Vol 16 No 9, September 2007
1009-1963/2007/16(09)/2825-05

Chinese Physics

© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

Amplitude death in steadily forced chaotic systems*

Feng Guo-Lin(封国林)[a)b)†] and He Wen-Ping(何文平)[c)]

[a)] *Key Laboratory of Regional Climate-Environment for Temperate East Asia, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing* 100029, *China*

[b)] *Laboratory for Climate Studies, National Climate Center, China Meteorological Administration, Beijing* 100081, *China*

[c)] *Department of Atmospheric Sciences, Lanzhou University, Lanzhou* 730000, *China*

(Received 26 August 2006; revised manuscript received 10 March 2007)

Steady forcing can induce the amplitude death in chaotic systems, which generally exists in coupled dynamic systems. Using the Lorenz system as a typical example, this paper investigates the dynamic behaviours of the chaotic system with steady forcing numerically, and finds that amplitude death can occur as the strength of the steady forcing goes beyond a critical constant.

Keywords: amplitude death, Lorenz system, steady forcing
PACC: 9260X

1. Introduction

Coupled dynamic systems can exhibit rich dynamic behaviours, and in recent years have been considerably studied and applied in a variety of disciplines, such as in physics, chemistry, biology and meteorology.[1−23] Coupled dynamic systems can display various complex phenomena: for example, synchronization, frequency-locking, and amplitude death. Moreover, the behaviours of coupled dynamic systems not only depend on the coupling strength, but also on the characteristics of the coupled systems.[24,25]

Amplitude death refers to the cease of oscillations under interactions, i.e. the coupled dynamic systems drive each other to fixed points, which are the states of stable equilibrium.[12,24] This phenomenon has been found in a variety of coupled oscillatory systems and successively observed in all kinds of experiments. Matthews and Strogatz[2] analysed collective behaviours of limit-cycle oscillators, which exhibited frequency locking, amplitude death, and incoherence depending on the choice of coupling strength and the spread of natural frequencies. Reddy *et al*[4] investigated the dynamic behaviour of two time-delay-coupled limit cycle oscillators and found that time delay can lead to amplitude death of the oscillators even if they have the same frequency. Such a time-delay-induced amplitude death was later also experimentally observed in two coupled limit-cycle oscillators.[25,26] Furthermore, amplitude death has been founded in coupled discrete-time dynamical systems.[27,28]

Differing from the coupled oscillators studied above, this paper introduces a no-feedback mechanism of action, i.e. investigates the dynamic behaviours of chaotic systems under steady external forcing. The results show that amplitude death can occur when the strength of the steady forcing goes beyond a critical constant. Namely, the systems transit directly or indirectly from chaos to fixed points, which is a quite general behaviour in the steady forced chaotic systems.

The paper is organized as follows. Using the Lorenz systems as the representative chaos model, we present the numerical results in Section 2, and give brief conclusions and remarks in Section 3.

2. Lorenz system with steady forcing

The Lorenz system[29] is a well-known maximally simplified climatic model, which has been widely studied in the past 40 years. In the present paper, we consider the Lorenz system driven by a steady forcing, and investigate the influences of the force on behaviours of the system, which can be denoted with the following equations:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\sigma x + \sigma y,$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = rx - y - xz,$$

*Project supported by the National Natural Science Foundation of China (Grant No 40325015), the National Key Program Development for Basic Research (Grant No 2006CB400503).

†Corresponding author. E-mail: fenggl@cma.gov.cn

$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - bz + A, \tag{1}$$

where x, y and z are dimensionless variables, and A is a variable for the steady forcing strength. We study the 'canonical' case,[29] and take the values of parameters as $\sigma = 10$ and $b = 8/3$, the Rayleigh number $r = 28$. Obviously, the system is chaotic in absence of forcing. It is easy to obtain the three equilibrium points of the system $E_1\,(0.0, 0.0, A/b)$, $E_2(-\sqrt{b(r-1)-A}, \quad -\sqrt{b(r-1)-A}, \quad r-1)$, $E_3(\sqrt{b(r-1)-A}, \ \sqrt{b(r-1)-A}, \ r-1)$, and they are all unstable when we take $A = 0$.

