Vol 16 No 9, September 2007
1009-1963/2007/16(09)/2689-12

Chinese Physics

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Intermittency in 3.7 A GeV ^{16}O-emulsion interactions*

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(Received 12 November 2006; revised manuscript received 25 December 2006)

The intermittency effect has been studied for an interaction of 3.7 A GeV ^{16}O with emulsion using the distributions of both the pseudorapidity intervals and the azimuthal angle intervals of the shower particles emitted in a central rapidity region. The scaled factorial moments, reduced scaled factorial moments and multifractal moments as functions of the bin size in pseudorapidity and in azimuthal angle have been calculated and have revealed the presence of an intermittent behaviour which may be due to the random cascading property of the reaction. The anomalous fractal dimension has been found to increase with the increase of rank of the moment.

Keywords: relativistic heavy-ion collision, intermittency, nuclear emulsion
PACC: 2570N, 2940R

1. Introduction

The study of nonstatistical fluctuations in relativistic nuclear collisions has recently attracted a great deal of attention due to the possibility of disentangling important information about the mechanism of multiparticle production in such collisions. Investigations relating to nonlinear phenomenon in high energy collisions have aroused considerable interest since the observation of the unexpectedly large local fluctuations in a very high multiplicity event recorded by the Japanese–American Cooperative Emulsion Experiment (JACEE) Collaboration.[1] In a series of papers, Bialas and Peschanski[2] have developed a method of analysing the data on high energy interactions in terms of scaled factorial moments (SFMs) as a function of decreasing rapidity intervals to search for nonstatistical fluctuations. They have suggested that if intermittency exists, then the normalized factorial moments of the multiplicity distribution should exhibit a characteristic power-law dependence. Such a dependence of the factorial moments has been observed not only in nuclear collisions[3−9] but also in leptonic and hadronic collisions.[10−15] The intermittency may be due to the self-similar structures in the random cascading for elementary processes or in nuclear collisions by the second-order-phase transition from the quark–gluon plasma (QGP) to the final hadronic state. What causes the intermittency may lead to the fundamental question about the production mechanism of these particles, which is highly desirable.

It is believed that in the pionization process for ultra-relativistic nucleus–nucleus collisions, the presence of intermittent-type fluctuations is due to the QGP phase transition. But in low energy nucleus–nucleus collisions, also, intermittent-type fluctuations are observed which cannot be explained by the QGP phase transition. So, it is obvious that the QGP phase transition cannot be the only cause for intermittency. One possible origin of intermittency is Bose–Einstein (B–E) interference. There is a strong feeling that the B–E interference can play a role in dynamical fluctuation. B–E correlations arise because the wave functions of identical bosons are symmetric. This correlation between equally charged particles can produce rising factorial moments with bin size decreasing. It will make intermittency stronger for equally charged particles than for other charged particles. The data from NA22 Collaboration[14] support the idea of B–E interference, but the results from e^+e^- data[10,11] and $\mu - p$ data[15] seem to be contrary to it. These observations conclude that the B–E effect cannot be

*Project supported by the National Natural Science Foundation of China (Grant No 10475054), the Key Program of Ministry of Education, China (Grant No 205026), the Natural Science Foundation of Shanxi Province, China (Grant No 20021007) and Shanxi Provincial Foundation for Returned Scholars, China (Grant No 20031046).
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considered as the main source of intermittency. However, it has been pointed out[16] that the slopes obtained from EMU01 data decrease with multiplicity increasing when target and projectile are fixed, excluding B–E correlations as a dominant source of intermittency in heavy-ion collisions.[17] It is believed that intermittency can be explained in terms of conventional short-range correlations. Calculations from NA22 and UA5 data show that intermittent rise of factorial moments can be explained reasonably in terms of short-range correlations for lower order moments. However, calculations seem to underestimate the factorial moment values of higher moments for very small bin sizes. The results from $\mu-p$ collision[15] show that there is a significant discrepancy between the experimental and estimated results at high resolutions. So, this approach is interesting but not beyond ambiguity.

