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Analytical structure of Hermite Gaussian beam in far field*

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Based on the vectorial structure of electromagnetic beam and the method of stationary phase, the analytical structure of Hermite Gaussian beam in far field is presented. The structural energy flux distributions are also investigated in the far field. The structural pictures of some Hermite Gaussian beams are depicted in the far field. As the structure of Hermite Gaussian beam is dominated by the transverse mode numbers and the initial transverse Gaussian half width, it is more complex than that of Gaussian beam. The ratios of the structural energy fluxes to the whole energy flux are independent of the transverse mode numbers and the initial transverse Gaussian half width. The present research reveals the internal vectorial structure of Hermite Gaussian beam from other viewpoint.

Keywords: Hermite Gaussian beam, vectorial structure, method of stationary phase, beam propagation
PACC: 4110H, 4225B

1. Introduction

The electromagnetic beam, which is described by the solution of Maxwell's equations, has an internal structure and is composed of the TE and TM terms.[1−3] The TE term denotes the electric field transverse to the propagation axis, and the TM term represents the associated magnetic field transverse to the propagation axis. The spatial azimuthal relation between the TE and TM terms is varied upon propagation. Only in the far field can the TE and TM terms be orthogonal to each other. According to this fact, the vectorial structure of non-paraxial linearly polarized Gaussian beam has been presented, and the influence of linearly polarized angle on vectorial structure has been also investigated.[4] Combined with non-paraxial vectorial moment theory of light beam propagation, the relationship among the beam waists, the divergence angles and the beam propagation factors of non-paraxial linearly polarized Gaussian beam, its TE and TM term have been studied.[5] Therefore, the characteristics of vectorial structure and structural propagation of Gaussian beam have been well revealed. However, the Gaussian beam only represents the structure of a fundamental mode of laser cavity.[6] One may be curious to know the vectorial structures of the high order modes. The beam of high order mode with Cartesian symmetry is Hermite Gaussian beam.[7] For simplicity, the polarization of Hermite Gaussian beam considered here is parallel to the x-axis. The properties of Hermite Gaussian beam have been studied extensively.[8−10] The purpose of the present paper is just to exhibit the internal structure of Hermite Gaussian beam and investigate the influence of the transverse mode numbers and the initial transverse Gaussian half width on the structure.

2. Analytical structure in the far field

In the Cartesian coordinate system, the half space $z \geq 0$ is filled with a homogeneous, isotropic, non-conducting and transparent medium with electric permittivity ε and magnetic permeability μ. A Hermite Gaussian beam propagates toward the half space $z \geq 0$, and the z-axis is taken as the propagation axis. The initial transverse electric field of Hermite Gaussian beam in the source plane $z = 0$ is described by

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$$\begin{pmatrix} E_x(x,y,0) \\ E_y(x,y,0) \end{pmatrix} = \begin{pmatrix} H_m\Big(\dfrac{\sqrt{2}x}{w_0}\Big)H_n\Big(\dfrac{\sqrt{2}y}{w_0}\Big)\exp\Big(-\dfrac{\rho^2}{w_0^2}\Big) \\ 0 \end{pmatrix}, \tag{1}$$

where w_0 is the initial transverse Gaussian half width; H_j is the jth order Hermite polynomial; m and n are the transverse mode numbers in the x- and y- directions, respectively; $\rho = (x^2 + y^2)^{1/2}$. The initial longitudinal electric field can be given by the divergence theorem of electric field. According to the full vector angular spectrum representation of electromagnetic field,[11,12] the propagating electric field of Hermite Gaussian beam at a point with position vector $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ is given by

$$E_x(\boldsymbol{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_x(p,q,\gamma)\exp(\mathrm{i}k\boldsymbol{r}\cdot\boldsymbol{s})\mathrm{d}p\,\mathrm{d}q, \tag{2a}$$

$$E_y(\boldsymbol{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_y(p,q,\gamma)\exp(\mathrm{i}k\boldsymbol{r}\cdot\boldsymbol{s})\mathrm{d}p\,\mathrm{d}q, \tag{2b}$$

$$E_z(\boldsymbol{r}) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Big(\frac{p}{\gamma}A_x(p,q,\gamma) + \frac{q}{\gamma}A_y(p,q,\gamma)\Big) \times \exp(\mathrm{i}k\boldsymbol{r}\cdot\boldsymbol{s})\mathrm{d}p\,\mathrm{d}q, \tag{2c}$$

where $\gamma = (1-p^2-q^2)^{1/2}$; $\boldsymbol{s} = p\boldsymbol{i}+q\boldsymbol{j}+\gamma\boldsymbol{k}$; $k = 2\pi/\lambda$ is the wave number; $\lambda = 2\pi/\omega(\mu\varepsilon)^{1/2}$ is the light wavelength in the medium; ω is the circular frequency. The dot denotes scalar product. According to the initial boundary condition of the transverse electric field, the angular spectrum is given by

$$\begin{aligned} A_x(p,q,\gamma) &= \frac{1}{\lambda^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_x(x,y,0) \\ &\quad\times\exp[-\mathrm{i}k(px+qy)]\mathrm{d}x\mathrm{d}y \\ &= \frac{(-\mathrm{i})^{m+n}}{4\pi f^2}\exp\Big(-\frac{p^2+q^2}{4f^2}\Big) \\ &\quad\times H_m\Big(\frac{p}{\sqrt{2}f}\Big)H_n\Big(\frac{q}{\sqrt{2}f}\Big), \end{aligned} \tag{3a}$$