Figure 1 shows the bifurcation diagrams of the steadily forced Lorenz system for finite strength, where the strength of steady forcing A is used as an alterable bifurcation parameter. Variables x and z present irregularly oscillatory behaviour for $A < A_\mathrm{c}$, where A_c is a critical constant, and Fig.1(c) is an enlarged view of the variable x near the bifurcation point A_c, which shows $A_\mathrm{c} \approx 9.697$. The system remains chaotic in $A \in [0, 9.697]$. When the strength of steady forcing goes beyond the critical constant A_c, the motion of the driven system becomes stable fixed point, namely amplitude death occurs. This transition is direct, which is very similar to the coupled identical Lorenz oscillators.[10] Variable z is a constant ($z = r - 1$) in $A \in [9.697, 72]$. Further increasing the steady forcing strength, variable z linearly evolves with the forcing strength for $A > 72$. In this case, z takes value $r - 1$. Differing from variable z, x is proportional to the external forcing strength, i.e. x monotonically increases with the increase of the strength about in the range $A \in [9.697, 50.315]$, where x equals to $-\sqrt{b(r-1)-A}$. When the strength A increases to 50.315, x abruptly jumps from $-\sqrt{b(r-1)-A}$ to $\sqrt{b(r-1)-A}$, and then gradually decreases with the increase of A in the range $A \in [50.315, 72]$. After A above 72, the variable x reduces to zero, and hereafter does not change with variable A. The bifurcation diagrams of variable y is very similar to that of variable x, and the both completely coincide with each other after $A > A_\mathrm{c}$ (figure is omitted).

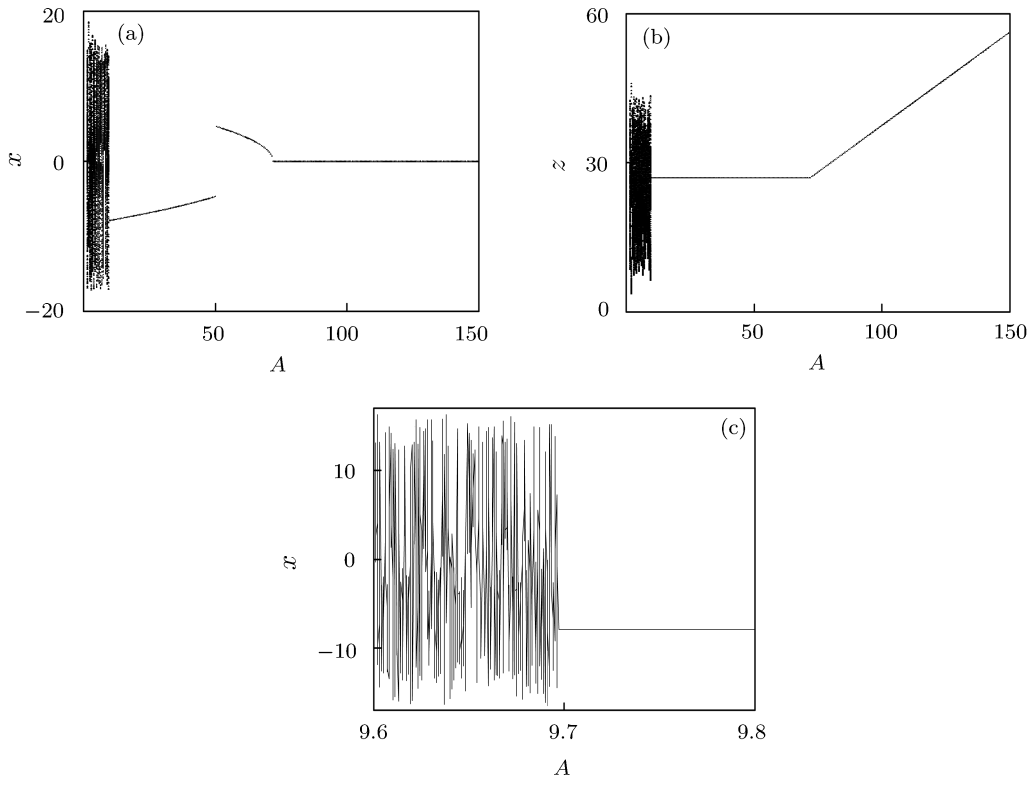


Fig.1. Bifurcation diagrams for the steadily forced Lorenz system: (a) and (c) for variable x, (b) for variable z, and the strength of steady forcing A is used as a bifurcation parameter.

