \tag{9}$$

These $\psi^I(z,t)$ play the role of wavefunction in real space.

To study the energy split of the atom, we consider a special case that, initially, the atom has a definite momentum $|p_0\rangle$ and the field is in a Fock state $|m\rangle$, i.e., $f(p)=\delta(p-p_0)$, $c_n=\delta_{m,n}$. The wavefunction in coordinate space becomes

$$\psi^0(z,t) = \frac{1}{\sqrt{2\pi}}\sum_{j=1}^{3} C'_{j,m}(p_0)\exp(-\mathrm{i}(E_0^j t - p_0 z)),$$
$$\psi^1(z,t) = \frac{1}{\sqrt{2\pi}}\sum_{j=1}^{3} C_{j,m}(p_0)\exp(-\mathrm{i}[E_1^j t - (p_0-k)z]),$$
$$\psi^2(z,t) = \frac{1}{\sqrt{2\pi}}\sum_{j=1}^{3} C''_{j,m}(p_0)\exp(-\mathrm{i}[E_2^j t - (p_0-k)z]), \tag{10}$$

where

$$E_0^1 = \lambda_m^1(p_0) + \frac{p_0^2}{2M} + \omega_0,$$
$$E_0^2 = \lambda_m^2(p_0) + \frac{p_0^2}{2M} + \omega_0,$$
$$E_0^3 = \lambda_m^3(p_0) + \frac{p_0^2}{2M} + \omega_0;$$
$$E_1^1 = \lambda_m^1(p_0) + \frac{(p_0-k)^2}{2M} + \omega_1,$$
$$E_1^2 = \lambda_m^2(p_0) + \frac{(p_0-k)^2}{2M} + \omega_1,$$
$$E_1^3 = \lambda_m^3(p_0) + \frac{(p_0-k)^2}{2M} + \omega_1;$$
$$E_2^1 = \lambda_m^1(p_0) + \frac{(p_0-k)^2}{2M} + \omega_2,$$
$$E_2^2 = \lambda_m^2(p_0) + \frac{(p_0-k)^2}{2M} + \omega_2,$$
$$E_2^3 = \lambda_m^3(p_0) + \frac{(p_0-k)^2}{2M} + \omega_2. \tag{11}$$

They are all sums of the interaction energy, the center-of-mass kinetic energy, and the internal energy. From Eq.(10), we find that the wave function for each atomic states is a superposition of three plane waves with the same momenta but different energies. This fact shows that when the atom is in some state, its momentum is definite, while its energy has three possible values. The energy distribution of the atom in its internal states is as follows: the ground state $|0\rangle$ splits into three sublevels with energy E_0^1, E_0^2 and E_0^3, the excited state $|1\rangle$ splits into three sublevels with energies E_1^1, E_1^2 and E_1^3, the excited state $|2\rangle$ splits into three sublevels with energies E_2^1, E_2^2 and E_2^3, respectively; the energy separation between split sublevels is determined by the difference of $\lambda_m^i(p_0)$. In the absence of the fine structure, each atomic level splits into two sublevels; in the presence of the fine structure, each atomic level splits into three sublevels, even for the ground level.

Ⅳ. ATOMIC POPULATION AND MEAN MOMENTUM

We assume that the momentum distribution is a Gaussian wavepacket centered at p_0 with a width Δp, i.e., $f(p) = \frac{1}{\sqrt{2\pi}\Delta p}\exp\left[-\frac{(p-p_0)^2}{2\Delta p}\right]$, and that the initial field is in a coherent state $|\alpha\rangle$, i.e., $c_n = \exp(-|\alpha|^2/2)\frac{\alpha^n}{\sqrt{n!}}$.

The occupation probability of the atomic state $|I\rangle$ reads

$$P_I(t) = \sum_n \int \mathrm{d}p |\phi_n^I(p,t)|^2 \qquad (I=0,1,2). \tag{12}$$

It has been shown[7] that the atomic motion induces a momentum-dependent detuning which holds back the atomic transition, and leads to a momentum-dependent Rabi frequency which causes an overlapping among Rabi oscillations. When the field is in a Fock state, the atomic population oscillates at certain frequency with some damping, the damping rate is related to the momentum distribution; the collapse-revival of the atomic population appears for some special momentum distribution. When the field is in a superposition state, the collapse-revival phenomenon is modified by the momentum distribution.

Numerical study shows that similar behaviors, such as the damping oscillation, the modified collapse-revival, appear in the evolution of $P_I(t)$. When the field is in a Fock state, the collapse-revival of $P_I(t)$ appears for some atomic momentum wavepackets. When the field is in a coherent state, the collapse-revival

of the atomic population exhibits a large damping rate and disappears in the case of wider momentum wavepackets. We notice that the behavior of $P_1(t)$ and that of $P_2(t)$ are very close to each other but keep an out-of-phase relation, their steady values in the collapse region are affected by the fine structure as well as the momentum distribution, and their difference from that of $P_0(t)$ is evident. For we deal with an initially deexcited atom, this difference can be easily explained by the detuning induced by the atomic motion. In order to clearly show the influence of the fine structure, we plot $P_0(t)$ in Fig.2 for several Δ_ω when the field is in a Fock state $|n=10\rangle$ with $\Delta p = 0.1\lambda$. We also plot $P_0(t)$ in Fig.3 for the field being in a coherent state for several atomic momentum wavepackets and different Δ_ω.