$$\begin{aligned} A_y(p,q,\gamma) &= \frac{1}{\lambda^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_y(x,y,0) \\ &\quad\times\exp[-\mathrm{i}k(px+qy)]\mathrm{d}x\mathrm{d}y = 0, \end{aligned} \tag{3b}$$

where $f = 1/kw_0$. The whole vector angular spectrum $\boldsymbol{A}(p,q,\gamma)$ is identical and expressed as

$$\begin{aligned} \boldsymbol{A}(p,q,\gamma) &= A_x(p,q,\gamma)\boldsymbol{i} + A_y(p,q,\gamma)\boldsymbol{j} \\ &\quad-\Big(\frac{pA_x(p,q,\gamma)+qA_y(p,q,\gamma)}{\gamma}\Big)\boldsymbol{k} \\ &= A_x(p,q,\gamma)\Big(\boldsymbol{i}-\frac{p}{\gamma}\boldsymbol{k}\Big). \end{aligned} \tag{4}$$

By substituting Eq.(3) into Eqs.(2), the propagating electric field of Hermite Gaussian beam, $\boldsymbol{E}(\boldsymbol{r}) = E_x(\boldsymbol{r})\boldsymbol{i}+E_y(\boldsymbol{r})\boldsymbol{j}+E_z(\boldsymbol{r})\boldsymbol{k}$, can be obtained. By resolving Maxwell's equations, the corresponding magnetic field of Hermite Gaussian beam is found to be

$$\begin{aligned} \boldsymbol{H}(\boldsymbol{r}) = &-\sqrt{\frac{\varepsilon}{\mu}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Big(\frac{pq}{\gamma}\boldsymbol{i} - \frac{1-q^2}{\gamma}\boldsymbol{j} + q\boldsymbol{k}\Big) \\ &\times A_x(p,q,\gamma)\exp(\mathrm{i}k\boldsymbol{r}\cdot\boldsymbol{s})\mathrm{d}p\,\mathrm{d}q. \end{aligned} \tag{5}$$

In the paraxial case, γ in the exponent can be expanded as follows:

$$\gamma = 1 - \frac{p^2+q^2}{2}. \tag{6}$$

Therefore, Eq.(2a) reduces to a conventional Hermite Gaussian beam[13]

$$\begin{aligned} E_x(\boldsymbol{r}) = &\frac{1}{1+\mathrm{i}z/z_r}H_m\Big(\frac{\sqrt{2}x}{w_0\sqrt{1+\mathrm{i}z/z_r}}\Big) \\ &\times H_n\Big(\frac{\sqrt{2}y}{w_0\sqrt{1+\mathrm{i}z/z_r}}\Big) \\ &\times\exp\Big[-\frac{\rho^2}{w_0^2(1+\mathrm{i}z/z_r)} \\ &-\mathrm{i}(m+n)\mathrm{arc}\tan^{-1}\Big(\frac{z}{z_r}\Big)+\mathrm{i}kz\Big], \end{aligned} \tag{7}$$

where $z_r = kw_0^2/2$ is the confocal parameter. As both the non-paraxial and paraxial cases are under our consideration, the electromagnetic fields of Hermite Gaussian beam should be expressed in the integral form.

Note that the electromagnetic field of Hermite Gaussian beam is obtained by Fourier transform. In the frequency domain, two unit vectors $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ can be defined as follows:[1−3]

$$\begin{aligned} \boldsymbol{e}_1 &= \frac{q}{(p^2+q^2)^{1/2}}\boldsymbol{i} - \frac{p}{(p^2+q^2)^{1/2}}\boldsymbol{j}, \\ \boldsymbol{e}_2 &= \frac{p\gamma}{(p^2+q^2)^{1/2}}\boldsymbol{i} + \frac{q\gamma}{(p^2+q^2)^{1/2}}\boldsymbol{j} - (p^2+q^2)^{1/2}\boldsymbol{k}. \end{aligned} \tag{8}$$

Thus, the three unit vectors $\boldsymbol{s}$, $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ form a mutually perpendicular right-handed system

$$\boldsymbol{s}\times\boldsymbol{e}_1 = \boldsymbol{e}_2, \quad \boldsymbol{e}_1\times\boldsymbol{e}_2 = \boldsymbol{s}, \quad \boldsymbol{e}_2\times\boldsymbol{s} = \boldsymbol{e}_1. \tag{9}$$

In this system, the vector angular spectrum $\boldsymbol{A}(p,q,\gamma)$ can be decomposed into two terms

$$\boldsymbol{A}(p,q,\gamma)=[\boldsymbol{A}(p,q,\gamma)\cdot\boldsymbol{e}_1]\boldsymbol{e}_1+[\boldsymbol{A}(p,q,\gamma)\cdot\boldsymbol{e}_2]\boldsymbol{e}_2. \quad (10)$$