Since Bialas and Peschanski introduced the idea of intermittency, the values of the intermittency exponents have been found not only in one-dimensional distributions of pseudorapidity (η) and the azimuthal angle (ϕ) but also in the two-dimensional (η, ϕ) distributions. But up to now the origin of intermittency has not been clear yet. For a better understanding of the origin of intermittency, the analysis of more and more systematic data should be made in η-, ϕ-, and (η,ϕ) phase spaces.

In this article, we analyse different aspects of intermittency of our pionization data of ^{16}O-Em interactions at 3.7 A GeV. To study the incident energy dependence, we compare our data with other available data of ^{16}O+AgBr interactions at different energies.

2. Experimental details

Stacks of NIKFI BR-2 nuclear emulsion were exposed to a 4.5 A GeV/c oxygen beam at the Joint Institute for Nuclear Research (JINR), Dubna, Russia. The volume of the stacks is 10 cm × 10 cm × 0.06 cm. The double scanning, along the track, fast in the forward and slow in the backward direction, was carried out. Details of the scanning and classification of events can be found in our previous paper.[18−21] A total of 2960 interactions of oxygen with the nuclei of the emulsion were observed by following a primary track length of 35768.9 cm, which led to a mean free path $\lambda = (12.08 \pm 0.22)$ cm. Among these interactions, 152 events were found to be produced by electromagnetic interactions and 149 events were due to elastic interactions. Out of these 2960 interactions, 2044 inelastic interactions were picked up without any bias for the final analysis. Interactions which were within 30 μm from the top or bottom surface of the emulsion were not taken into the consideration for the final analysis.

The secondary particles in each event are classified according to traditional emulsion terminology as:

i) Shower particles, which are singly charged relativistic particles with a relative ionization $I/I_0 < 1.4$ where I_0 is the plateau ionization for singly charged minimum ionizing particles. They are mainly due to the produced charged pions having a relative speed $\beta = v/c \geq 0.7$. Their multiplicity is denoted by $n_{\rm s}$ (not including the stripped projectile protons).

ii) Grey particle tracks, which are those tracks having the values of range $R > 3$ mm in emulsion and relative ionization $I/I_0 > 1.4$. They are mostly due to protons with kinetic energies ranging from 26 to 400 MeV. The multiplicity of these grey tracks is denoted by $n_{\rm g}$.

iii) Black particle tracks, which are those that have the values of range $R \leq 3$ mm, i.e. short ranges, in emulsion (corresponding to proton kinetic energy < 26 MeV). Their multiplicity is denoted by $n_{\rm b}$. Grey and black tracks together are usually considered to be due to heavily ionizing particles. Their multiplicity is denoted by $n_{\rm h} = n_{\rm g} + n_{\rm b}$.

iv) Any charged particle, which is emitted at the values of angle $\theta \leq 3^\circ$ with respect to the incident direction and characterized by no change in its ionization for at least 2 cm away from the interaction point. These particles are regarded as a noninteracting (stripped) projectile fragment (PF).

3. Method of analysis

In the selected interactions, the shower particles have been identified and their polar angle θ and azimuthal angle ϕ have been measured. For each shower particle (pion), the pseudorapidity η can be determined from the relation

$$\eta = -\ln(\tan(\theta/2)), \tag{1}$$

where θ is the emission angle of the pion in the laboratory system.

For the intermittency analysis, the pseudorapidity range $\Delta\eta$ is divided into M bins of uniform width $\delta\eta = \Delta\eta/M$. For a single event containing N tracks

in the window $\Delta\eta$, the qth scaled factorial moment is

$$F_q = M^{q-1} \sum_{m=1}^{M} \frac{k_m(k_m-1)\cdots(k_m-q+1)}{N(N-1)\cdots(N-q+1)}, \quad (2)$$

where k_m is the number of particles in the mth bin. The behaviour of F_q as a function of $\delta\eta$ (hence of M) is an indicator of the correlation length for fluctuations. For example, one might expect that an increment of F_q due to the decrease of $\delta\eta$ is chosen to be smaller than the scale length for fluctuations, such that at that point F_q becomes independent of $\delta\eta$.

