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Potential fields of merging and splitting filaments in air*

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Two interacting light filaments with different initial phases propagating in air are investigated numerically by using a ray tracing method. The evolution of the rays of a filament is governed by a potential field. During propagation, the two potential wells of the two filaments can merge into one or repel each other, depending on the initial phase difference between the two filaments. The study provides a simple description of the interacting filaments.

Keywords: potential fields, filaments
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1. Introduction

In the past decade, there has been considerable progress in the field of high-power ultrashort laser pulse propagation in air, which involves interesting phenomena such as self-focusing, conical generation,[1] white light generation,[1] pulse splitting and merging,[2] and high-harmonic emission.[3,4] The induced filaments open the new way for lightning control and atmospheric remote sensing.[5,6] The physics governing the propagation of short intense laser pulses in transparent media is now well understood. When the peak power of the laser pulse is high enough to initiate nonlinear effects, the refractive index of the medium will be modified. The change of the refractive index, $\Delta n = n_2 I$, where n_2 is the nonlinear refractive index and I is the pulse intensity, is governed by the transverse intensity profile of the laser pulse. As a result, the medium behaves as a focusing lens for a Gaussian intensity profile of the laser pulse. When the power exceeds a critical power of $P_{\rm cr} = \lambda_0^2/2\pi n_0 n_2$, where n_0 is the refractive index of the medium and λ_0 is the laser wavelength in vacuum, the Kerr focusing effect overcomes diffraction. The resulting high light intensity ionizes the medium and creates an electron plasma. Multiphoton ionization (MPI) is dominant for an intensity of less than 10^{14} W/cm^2[7] and the resulting electron density is about 10^{16} to 10^{17} cm^{-3}. The refractive index change induced by electrons is negative and therefore defocuses the laser beam. The balance between the Kerr effect and the plasma defocusing effect results in a dynamic self-guided filament. When the laser power does not exceed a few tens of $P_{\rm cr}$, the beam may produce just a few filaments.[2] Otherwise, many of them can appear.[8,9] Thus it is of interest to study the interaction of the filaments.

Bergé *et al*[10] have proposed theoretically that two in-phase interacting filaments will fuse when their powers lie between $P_{\rm cr}/4$ and $P_{\rm cr}$ and their initial separation is appropriate, and two light filaments with opposite phases will never coalesce. However, they took into account only the diffraction and the self-focusing effect. Couairon *et al*[11] have analytically estimated the process by considering MPI as a perturbation. Recently, Xi *et al*[12] have numerically simulated the interaction between two light filaments in air and observed features such as attraction, fusion, repulsion and spiral propagation, depending on the relative phase and the crossing angle between them. In this paper, we will analytically discuss this phenomenon and propose a potential-field method so that the interaction between the light filaments can be understood by referring to a scenario of rays moving in the evolving potential fields. This approach is different from the common paraxial approximation, which treats each pulse, instead of each ray, as a particle.[13] Our approach is based on the variational method that has been successfully used to discuss one-pulse prop-

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agation in nonlinear medium.[14,15]

2. Numerical simulation

The propagation of laser pulse in air is governed by an extended nonlinear Schrödinger (NLS) equation together with the evolution equation for the electrons from multiphoton ionization[15]

$$2k_0\frac{\partial E}{\partial z} = \mathrm{i}\Big[\Delta_\perp E - \frac{k''}{2}\frac{\partial^2 E}{\partial t^2} + k_0^2 n_2|E|^2 E - k_0^2\frac{\omega_{\mathrm{pe}}^2(\rho)}{\omega_0^2}E\Big], \quad (1)$$

$$\frac{\partial \rho}{\partial t} = \frac{\beta^{(K)}}{K\hbar\omega_0}|E|^{2K}\Big(1-\frac{\rho}{\rho_{\mathrm{at}}}\Big), \quad (2)$$