Therefore, increasing the forcing strength, the forced Lorenz system is driven directly from chaotic state to fixed points, namely the steady forcing induces amplitude death of the system. Various states for different forcing strength are observed. The system is chaotic when $A < A_c$, and the forcing strength continuously increases after $A > A_c$, the system eventually is driven to different fixed points, i.e. the equilibrium points of the forced system.

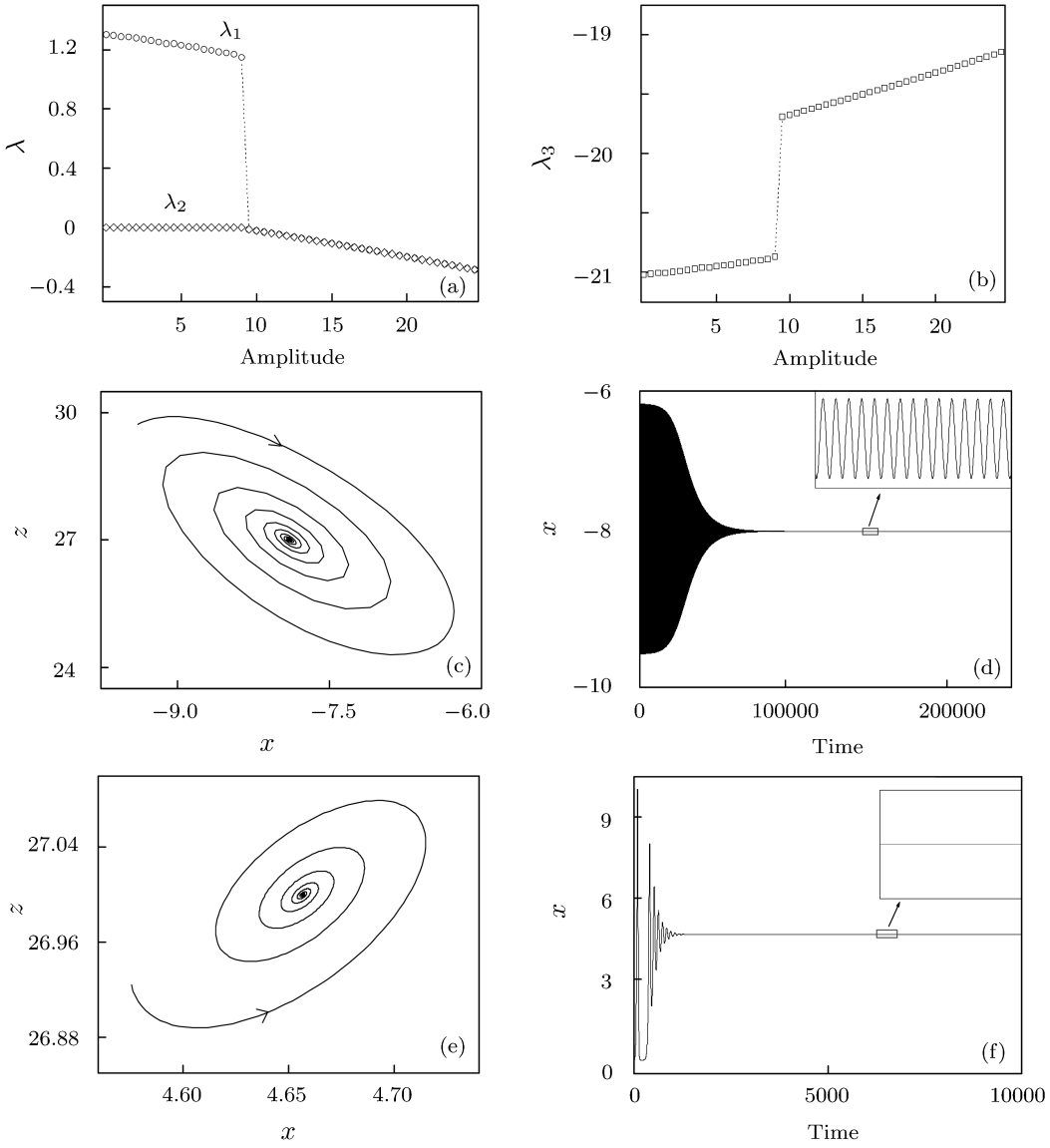


Fig.2. The spectrum of Lyapunov exponents (a) and (b), and circle, rhombus and square correspond to λ_1, λ_2, and λ_3 respectively. The trajectories of the oscillator (1) in phase space (left panel) and with time (right panel) (c) and (d) at $A = 9.6974$, (e) and (f) at $A = 50.315$, respectively. The insets in (d) and (f) enlarge the corresponding marked boxes.

The transitions, i.e. the forced Lorenz system is driven from chaotic state to fixed points, are well confirmed by the spectrum of the Lyapunov components shown in Figs.2(a) and 2(b). We can find in Fig.2(a) that the largest nontrivial Lyapunov component λ_1 of the forced system abruptly becomes negative when the strength of steady forcing goes beyond the critical constant A_c, which indicates the state transition of the forced system from chaotic dynamics to deterministic motion. The second Lyapunov component λ_2 has also a change in slope at $A = A_c$, and afterwards the curves of λ_1 and λ_2 almost completely coincide with

each other in the amplitude death range, i.e. $\lambda_1 \approx \lambda_2$. There is also an abrupt change in the slope of the minimum Lyapunov exponent λ_3 at point A_c, as can be seen in Fig.2(b).

In order to gain a detailed account for the state of the forced Lorenz system, the phase diagrams in phase space and time series of x are shown from Fig.2(c) to 2(f). It can be seen from the figures that the transient trajectories of the forced system eventually spiral into fixed points, i.e. amplitude death occurs. Furthermore, when the strength of the steady forcing changes, the fixed points which are stabilized by the external steady forcing are also different. However, the fixed point is one of the equilibrium points of the forced system for a given parameter A. It is easy to find that the fixed points are E_2 and E_3 for the given strength $A = 9.697$ and $A = 50.315$ respectively. The forced system presents the periodic motion with a very small amplitude at $A = 9.697$, but in the latter case the oscillator stays at quenched state.