The preceding calculation applies to single event, but in practice it is necessary to take an average over an ensemble of events. Equation (2) can be generalized to the case including events of varying multiplicity in several ways. The simplest approach is to use Eq.(2) averaged over the events in a sample, i.e. an exclusive average. This procedure neglects the influence of varying multiplicities on the SFMs, so an inclusive average, where $N(N-1)\cdots(N-q+1)$ is replaced with $< N >^q$, has been the preferred method. The latter approach is referred to as the 'horizontal' analysis. The moment is then given by

$$< F_q >_H = \frac{1}{N_{\rm ev}} \sum_{i=1}^{N_{\rm ev}} M^{q-1} \sum_{m=1}^{M} \frac{k_{m,i}(k_{m,i}-1)\cdots(k_{m,i}-q+1)}{< N >^q}, \quad (3)$$

where $N_{\rm ev}$ is the number of events in a sample, $k_{m,i}$ is the content of bin m in event i, and $< N >$ is the average multiplicity in pseudorapidity window $\Delta\eta$. Results using this mode of analysis[3,6,14] suggest power-law behaviour of moments down to the experimental resolution limit $\delta\eta = 0.1$.

With the horizontal analysis, all of the bins are treated uniformly, so some of the variation in the factorial moments is due to η-dependence of the average multiplicity within a bin. The pseudorapidity density distribution is far from uniform as can be seen in the typical η-distribution.

Fialkowski *et al*[22] have shown that the horizontal moments can be corrected for the nonuniform shape of the rapidity distributions by being divided by the factor

$$R_{\rm F} = \frac{1}{M} \sum_{m=1}^{M} M^q \frac{< k_m^q >}{< N >^q}. \quad (4)$$

Thus, the reduced scaled factorial moments (RSFM) are given by

$$< F_q >^{\rm corr} = \frac{< F_q >}{R_{\rm F}}. \quad (5)$$

To make correction for this variation, one may adopt an alternate approach referred to as the 'vertical' analysis, in which

$$< F_q >_{\rm V} = \frac{1}{M} \sum_{m=1}^{M} \frac{1}{N_{\rm ev}} \sum_{i=1}^{N_{\rm ev}} \frac{k_{m,i}(k_{m,i}-1)\cdots(k_{m,i}-q+1)}{< k_m >^q}, \quad (6)$$

where

$$< k_m > = \frac{1}{N_{\rm ev}} \sum_{i=1}^{N_{\rm ev}} k_{m,i} \quad (7)$$

the average content of bin m over the ensemble of events, is substituted for $< N/M >$. In this formulation the factorials are weighted by the average multiplicity for a given bin rather than the overall average bin multiplicity.

It has been shown[2] that for a smooth rapidity distribution, without exhibiting any fluctuations other than the statistical ones, $< F_q(\delta\eta) >$ is essentially independent of the resolution $\delta\eta$ in the limit $\delta\eta \to 0$. However, if there exists any dynamical contribution to these fluctuations, the scaled factorial moment is believed to obey the following power law:

$$< F_q > = \left(\frac{\Delta\eta}{\delta\eta}\right)^{a_q} \quad \text{for} \quad \delta\eta \to 0. \quad (8)$$

For a situation where the above power law holds, the particle spectrum is said to be intermittent. The positive constants of a_q are referred to as the 'intermittency indices'; a_q characterizes the strength of the effect. It may thus be stated that for an intermittent behaviour of the particle spectra, $\ln < F_q >$ should exhibit a linear rise with $-\ln(\delta\eta)$ for comparatively small bin widths.