Accordingly, the propagating electric field of Hermite Gaussian beam can be decomposed into the TE and TM terms

$$\boldsymbol{E}(\boldsymbol{r})=\boldsymbol{E}_{\mathrm{TE}}(\boldsymbol{r})+\boldsymbol{E}_{\mathrm{TM}}(\boldsymbol{r}), \quad (11)$$

with

$$\begin{aligned}\boldsymbol{E}_{\mathrm{TE}}(\boldsymbol{r})&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[\boldsymbol{A}(p,q,\gamma)\cdot\boldsymbol{e}_1]\boldsymbol{e}_1\exp(\mathrm{i}k\boldsymbol{r}\cdot\boldsymbol{s})\mathrm{d}p\,\mathrm{d}q\\&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{qA_x(p,q,\gamma)}{(p^2+q^2)}(q\boldsymbol{i}-p\boldsymbol{j})\\&\quad\times\exp(\mathrm{i}k\boldsymbol{r}\cdot\boldsymbol{s})\mathrm{d}p\,\mathrm{d}q, \quad (12)\\\boldsymbol{E}_{\mathrm{TM}}(\boldsymbol{r})&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[\boldsymbol{A}(p,q,\gamma)\cdot\boldsymbol{e}_2]\boldsymbol{e}_2\exp(\mathrm{i}k\boldsymbol{r}\cdot\boldsymbol{s})\mathrm{d}p\,\mathrm{d}q\\&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{pA_x(p,q,\gamma)}{(p^2+q^2)}\Big(p\boldsymbol{i}+q\boldsymbol{j}-\frac{p^2+q^2}{\gamma}\boldsymbol{k}\Big)\\&\quad\times\exp(\mathrm{i}k\boldsymbol{r}\cdot\boldsymbol{s})\mathrm{d}p\,\mathrm{d}q. \quad (13)\end{aligned}$$

As the divergence condition of the electric field should be satisfied and the polarized direction of each plane wave component must be perpendicular to its own wave vector, the decomposition of electric field into the TE and TM terms is unique. Similarly, the propagating magnetic field of Hermite Gaussian beam can also be expressed as a sum of the corresponding TE and TM terms

$$\boldsymbol{H}(\boldsymbol{r})=\boldsymbol{H}_{\mathrm{TE}}(\boldsymbol{r})+\boldsymbol{H}_{\mathrm{TM}}(\boldsymbol{r}), \quad (14)$$

with

$$\begin{aligned}\boldsymbol{H}_{\mathrm{TE}}(\boldsymbol{r})&=\sqrt{\frac{\varepsilon}{\mu}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{qA_x(p,q,\gamma)}{\gamma(p^2+q^2)}\\&\quad\times\Big(p\boldsymbol{i}+q\boldsymbol{j}-\frac{p^2+q^2}{\gamma}\boldsymbol{k}\Big)\\&\quad\times\exp(\mathrm{i}k\boldsymbol{r}\cdot\boldsymbol{s})\mathrm{d}p\,\mathrm{d}q, \quad (15)\\\boldsymbol{H}_{\mathrm{TM}}(\boldsymbol{r})&=-\sqrt{\frac{\varepsilon}{\mu}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{pA_x(p,q,\gamma)}{\gamma(p^2+q^2)}(q\boldsymbol{i}-p\boldsymbol{j})\\&\quad\times\exp(\mathrm{i}k\boldsymbol{r}\cdot\boldsymbol{s})\mathrm{d}p\,\mathrm{d}q. \quad (16)\end{aligned}$$

The time-dependent factor $\exp(-\mathrm{i}\omega t)$ has been omitted in the representations of the electromagnetic fields. Here, the TE and TM terms denote that the z-components of the electric and the magnetic fields are both equal to zero. We can conclude that the electric field of Hermite Gaussian beam can be decomposed into E_x, E_y and E_z in the spatial domain, while the decomposition of the electric field into the TE and TM terms is carried out in the frequency domain.

When z is larger than several times the light wavelength, the evanescent plane waves have almost fallen off. As z is large enough in the far field, the evanescent plane waves have completely disappeared. Thus, the integrals in Eqs.(12), (13), (15) and (16) should be restricted to a range of $0\le p^2+q^2\le 1$. However, the method of stationary phase is used to obtain the analytical propagating electromagnetic fields of TE and TM terms in the far field. Therefore, the limits of integral are still taken as negative infinity and positive infinity. In the far field regime, the condition $kr=k(x^2+y^2+z^2)^{1/2}\to\infty$ is satisfied. If the electric field is expressed in the vector angular representations as

$$\boldsymbol{E}(\boldsymbol{r})=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\boldsymbol{A}(p,q,\gamma)\exp(\mathrm{i}k\boldsymbol{r}\cdot\boldsymbol{s})\mathrm{d}p\,\mathrm{d}q, \quad (17)$$

the electric field in the far field can be given in the analytical form by using the method of stationary phase[14,15]

$$\boldsymbol{E}(\boldsymbol{r})=-\frac{\mathrm{i}\lambda z}{r^2}\boldsymbol{A}\Big(\frac{x}{r},\frac{y}{r},\frac{z}{r}\Big)\exp(\mathrm{i}kr). \quad (18)$$