Figure 3 describes the relation between critical amplitude A_c of steady forcing and Rayleigh number r. It is easy to observe that the critical amplitude A_c of the force is proportional to r. The region marked C corresponds to chaotic states while the fixed points are shown in the region marked AD, wherein the systems with different constant forcing strength will be driven to different fixed points.

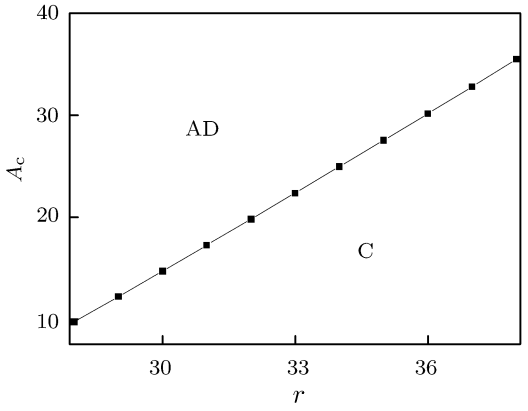


Fig.3. The critical amplitudes of the steady forcing vs Rayleigh number r.

It is known from the above analyses on bifurcation diagrams, Lyapunov spectrum, and transient trajectories in the phase space as well as trajectories with time of the forced Lorenz systems that amplitude death phenomenon can occur in the steadily forced chaotic systems. The point, whereat the Lyapunov spectrum abruptly changes, coincides with the bifurcation point of the system. When amplitude death occurs, the largest Lyapunov exponent of the forced system is negative, and almost completely equal to the second Lyapunov exponent, i.e. $\lambda_1 \approx \lambda_2$.

The numerical tests indicate that if the steady forcing term is applied to the first variable or the second variable in Eq.(1), the similar results can be obtained, and in both cases the steady forcing can induce amplitude death. It is easy to find that this behaviour is a general phenomenon, but the manner of the transition from chaotic state to fixed points may be different. For example, we have applied the steady forcing to the third variable of the Rössler system, and found that the transition is indirect, namely the steady forcing eventually induced amplitude death via a limit cycle, which is very similar to the coupled Rössler system.[10] It surprises us that no matter how strong the strength was, amplitude death did not occur in the forced Rössler system when the forced term was applied to other variables.