Once the power law is observed to be valid in the data, indicating the self-similarity property, the analysis of data in terms of fractal parameters[23] is imperative. The generalized dimensions D_q of fractal and multifractal are related to the intermittency exponent[24] by

$$D_q = 1 - a_q/(q-1), \tag{9}$$

where

$$d_q = a_q/(q-1) \tag{10}$$

is known as anomalous fractal dimensions. Although this relationship between intermittency and fractality or multifractality has not been fully understood as yet, attempts have been made to interpret data in terms of anomalous dimensions. As suggested by Bialas and Hwa,[25] if a second-order phase transition takes place from a quark-gluon plasma to a hadron phase in the thermodynamic equilibrium, then the produced particles will show intermittency with an anomalous dimension d_q independent of q. On the other hand, if hadronization takes place through the cascading process, d_q is expected to be linear in q.

The power-law behaviour of the scaled factorial moments, indeed, implies the existence of some kind of fractal pattern[26] in the dynamics of the particles produced in their final state. Therefore, it is natural to introduce the concept of self-similarity under the perspective of fractal properties of the hadronic matter in multiparticle production. A formalism suitable for investigating quantitatively any single particle density distribution in the framework of multifractal characteristics has been developed by Hwa[26] and successfully applied to the study of intermittent behaviour in turbulent fluids and other transitions to chaos. In multifractal analysis, the inelastic collisions can be considered as purely geometrical objects with noninteger dimensions.

In order to examine the dependence of multifractal moments G_q on η, a given pseudorapidity range $\Delta\eta = \eta_{\max} - \eta_{\min}$ is divided into M_0 bins of width $\delta\eta = \Delta\eta/M_0$. Let n_i denote the multiplicity of particles detected in the ith bin. Since some of the bins may be empty, we define M as the number of nonempty bins, which constitutes a fractal set. According to the nomenclature of Hwa[26] a multifractal moment is given by

$$G_q = \sum_{j=1}^{M} p_j^q, \tag{11}$$

where, $p_j = n_j/n$, such that $n = n_1 + n_2 + \cdots + n_M$; q is a real number either positive or negative; the summation is performed only over the nonempty content of the bins. The vertical average of the horizontal moments is given by

$$< G_q >= \frac{1}{N_{\rm ev}} \sum_{1}^{N_{\rm ev}} G_q, \tag{12}$$

where $N_{\rm ev}$ denotes the total of events in a given data set. If there exists a self-similarity in the production of particles, then G_q moments can be expressed in the form of a power law as follows:

$$< G_q >= (\delta\eta)^{\tau_q}, \tag{13}$$

where τ_q are the mass exponents. The linear dependence of $< \ln G_q >$ on $\ln(\delta\eta)$ over all the windows is expressed as

$$\tau_q = \lim_{\delta\eta\to 0} (\triangle < \ln G_q > / \Delta \ln(\delta\eta)). \tag{14}$$

The generalized dimensions D_q are related to the mass exponents τ_q by an expression

$$D_q = \frac{\tau_q}{q-1}, \tag{15}$$

where, D_0, D_1 and D_2 are known as the fractal, information and correlation dimensions, respectively.

4. Experimental results

Figure 1 presents the η-distribution for the selected interactions of ^{16}O with AgBr emulsion nuclei at 3.7 A GeV. For comparison, the results for ^{16}O with AgBr emulsion nuclei at 60 and 200 A GeV are also presented in the figure. Table 1 shows the data structure used in this paper.

Table 1. The data structure used in this paper.