Therefore, the analytical electromagnetic field of TE term in the far field is found to be

$$\boldsymbol{E}_{\mathrm{TE}}(\boldsymbol{r})=(-\mathrm{i})^{m+n+3}\frac{z_r yz}{r^2\rho^2}\exp\Big(-\frac{\rho^2}{4f^2r^2}+\mathrm{i}kr\Big)H_m\Big(\frac{x}{\sqrt{2}fr}\Big)H_n\Big(\frac{y}{\sqrt{2}fr}\Big)(y\boldsymbol{i}-x\boldsymbol{j}), \quad (19)$$

$$\boldsymbol{H}_{\mathrm{TE}}(\boldsymbol{r})=(-\mathrm{i})^{m+n+3}\sqrt{\frac{\varepsilon}{\mu}}\frac{z_r yz}{r^3\rho^2}\exp\Big(-\frac{\rho^2}{4f^2r^2}+\mathrm{i}kr\Big)H_m\Big(\frac{x}{\sqrt{2}fr}\Big)H_n\Big(\frac{y}{\sqrt{2}fr}\Big)(xz\boldsymbol{i}+yz\boldsymbol{j}-\rho^2\boldsymbol{k}). \quad (20)$$

Similarly, the analytical electromagnetic field of TM term in the far field is given by

$$\boldsymbol{E}_{\mathrm{TM}}(\boldsymbol{r})=(-\mathrm{i})^{m+n+3}\frac{z_r x}{r^2\rho^2}\exp\Big(-\frac{\rho^2}{4f^2r^2}+\mathrm{i}kr\Big)H_m\Big(\frac{x}{\sqrt{2}fr}\Big)H_n\Big(\frac{y}{\sqrt{2}fr}\Big)(xz\boldsymbol{i}+yz\boldsymbol{j}-\rho^2\boldsymbol{k}), \quad (21)$$

$$\boldsymbol{H}_{\rm TM}(\boldsymbol{r}) = (-{\rm i})^{m+n+1}\sqrt{\frac{\varepsilon}{\mu}}\frac{z_r x}{r\rho^2}\exp\Big(-\frac{\rho^2}{4f^2r^2}+{\rm i}kr\Big)H_m\Big(\frac{x}{\sqrt{2}fr}\Big)H_n\Big(\frac{y}{\sqrt{2}fr}\Big)(y\boldsymbol{i}-x\boldsymbol{j}). \tag{22}$$

Apparently, the electromagnetic fields of TE and TM terms are orthogonal to each other in the far field. As the range of w_0 is not limited, the analytical TE and TM terms of Hermite Gaussian beam are applicable not only to the paraxial case, but also to the non-paraxial case.

3. Energy flux distributions in the far field

According to the above analytical electromagnetic representations of TE and TM terms, their energy flux distributions in the far field can be investigated. The energy flux distribution in the plane $z =$ constant is described by the z-component of the time-average Poynting vector

$$\langle S_z\rangle = {\rm Re}[\boldsymbol{E}(\boldsymbol{r})\times\boldsymbol{H}^*(\boldsymbol{r})]_z, \tag{23}$$

where Re means taking the real part of an expression, and the asterisk denotes complex conjugation. Therefore, the energy flux distributions of TE and TM terms in the far field plane $z =$ constant are respectively given by

$$\langle S_z\rangle_{\rm TE} = \sqrt{\frac{\varepsilon}{\mu}}\frac{z_r^3y^2z^3}{r^5\rho^2}\exp\Big(-\frac{\rho^2}{2f^2r^2}\Big) \times H_m^2\Big(\frac{x}{\sqrt{2}fr}\Big)H_n^2\Big(\frac{y}{\sqrt{2}fr}\Big), \tag{24}$$

$$\langle S_z\rangle_{\rm TM} = \sqrt{\frac{\varepsilon}{\mu}}\frac{z_r^2x^2z}{r^3\rho^2}\exp\Big(-\frac{\rho^2}{2f^2r^2}\Big) \times H_m^2\Big(\frac{x}{\sqrt{2}fr}\Big)H_n^2\Big(\frac{y}{\sqrt{2}fr}\Big). \tag{25}$$

The energy flux distribution of Hermite Gaussian beam yields

$$\begin{aligned}\langle S_z\rangle &= {\rm Re}[(\boldsymbol{E}_{\rm TE}+\boldsymbol{E}_{\rm TM})\times(\boldsymbol{H}_{\rm TE}^*+\boldsymbol{H}_{\rm TM}^*)]_z\\ &= {\rm Re}(\boldsymbol{E}_{\rm TE}\times\boldsymbol{H}_{\rm TE}^*+\boldsymbol{E}_{\rm TM}\times\boldsymbol{H}_{\rm TM}^*\\ &\quad+\boldsymbol{E}_{\rm TM}\times\boldsymbol{H}_{\rm TE}^*+\boldsymbol{E}_{\rm TE}\times\boldsymbol{H}_{\rm TM}^*)_z\\ &= {\rm Re}(\boldsymbol{E}_{\rm TE}\times\boldsymbol{H}_{\rm TE}^*+\boldsymbol{E}_{\rm TM}\times\boldsymbol{H}_{\rm TM}^*)\\ &= \langle S_z\rangle_{\rm TE}+\langle S_z\rangle_{\rm TM}\\ &= \sqrt{\frac{\varepsilon}{\mu}}\frac{z_r^2z}{r^3\rho^2}\exp\Big(-\frac{\rho^2}{2f^2r^2}\Big)H_m^2\Big(\frac{x}{\sqrt{2}fr}\Big)\\ &\quad\times H_n^2\Big(\frac{y}{\sqrt{2}fr}\Big)\Big(y^2\frac{z^2}{r^2}+x^2\Big).\end{aligned} \tag{26}$$

The crossed terms are equal to zero because the TE and TM terms are orthogonal to each other.