where $E(x,y,z,t)$ is the electric field envelope, ρ is the electron density. These equations are expressed in the reference frame moving with the pulse group velocity, characterized by the central wave number $k_0 = 2\pi/\lambda_0$, where $\lambda_0 = 800$ nm is the central wavelength of the laser beam in air. The terms on the right-hand sides of Eqs.(1) and (2) account for the transverse diffraction with $\Delta_\perp \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2$, the group velocity dispersion with coefficient $k'' = 0.2$ fs^2/cm at 800 nm, the self-focusing due to the Kerr effect with the nonlinear refractive index of air at atmosphere pressure $n_2 = 3.2\times10^{-19}$ cm^2/W, and the defocusing induced by MPI. The effect of multiphoton absorption is neglected.[2] The plasma frequency is $\omega_{\mathrm{pe}} = \sqrt{q_{\mathrm{e}}^2\rho/m_{\mathrm{e}}\varepsilon_0}$ ($q_{\mathrm{e}}, m_{\mathrm{e}}$, and ρ are the electron charge, mass, and density, respectively), and the density of neutral atoms is $\rho_{\mathrm{at}} = 2.7\times10^{19}$ cm^{-3}.

The laser power is about twice P_{cr}. We ignore the group velocity dispersion, which can significantly affect the beam dynamics only if the power is far beyond the ionization threshold.[2] Assuming $E(x,y,z,t) = A\mathrm{e}^{\mathrm{i}\phi}$, Eq.(1) can be split into two coupled equations for the absolute amplitude A and the phase ϕ.[13,14] The trajectory of the laser pulse can be described by[12]

$$\frac{\partial\phi}{\partial z} + \frac{1}{2k_0}\Big[\Big(\frac{\partial\phi}{\partial x}\Big)^2 + \Big(\frac{\partial\phi}{\partial y}\Big)^2\Big] - \frac{1}{2k_0}\Big[\frac{1}{A}\Big(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}\Big) - \frac{\rho}{\rho_{\mathrm{c}}} + k_0^2 n_2|A|^2\Big] = 0, \quad (3)$$

where the critical plasma density is $\rho_{\mathrm{c}} \simeq 1.8\times10^{21}$ cm^{-3}. We can treat Eq.(3) as a Hamilton–Jacobi equation which describes the motion of a particle in a potential. The corresponding Hamiltonian is then

$$H = \frac{1}{2k_0}\Big[\Big(\frac{\partial\phi}{\partial x}\Big)^2 + \Big(\frac{\partial\phi}{\partial y}\Big)^2\Big] + V, \quad (4)$$

where the effective potential V is

$$V = -\frac{1}{2k_0}\Big[\frac{1}{A}\Big(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}\Big) - \frac{\rho}{\rho_{\mathrm{c}}} + k_0^2 n_2|A|^2\Big]. \quad (5)$$

The terms on the right-hand side represent the contributions of diffraction, defocusing and focusing. The trajectory of a ray is then analogous to the motion of a particle in the potential V. A ray is thus like a photon trajectory and the evolution of the laser pulse is approximated by a large number of rays. The potential then governs the trajectory (x,y) of the photons. The trajectory of a ray is then given by

$$\frac{\mathrm{d}^2 x}{\mathrm{d}z^2} = -\frac{1}{k_0}\frac{\partial V}{\partial x}, \quad (6)$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}z^2} = -\frac{1}{k_0}\frac{\partial V}{\partial y}, \quad (7)$$

so that the cross section of the laser beam can be estimated if the potential is known.

Note that the present approach is different from the traditional method of solving the two-dimensional NLS equation that describes the interaction between the slightly overlapped filaments.[13] The potential energy in the latter case, based on a perturbation theory, is the overlapping term (with respect to the two lasers) of the full Hamiltonian of the system. Ignoring the deformation of the filaments due to the interaction, the method treats the entire filament as a particle and the potential energy governs the filament's movement. The potential energy is a function of the separation between the two filaments. If the potential energy has a minimum, a bound state can be dealt with two solitons in the interaction. It has been applied to study the attractive and repulsive interactions when the system is unstable.[16] However, in our method each ray is treated as a particle and the potential governs the ray's movement. The common ground of the two methods is that the minimum of the potential energy gives a most stable state.

3. Results and discussion

We follow the simulation in Ref.[12] and calculate the potential field. In our simulation, the transverse waist of the filament is 0.1 mm. Figure 1 shows the spatiotemporal intensity pattern with a peak intensity of 5.75×10^{13} W/cm^2 and the calculated potential profile of the two initial light filaments with a 0.5 mm separation in between. Two potential wells are formed around the peaks of the laser pulses, and there are two

small potential barriers at the ascending part of the pulses as shown in Fig.1(b). This is because the high intensity in the centre of the each pulse makes a prominent contribution to the third term in the expression of the potential, which represents the focusing effect, forming two deep potential wells. When the peak intensity is high enough, the air molecules are ionized and the resulting free electrons contribute positively to the formation of potential. This leads to the two small potential barriers in the ascending part of the pulses in Fig.1(b), which is correspondingly to de-focusing.