Type of events	Number of events	Number of events with $n_s \geq 10$	range of $\triangle\eta$	$< n_s >$ in $\triangle\eta$	Reference
3.7 A GeV ^{16}O+AgBr	417	406	0.5–3.5	18.6 ± 0.3	Present work
3.7 A GeV ^{16}O+AgBr	417	406	0.5–4.5	19.6 ± 0.3	Present work
60 A GeV ^{16}O+AgBr	147	147	0.5–4.5	94.4 ± 8.0	[3]
200 A GeV ^{16}O+AgBr	124	124	0.5-5.5	152.0 ± 12.0	[3]

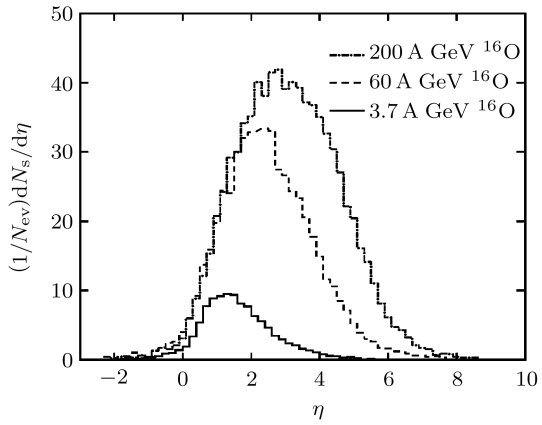


Fig.1. Pseudorapidity distributions of the produced charged pions in ^{16}O+AgBr interactions at 3.7, 60 and 200 A GeV

The dependence of $< F_q >$ on the size of bin $\delta\eta$ is presented in Fig.2, where plotted are $\ln < F_q >$ against $-\ln(\delta\eta)$, for $q = 2, 3, 4, 5$ and 6, along with corresponding best fit curves for $\triangle\eta = 0.5-3.5$ (a) and $\triangle\eta = 0.5-4.5$ (b) respectively. The positive slopes of the plots as evidenced in Fig.2, show the qualitative agreement with the power law Eq.(8), indicating the self-similarity in the distribution of produced charged pions. The intermittency indices (fitting slopes) a_q, as presented in Table 2, increase not only with the increase of the value of q but also with the increase of the range of pseudorapidity $\triangle\eta$. For comparison, the results for ^{16}O–Em interactions at different energies are also presented in the table. It can be seen that for different experiments the experimental limits are different. For the energies less than 60 A GeV, the intermittency exponent increases with the decrease of beam energy.

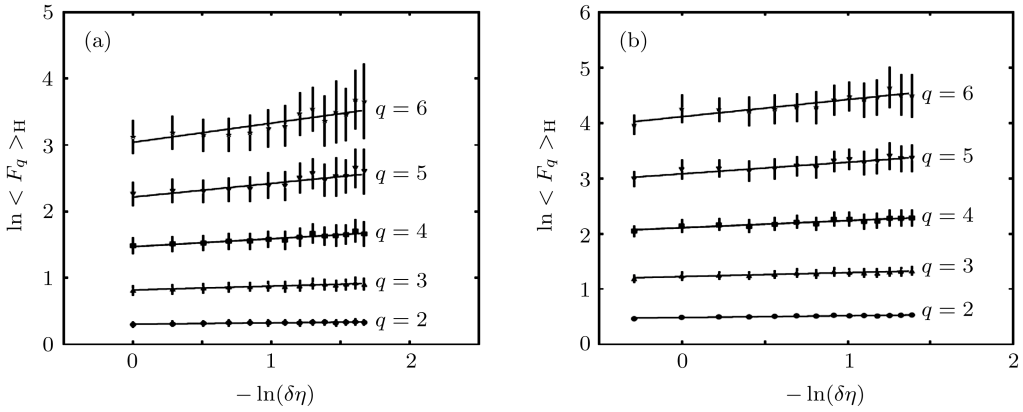


Fig.2. Average scaled factorial moments as a function of the bin size $\delta\eta$ for the produced charged pions in ^{16}O+AgBr interactions at 3.7 A GeV, (a) $\triangle\eta = 0.5$–3.5, (b) $\triangle\eta = 0.5$–4.5.