If $m = n = 0$, the Hermite Gaussian beam reduces to a Gaussian beam. In this case, the formulae are the same as those in Ref.[4] with a linearly polarized angle $\alpha = 0°$. The structural energy flux distributions of different Hermite Gaussian beams in a reference plane $z = 500\lambda$ are depicted in Figs.1–3. w_0 is set to be 5λ, which corresponds to the paraxial case. As the medium is homogeneous, ε/μ is set to be 1. In Figs.1–4, subfigures (a) and (b) represent the energy flux distributions of the TE term and TM term respectively, and subfigure (c) refers to the sum of energy flux distributions (a) and (b), i.e. the energy flux distribution of the whole beam. In Fig.1 $m = 1$ and $n = 0$. In Fig.2 $m = n = 1$. In Fig.3 $m = 2$ and $n = 0$. As shown in Fig.1, the beam spot of TE term is composed of four lobes. Moreover, the orientation of the two lobes is at 45° with respect to the x-axis, and that of the other two lobes, at 135° with respect to the x-axis. The beam spot of TM term is comprised of two lobes and located at the x-axis. The energy flux intensity of TM term is larger than that of TE term. The beam spot of $\rm TEM_{10}$ mode is composed of two lobes and located at the x-axis. When $m = n = 1$, the beam spots of TE and TM terms each consist of four lobes. Moreover, the energy flux intensity of TE term is identical to that of TM term. However, the spatial orientation of TE term is different from that of TM term. The TE term is nearer to the y-axis than to the x-axis, and the TM term is nearer to the x-axis than to the y-axis. When the energy fluxes of TE and TM terms are superposed, the distance from the whole beam to the x-axis is identical to that from the whole beam to the y-axis. As to $\rm TEM_{20}$ mode, the TE term is composed of four side lobes and one central lobe. The energy fluxes of side lobes are very small. The shape of central lobe is similar to 8-figure. Three lobes constitute a TM term. The central lobe of TM term is similar to "∞" in shape. The energy flux of TE term is smaller than that of TM term. If w_0 is smaller than or of the order of the wavelength, the spot shapes of TE and TM terms will vary though the

m and n are kept unchanged as shown in Fig.4 where w_0 is equal to 0.2λ, which represents the non-paraxial case, $m = 2$, and $n = 0$. Apparently, Fig.4 is different from Fig.3. In the non-paraxial case, the structure of Hermite Gaussian beam is dominated mainly by the Gaussian function, and the effect of Hermite polynomial is trivial.

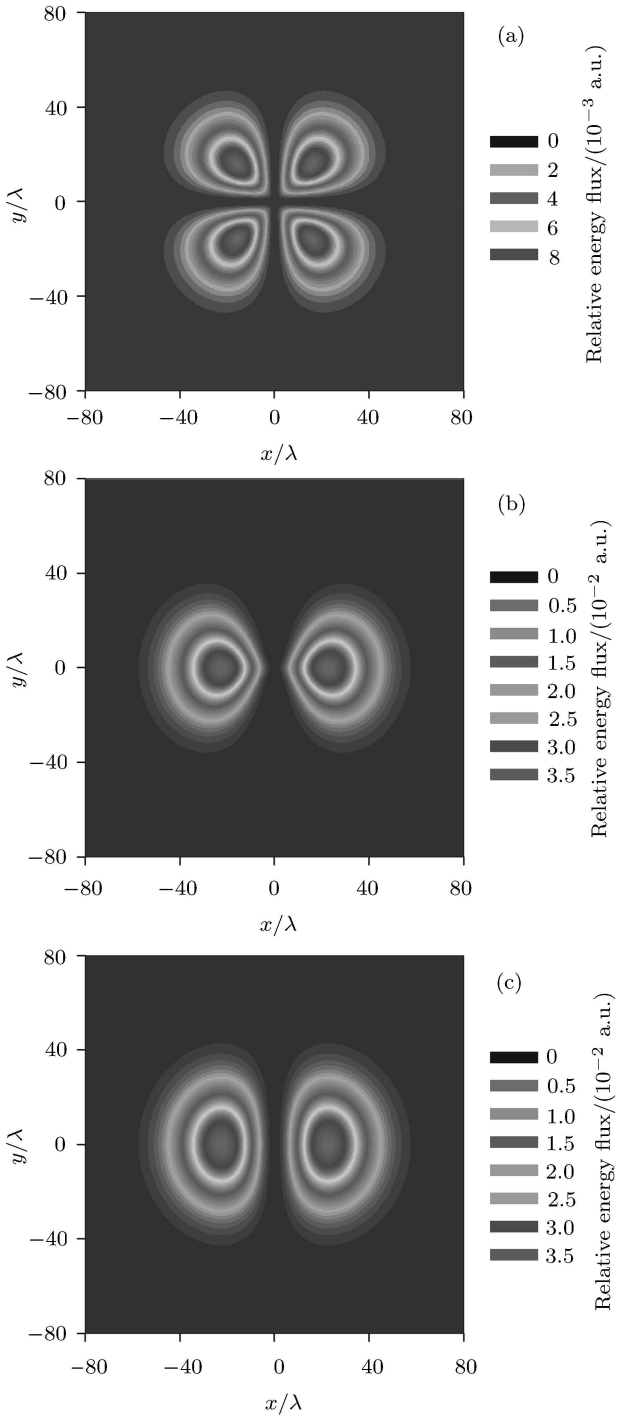


Fig.1. The energy flux distribution in the plane $z = 500\lambda$. $w_0 = 5\lambda$, $m = 1$, and $n = 0$. (a) The TE term, (b) the TM term, (c) the whole beam.