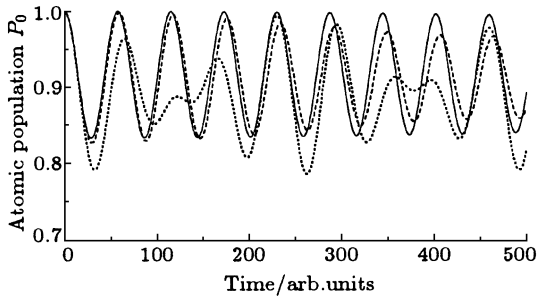


Fig.2. Time evolution of the atomic population $P_0(t)$ when the field is in a Fock state $|n=10\rangle$ for $\Delta p = 0.1\lambda$ and different $\Delta_\omega = 0.01\omega_{\rm L}$ (real line), $\Delta_\omega = 0.1\omega_{\rm L}$ (dashed line), and $\Delta_\omega = 0.5\omega_{\rm L}$ (dotted line). The laser is modulated as $\Delta_1 = 0$.

The mean momentum of the atom reads

$$\overline{P}(t) = \sum_n \int \mathrm{d}p\{(p-k)[|\phi^1_{n-1}(p-k,t)|^2 + |\phi^2_{n-1}(p-k,t)|^2] + p|\phi^0_n|(p,t)|^2\}. \quad (13)$$

When the field is in a Fock state, the mean momentum exhibits damping oscillation and in a long time the collapse-revival also appears for several atomic momentum distributions; when the field is in a superposition state, the mean momentum exhibits damped collapse-revival. The fine structure increases the damping rate induced by the atomic motion. The $\overline{P}(t)$ keeps an in-phase relation to the atomic population $P_0(t)$ because the atom jumps to its higher level at the expense of decrement of its momentum and the two excited levels have equal momentum. Its value in the collapse region is also affected by the fine structure and the momentum distribution.

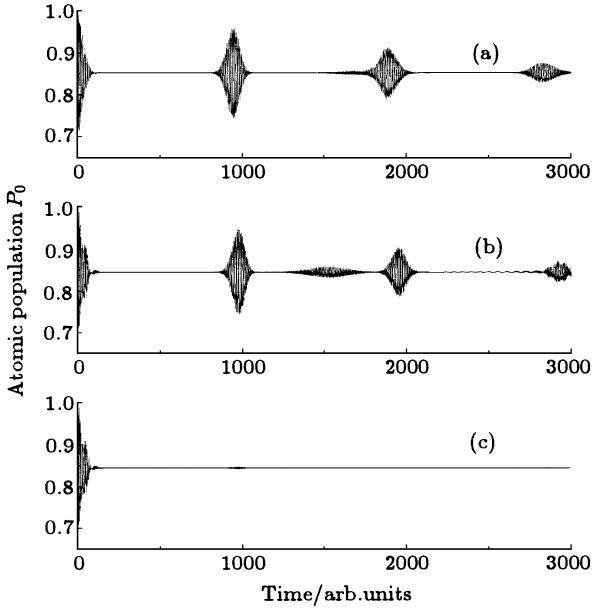


Fig.3. Time evolution of $P_0(t)$ for the field is in a coherent state with the mean photon number $\overline{n} = 20$: (a) $\Delta p = 0.01\lambda$, $\Delta_\omega = 0.1\omega_{\rm L}$; (b) $\Delta p = 0.05\lambda$, $\Delta_\omega = 0.1\omega_{\rm L}$; (c) $\Delta p = 0.05\lambda$, $\Delta_\omega = 0.2\omega_{\rm L}$. The laser is modulated as $\Delta_1 = 0$.

V. SUMMARY

The momentum and momentum distribution may affect the matter-field interaction, the internal and the external dynamics of the atom may be strongly modified. The atomic motion works in the following two manners: one is inducing an effective detuning, the other is leading to a momentum-dependent Rabi frequency. The atomic momentum distribution causes some damping in the atomic dynamics, and the fine structure increases the damping rate. When the field is in a Fock state, the Rabi oscillation becomes very complicated due to the influence of the atomic momentum distribution and the fine structure; the collapse-revival of the atomic population appears for some momentum distribution. When the field is in a coherent state, the collapse-revival of the atomic population exhibits a large damping rate and disappears for wider momentum wavepackets. The evolution of the mean momentum is very similar to that of the atomic population $P_0(t)$. In the presence of the fine structure, each atomic level splits into three sublevels, which is somewhat similar to a three-level V-type atom.[9] The energy difference between the split sublevels is determined by the atomic momentum, the field state, and the fine structure.

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