As can been seen from Fig.1, the pseudorapidity distribution of produced charged pions from ^{16}O–Em interactions at 3.7 A GeV is not flat within the studied region $\triangle\eta = 3.0$ and 4.0. Therefore the correction is needed for a nonflat shape of particle distribution in the phase space. In Fig.3, the variations in the values of $\ln < F_q >^{corr}$ with $-\ln(\delta\eta)$ are plotted for $q = 2, 3, 4$ and 5, and the size of $\triangle\eta = 3.0$ and 4.0, respectively. Nondependence on the size of $\triangle\eta$ is observed. The slopes (presented in Table 2), here, are the corrected intermittency exponents and, therefore, can be used for quantitative analysis.

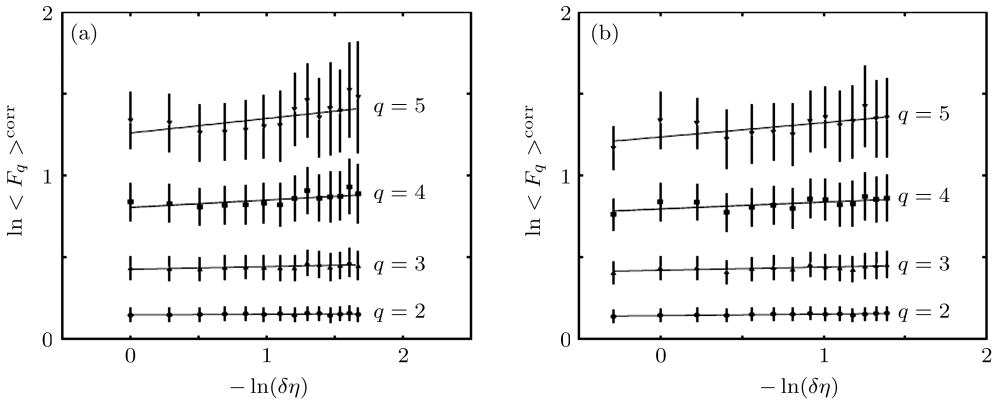


Fig.3. Average reduced scaled factorial moments as a function of the bin size $\delta\eta$ for the produced charged pions in ^{16}O+AgBr interactions at 3.7 A GeV, (a) $\triangle\eta = 0.5 - 3.5$, (b) $\triangle\eta = 0.5 - 4.5$.

Table 2. Values of the intermittency exponents for produced charged pions from the ^{16}O-Em interactions at 2.1, 3.7, 60 and 200 A GeV.