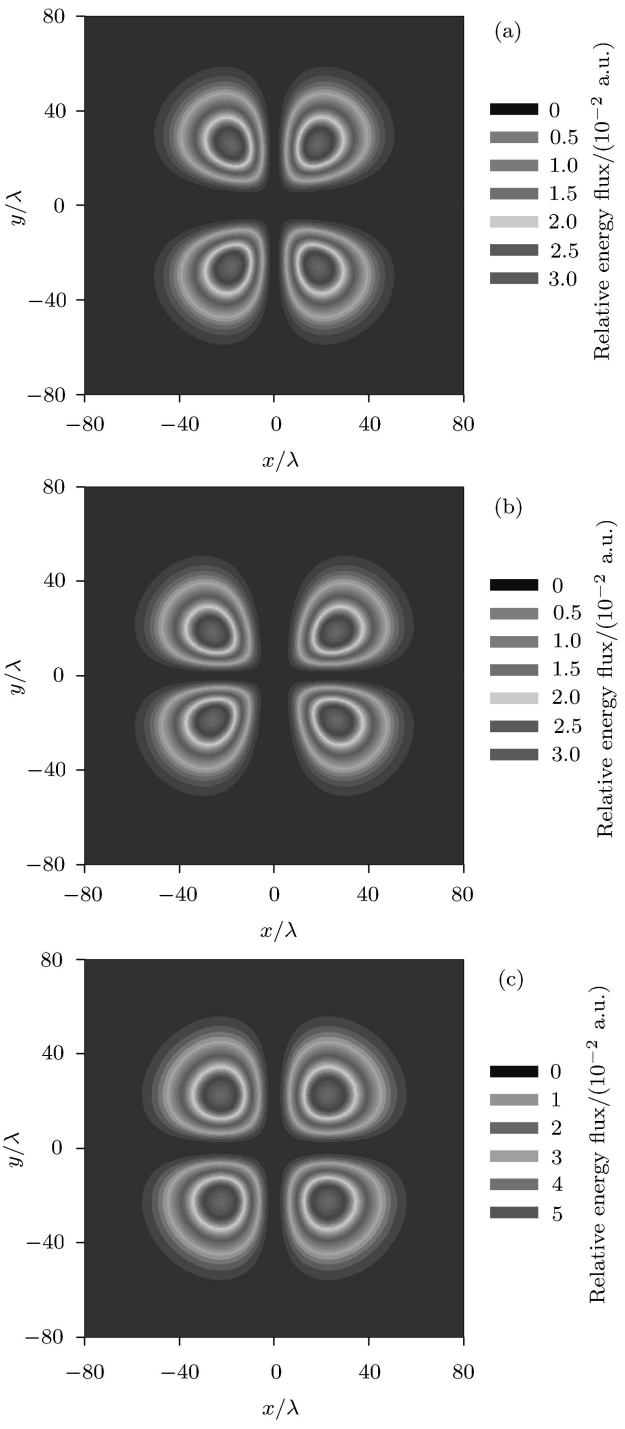


Fig.2. The energy flux distribution in the plane $z = 500\lambda$. $w_0 = 5\lambda$, and $m = n = 1$. (a) The TE term. (b) the TM term, (c) the whole beam.

Though the structures of Hermite Gaussian beams are different, depending on the values of m, n and w_0, the ratios of the energy fluxes of TE and TM terms to the whole energy flux are independent of the transverse mode numbers and the initial trans-

verse Gaussian half width as shown below:

$$T_1 = \frac{\langle S_z \rangle_{\mathrm{TE}}}{\langle S_z \rangle} = \frac{y^2 z^2}{\rho^2 (z^2 + x^2)},$$

$$T_2 = \frac{\langle S_z \rangle_{\mathrm{TM}}}{\langle S_z \rangle} = \frac{r^2 x^2}{\rho^2 (z^2 + x^2)}. \tag{27}$$

Figure 5 depicts the variations of T_1 and T_2 in a reference plane $z = 500\lambda$. The number in Figs.5(a) and 5(b) are the values of T_1 and T_2 respectively. The sum of T_1 and T_2 is just equal to unity.