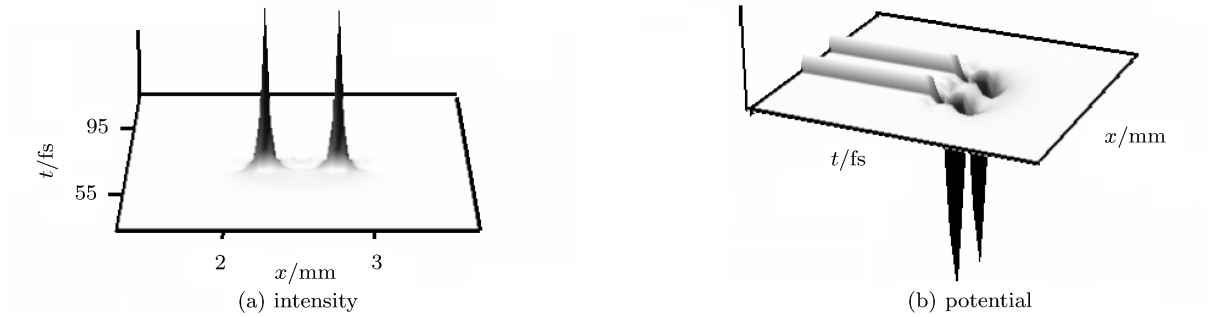


Fig.1. The spatiotemporal intensity. (a) and the potential profiles, (b) of two input light filaments with a peak intensity of 5.75×10^{13} W/cm^2.