3. Conclusions

In summary, we used the Lorenz system as a representative model, and have numerically investigated the dynamical behaviours of the forced system. Then we discovered that the steady forcing can induce amplitude death in the forced Lorenz system, i.e. the chaotic motion of the system is eventually replaced by fixed points, which are all the equilibrium points of the forced chaotic system. Further increasing the strength of the steady forcing, the different equilibrium points of the forced system will be stabilized successively.

We have shown that the largest Lyapunov exponent almost equals to the second one when amplitude death occurs, which is the same to the coupled Lorenz system.[10] Furthermore, the bifurcation diagram and Lyapunov spectrum can be well used to detect the occurrence of amplitude death of the forced system. The critical amplitude of the steady forcing for amplitude death linearly increases along with the increase of Rayleigh number r.

Acknowledgments

The authors would like to thank Professors Yang Jun-Zhong and Zhu Fu-Cheng for help, and especially thank the anonymous reviewers for their valuable comments.

References

[1] Pecora L M and Carroll T L 1990 *Phys. Rev. Lett.* **64** 821
[2] Matthews P C and Strogatz S H 1990 *Phys. Rev. Lett.* **65** 1701
[3] Ramaswamy R 1997 *Phys. Rev.* E **56** 7294
[4] Reddy R, Sen A and Johnson G L 1998 *Phys. Rev. Lett.* **80** 5109
[5] Vadivasova T E, Sosnovtseva O V, Balanov A G and Astakhov V V 2000 *Phys. Rev.* E **61** 4618
[6] Atay F M 2003 *Phys. Rev.* E **91** 94101
[7] Konishi K 2003 *Phys. Rev.* E **68** 67202
[8] Konishi K 2004 *Phys. Rev.* E **70** 66201
[9] Ozden I, Venkataramani S, Long M A, Connors B W and Nurmikko A V 2004 *Phys. Rev. Lett.* **93** 158102
[10] He W P, Feng G L, Gao X Q and Li J P 2006 *Acta Phys. Rev.* **55** 6192 (in Chinese)
[11] Liu W, Xiao J and Yang J 2005 *Phys. Rev.* E **72** 57201
[12] Bar-Eli K 1984 *J. Phys. Chem.* **88** 6174
[13] Bar-Eli K 1985 *Physica* (Amsterdam) D **14** 242
[14] Crowley M F and Epstein I R 1989 *J. Phys. Chem.* **93** 2496
[15] Ghosh A K, Chance B and Pye E K1971*Arch. Biochem. Biophys.* **145** 319
[16] Bahiana M and Massunaga M S O 1995 *Phys. Rev.* E **52** 321
[17] Cao H X and Feng G L 2000 *J. Appl. Meteor.* **11** 249 (in Chinese)
[18] Wang P and Dai X G 2004 *Chin. Phys.* **13** 1770
[19] Dai X G, Wang P and Chou J F 2004 *Progress in Chinese Science* **60** 544
[20] Dai X G, Fu C B and Wang P 2005 *Chin. Phys.* **14** 850
[21] Wang P and Dai X G 2005 *Acta Phys. Sin.* **54** 4961(in Chinese)
[22] Shi N, Gu J Q, Yi Y M and Lin Z N 2005 *Chin. Phys.* **14** 844
[23] Shi N, Yi Y M, Gu J Q and Xia D D 2006 *Chin. Phys.* **15** 2180
[24] Zhai Y, Kiss I Z and Hudson J L 2004 *Phys. Rev.* E **69** 26208
[25] Yamaguchi Y and Shimizu H 1984 *Physica* (Amsterdam) D **11** 212
[26] Reddy R, Sen A and Johnson G L 2000 *Phys. Rev. Lett.* **85** 3381
[27] Herrero R, Figueras M, Rius J, Pi F and Orriols G 2000 *Phys. Rev. Lett.* **84** 5312
[28] Konishi K 2003 *Phys. Rev.* E **67** 17201
[29] Lorenz E N 1963 *J. Atmos. Sci.* **20** 130