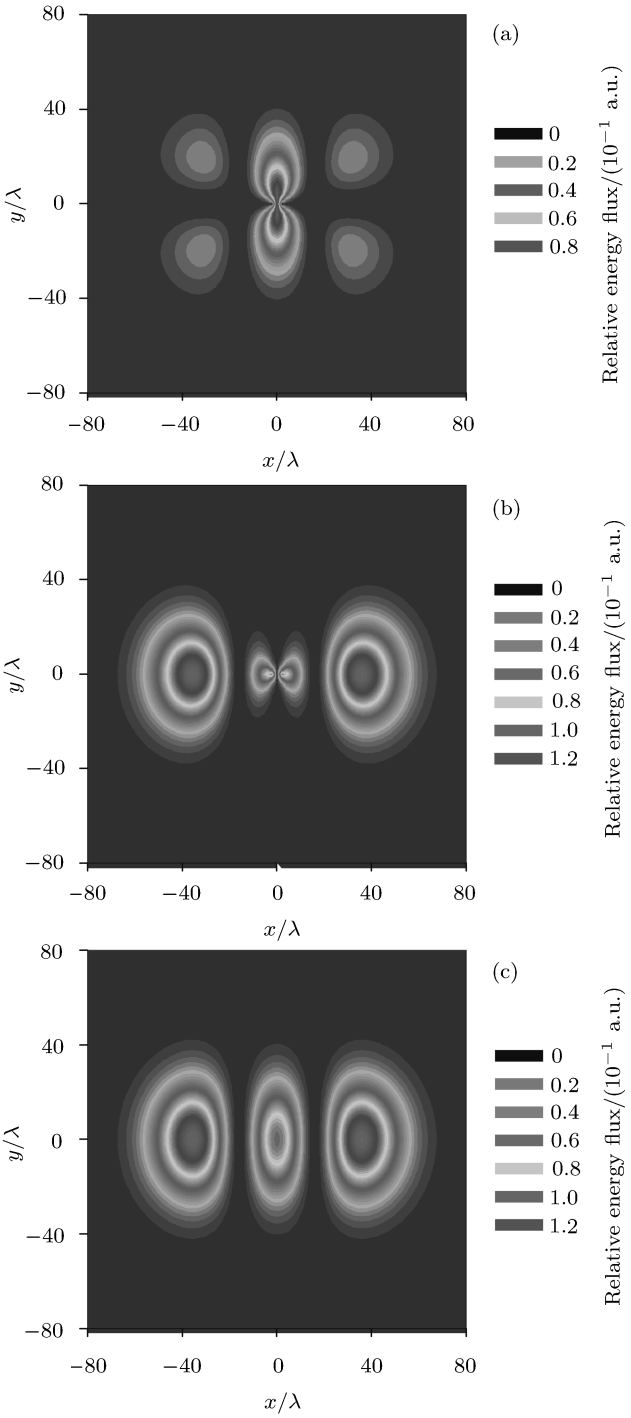


Fig.3. The energy flux distribution in the plane $z = 500\lambda$. $w_0 = 0.2\lambda$, $m = 2$, and $n = 0$. (a) The TE term, (b) the TM term, (c) the whole beam.

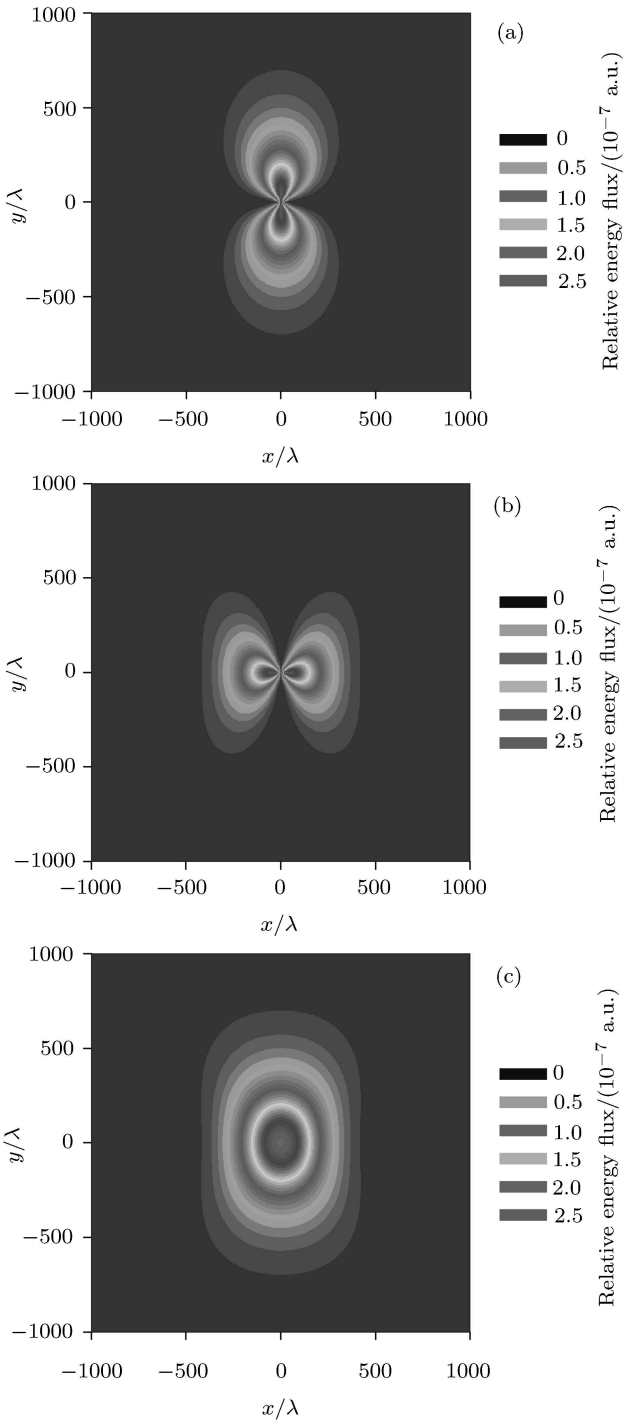


Fig.4. The energy flux distribution in the plane $z = 500\lambda$. $w_0 = 0.2\lambda$, $m = 2$, and $n = 0$. (a) The TE term, (b) the TM term, (c) the whole beam.