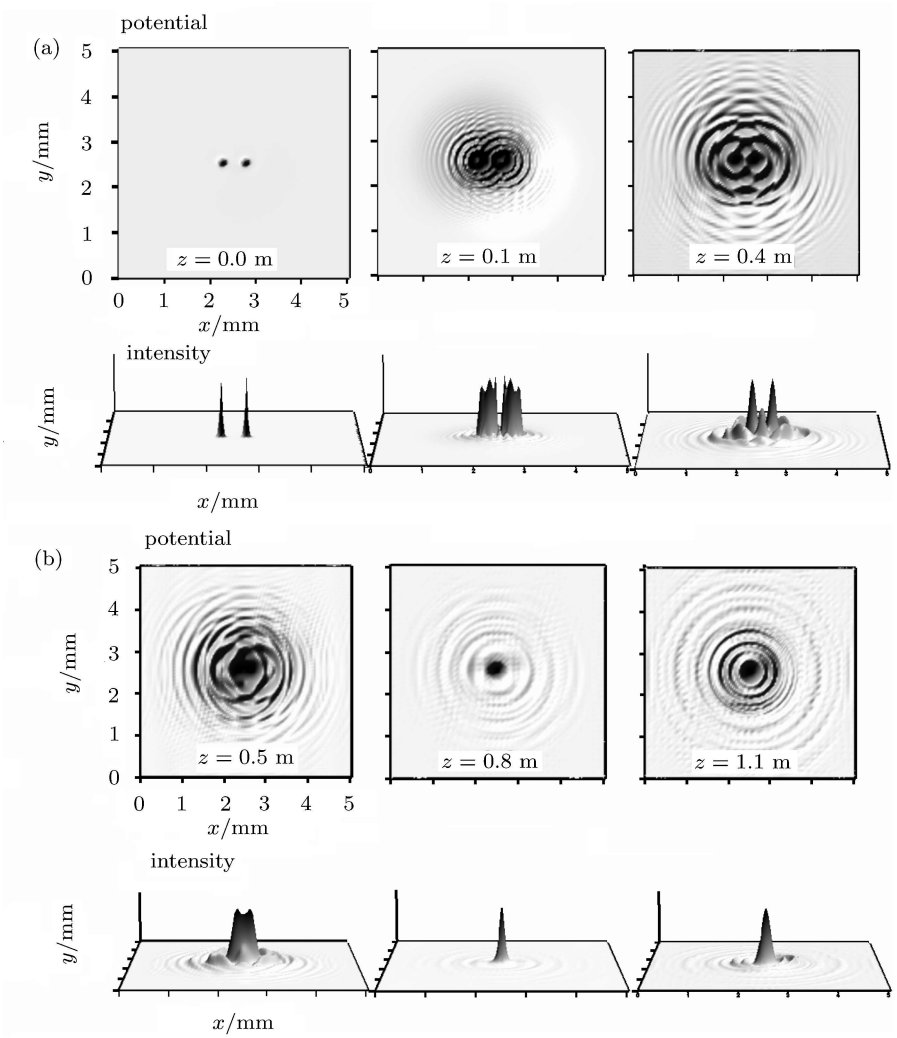


Fig.2. The potential field and the intensity profile (at $t = 70$ fs) produced by the two interacting in-phase filaments at the propagation distances of (a) 0, 0.1, 0.4, (b) 0.5, 0.8 and 1.1 m. The maximum intensities are 7.19×10^{12}, 4.3×10^{11}, 2.9×10^{11}, 3×10^{11}, 1.63×10^{12} and 5.4×10^{11} W/cm^2, respectively.

To see the evolution of the potential during the interaction between two in-phase light filaments, we show in Fig.2 the potential and intensity at different propagation distances. At the beginning, photons tend to fall into both of the original potential wells and their overlapping area, so that fewer photons appear near the walls of the wells but more photons are in the centre of the two wells. As a result, a small new potential well is formed in the centre. This can be understood in terms of the interference of the two in-phase pulses. The intensity of the superposition part is four times higher than the original, producing a potential well four times deeper than the original one in the overlapping region. The electrons created in the ionization stage suppress the growth of the laser intensity, so that the two deep wells become shallower and fatter. However, the small potential well in the centre can still accept more photons, so the energy will flow into the central well from the two wells. That is why the two wells tend to attract each other. In Fig.2, we see that the barrier between the two wells gradually disappears. The two potential wells merge into one and the two in-phase filaments coalesce, agreeing well with what are indicated in Ref.[12]. The coalescing process enhances the propagation distance compared with the original two filaments.

When the two filaments are out of phase and their separation is large enough, then they do not overlap in the transverse plane: the two potential wells develop without interaction between them. When the wells becomes fatter and somewhat overlap at around 0.15 m, as shown in Fig.3(a), a barrier is formed between the two potential wells because of the destructive interference of the two anti-phase filaments and the barrier remaining on further propagation. That is why the defocusing process can turn potential wells into fatter ones each, and they can expand without merging each other into one. As a result, from the development of the potential field, it is found that the two potential wells can only move away from each other, or the two filaments mutually repel and disperse quickly because of energy dissipation.

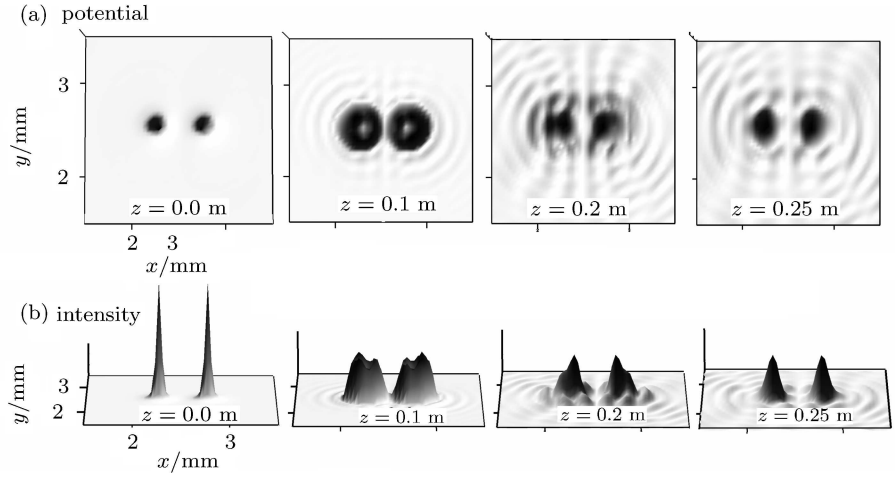


Fig.3. The potential field (a) and the following intensity profiles (b) at $t = 70$ fs formed by the two anti-phase filaments at 0, 0.1, 0.15 and 0.25 m, with the maximum intensities 2.3×10^{12}, 1.23×10^{12}, 1.39×10^{12} and 1.09×10^{12} W/cm^2, respectively.