A comparison shows that the structure of Hermite Gaussian beam is more complex than that of Gaussian beam. The transverse mode numbers can influence the spot shapes, spatial orientations and energy fluxes of TE and TM terms. The initial transverse Gaussian half width can also affect the spot shapes of TE and TM terms.

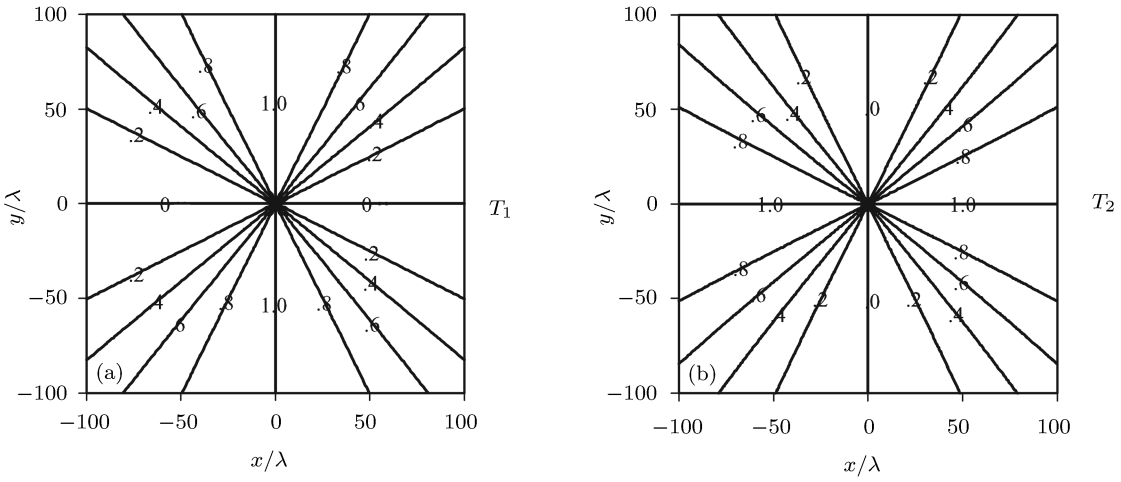


Fig.5. The ratios of the structural energy fluxes to the whole energy flux in a reference plane $z = 500\lambda$. The number in (a) and (b) are values of T_1 and T_2 respectively.

4. Conclusion

Based on the vectorial structure of electromagnetic beam and the method of stationary phase, the analytical structure of Hermite Gaussian beam in far field is presented. According to the analytical electromagnetic representations of TE and TM terms, the energy flux distributions of TE and TM terms in the far field are investigated. The formulae obtained are applicable not only to the paraxial case, but also to the non-paraxial case. The structural pictures of some Hermite Gaussian beams in the far field are depicted. The structure of paraxial Hermite Gaussian beam is dominated primarily by the transverse mode numbers, while that of non-paraxial Hermite Gaussian beam is dominated mainly by the initial transverse Gaussian half width. The ratios of the structural energy fluxes to the whole energy flux are independent of the transverse mode numbers and the initial transverse Gaussian half width. The decomposition of Hermite Gaussian beam starts from the frame of frequency domain. Therefore, the present research reveals the internal vectorial structure of Hermite Gaussian beam from other viewpoint. The results obtained here can be regarded as the supplement to the Hermite Gaussian, which is described by an approximate solution of the Helmholtz equation. Also, the results are helpful to the investigation of the propagation of elegant Hermite Gaussian beam and Hermite–Laguerre–Gaussian beam.[16−18]

References

[1] Martínez-Herrero R, Mejías P M, Bosch S and Carnicer A 2001 *J. Opt. Soc. Am.* A **18** 1678

[2] Mejías P M, Martínez-Herrero R, Piquero G and Movilla J M 2002 *Prog. Quantum Electron.* **26** 65

[3] Guo H M, Chen J B and Zhuang S L 2006 *Opt. Express* **14** 2095

[4] Zhou G Q, Chen L and Ni Y Z 2006 *Chin. Phys. Lett.* **23** 1180

[5] Zhou G Q 2006 *Opt. Commun.* **265** 39

[6] Kogelnik H and Li T 1966 *Proc. IEEE* **54** 1312

[7] Luk K M and Yu P K 1985 *J. Opt. Soc. Am.* A **2** 1818

[8] Kim H C and Lee Y H 1999 *Opt. Commun.* **169** 9

[9] Wang J X, Sceid W, Hoelss M and Ho Y K 2001 *Phys. Rev.* E **64** 066612

[10] Cai Y J and Lin Q 2003 *J. Opt. Soc. Am.* A **20** 1111

[11] Rhodes D R 1964 *Proc. IEEE* **52** 1013

[12] Rhodes D R 1966 *IEEE Trans. Antennas Propag.* **AP-14** 676

[13] Siegman A E 1986 *Lasers* (Mill Valley, CA: University Science Books)

[14] Carter W H 1972 *J. Opt. Soc. Am.* **62** 1195

[15] Mandel L and Wolf E 1995 *Optical Coherence and Quantum Optics* (Cambridge: Cambridge University Press)

[16] Wang B Z, Zhao Z G, Lü B D and Duan K L 2007 *Chin. Phys.* **16** 143

[17] Liu Z J and Lü B D 2003 *Chin. Phys.* **12** 879

[18] Dan Y Q and Zhang B 2006 *Acta Phys. Sin.* **55** 713 (in Chinese)