Now we investigate the energy exchange between the two light filaments when they are neither completely in phase nor out of phase. Taking a phase difference of $\pi/2$ for example, we show in Fig.4 the evolutions of the potential and the corresponding intensity distributions. The overlapping potential slightly weighs to the left. More photons then visit the left well, which will become deeper. This can be explained by resorting to the equiphase surface. The equiphase surface of the two initial pulses inclines towards the left one which lags in phase, so does the direction of the ray. The ionization there quickly suppresses the intensity growing as well as the well deepening. As a result, photons move into the right well and the said focusing and defocusing processes continue. As shown in Fig.4, the new potential well in the overlapping region is very small at $z = 0.2$ m, which has no tendency to become deeper. The energy exchange between the two potential wells repeats several times and eventually only one filament survives. From the computation above, it is found that the potential field disperses quickly with the increase of the phase dif-

ference. The influence of the phase difference can be understood from the Kerr term in Eq.(5)

$$V_{\mathrm{Kerr}} = -\frac{k_0 n_2}{2}[|A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos\phi], \quad (8)$$

where $E_1 = A_1 \mathrm{e}^{\mathrm{i}\phi_1}$, $E_2 = A_2 \mathrm{e}^{\mathrm{i}\phi_2}$, and $\phi = \phi_1 - \phi_2$ is the phase difference. In the overlapping part of the two filaments, when the phase difference decreases from π to 0, the potential well caused by the Kerr effect, V_{Kerr}, becomes deeper and will attract more photons. This shows that the light channel can travel longer if two pulses are in phase[12] because the Kerr-effect contributions dominantly to the potential field.

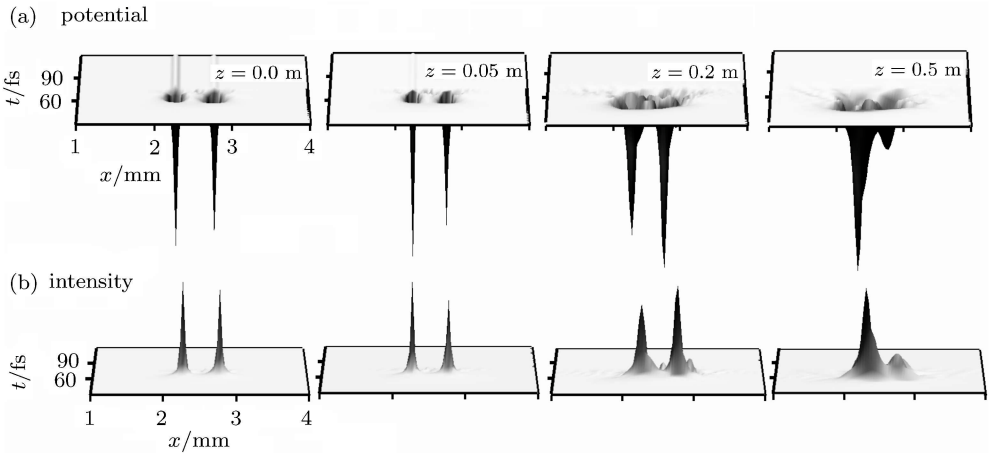


Fig.4. The spatiotemporal potential profiles (a) and intensity profiles (b) along the y-axis for two interacting filaments with a phase difference of $\pi/2$ at 0.05, 0.1, 0.2 and 0.5 m, with the maximum intensities 1.07×10^{13}, 9.5×10^{12}, 1.48×10^{13} and 4.15×10^{12} W/cm^2, respectively.

We now lower the separation between the two in-phase filaments to 0.28 mm. In this case the two filaments first coalesce into one at a distance of 0.2 m. Then the new filament again spreads its energy in both time and in space. As shown in Fig.5, the two filaments merge into one again at $z = 0.6$ m, but the energy still spreads in time. We choose the time interval in which one merged filaments spatially splits into two from $z = 0.3$ m to $z = 0.5$ m. Figure 6 shows the potential field and the intensity profiles. A small barrier gradually grows up and the potential well at $z = 0.3$ m gradually splits into two wells along the x-axis. The corresponding intensity also evolves from one peak to two peaks, corresponding to the evolution of the potential field.

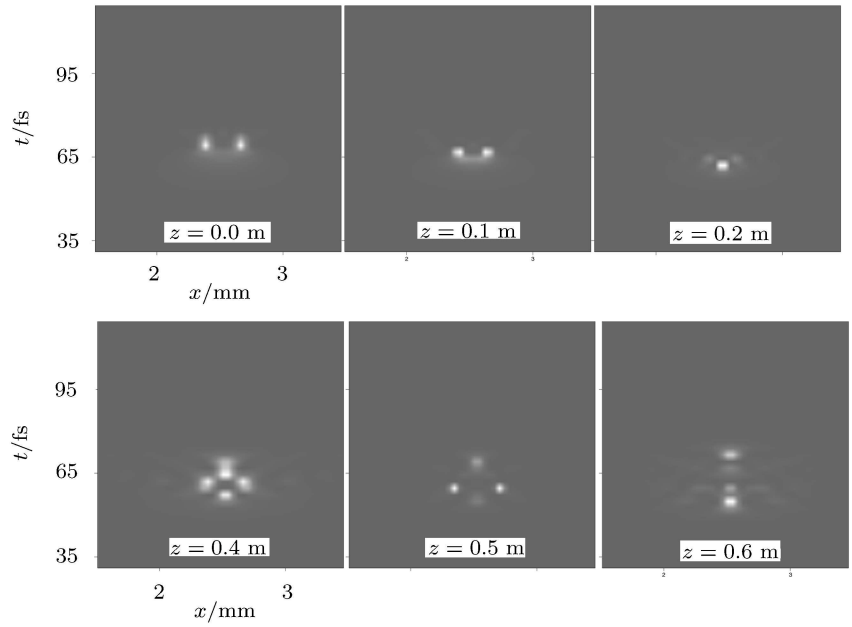


Fig.5. Intensity evolution of the two in-phase filaments with a separation of 0.28 mm.

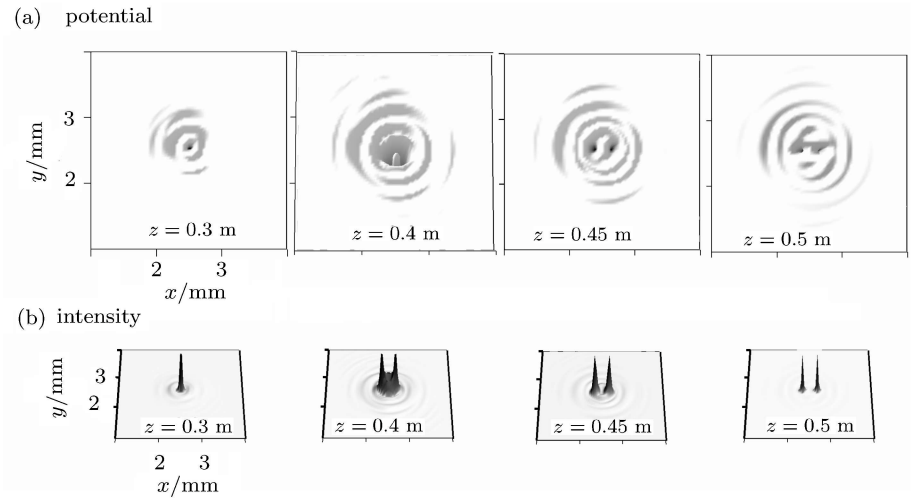


Fig.6. The potentials of the merging filament at $z = 0.3$, 0.4, 0.45, and 0.5 m. (b) The corresponding intensities with the maxima at 7.51×10^{12}, 1.99×10^{12}, 4.99×10^{12} and 1.57×10^{13} W/cm^2, respectively.

4. Conclusions

We have studied the potential field formed by merging and splitting filaments. The potential of the two filaments form two potential wells that can merge into one if the two filaments are in phase but repel each other if they are in anti-phase. The two filaments that have intermediate phase differences will lead to alternating potential well and energy exchanges. The potential well focuses and defocuses along the propagation direction and it will persist longer with the decrease of the phase difference between the two pulses. In addition, we have shown that the single potential well formed by merging filament can split into two and the filament will split into two accordingly. That is, by using the potential theory, we can predict the evolution of two (or more) laser pulses in a self-guided channel in air, in particular, their breakup or coalescence. The theory also provides an understanding of the energy exchange between the reservoir and the filaments as well as, if needed, the conditions necessary for very long distance laser channelling in air.

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