Energy	Target	Variable	Moment	a_2	a_3	a_4	a_5
2.1 A GeV[27]	AgBr	$\triangle\eta = 4.0$ $N_{\rm h} \geq 8$	SFM	$.141 \pm .015$	$.504 \pm .045$	$1.090 \pm .087$	
	AgBr	$\triangle\eta = 2.0$ $N_{\rm h} \geq 8$	SFM	$.137 \pm .013$	$.494 \pm .046$	$1.065 \pm .093$	
	AgBr	$\triangle\eta = 4.0$ $N_{\rm h} \geq 8$	RSFM	$.091 \pm .010$	$.373 \pm .041$	$0.827 \pm .098$	
	AgBr	$\triangle\eta = 2.0$ $N_{\rm h} \geq 8$	RSFM	$.091 \pm .011$	$.373 \pm .044$	$0.830 \pm .099$	
	AgBr	$\triangle\phi = 2\pi$ $N_{\rm h} \geq 8$	RSFM	$.184 \pm .021$	$.607 \pm .058$	$0.983 \pm .078$	
	AgBr	$\triangle\eta = 4.0$,$\triangle\phi = 2\pi$	RSFM	$.353 \pm .035$	$.785 \pm .102$	$1.088 \pm .130$	
	AgBr	$\triangle\eta = 4.0$,$n_{\rm s} > 8$	RSFM	$.084 \pm .011$	$.362 \pm .047$	$0.793 \pm .103$	
	AgBr	$\triangle\eta = 4.0$,$n_{\rm s} > 12$	RSFM	$.067 \pm .009$	$.334 \pm .050$	$0.741 \pm .111$	
3.7 A GeV	AgBr	$\triangle\eta = 3.0$ $n_{\rm s} \geq 10$	SFM	$.020 \pm .022$	$.058 \pm .040$	$0.121 \pm .070$	$.206 \pm .113$
	AgBr	$\triangle\eta = 3.0$ $n_{\rm s} \geq 10$	RSFM	$.004 \pm .022$	$.017 \pm .040$	$0.046 \pm .070$	$.088 \pm .113$
	AgBr	$\triangle\eta = 4.0$ $n_{\rm s} \geq 10$	SFM	$.032 \pm .021$	$.073 \pm .038$	$0.129 \pm .062$	$.210 \pm .093$
	AgBr	$\triangle\eta = 4.0$ $n_{\rm s} \geq 10$	RSFM	$.009 \pm .021$	$.019 \pm .038$	$0.042 \pm .062$	$.088 \pm .093$
	AgBr	$\triangle\phi = 2\pi$ $n_{\rm s} \geq 10$	SFM	$.048 \pm .009$	$.116 \pm .020$	$0.190 \pm .036$	$.263 \pm .061$
	Em	$\triangle\phi = 2\pi$ $n_{\rm s} \geq 10$	SFM	$.032 \pm .007$	$.087 \pm .015$	$0.145 \pm .028$	$.154 \pm .048$
	CNO	$\triangle\phi = 2\pi$ $n_{\rm s} \geq 10$	SFM	$.027 \pm .012$	$.037 \pm .023$	$0.051 \pm .033$	
	AgBr	$\triangle\eta = 4.0$,$\triangle\phi = 2\pi$	RSFM	$.061 \pm .008$	$.138 \pm .018$	$0.258 \pm .040$	
	Em	$\triangle\eta = 4.0$,$\triangle\phi = 2\pi$	RSFM	$.046 \pm .008$	$.092 \pm .018$	$0.168 \pm .039$	
60 A GeV[8]	AgBr	$\triangle\eta = 4.0$,$N_{\rm h} \geq 15$	RSFM	$.010 \pm .001$	$.024 \pm .005$	$0.049 \pm .014$	$.106 \pm .032$
	AgBr	$\triangle\phi = 2\phi$,$N_{\rm h} \geq 15$	RSFM	$.020 \pm .002$	$.059 \pm .005$	$0.115 \pm .012$	$.195 \pm .032$
	AgBr	$\triangle\eta, \triangle\phi$,$N_{\rm h} \geq 15$	RSFM	$.008 \pm .002$	$.045 \pm .008$	$0.191 \pm .022$	$.522 \pm .051$
60 A GeV[3]	Em	$\triangle\eta = 4.0$,$n_{\rm s} \geq 10$	SFM	$.017 \pm .002$	$.039 \pm .006$	$0.066 \pm .014$	
200 A GeV[8]	AgBr	$\triangle\eta = 4.0$,$N_{\rm h} \geq 15$	RSFM	$.007 \pm .001$	$.023 \pm .002$	$0.046 \pm .006$	$.086 \pm .012$
	AgBr	$\triangle\phi = 2\phi$,$N_{\rm h} \geq 15$	RSFM	$.025 \pm .001$	$.082 \pm .005$	$0.221 \pm .017$	$.517 \pm .047$
	AgBr	$\triangle\eta, \triangle\phi$,$N_{\rm h} \geq 15$	RSFM	$.013 \pm .002$	$.071 \pm .010$	$0.307 \pm .046$	$.823 \pm .098$
200 A GeV[3]	Em	$\triangle\eta = 5.0$,$n_{\rm s} \geq 10$	SFM	$.020 \pm .001$	$.052 \pm .004$	$0.099 \pm .008$	

From Fig.2 we find that $< F_q >$ exhibits a rise, consistent with the power-law dependence on $\delta\eta$, which is characteristic of intermittency. The other features, such as an increase in the slope or intermittency exponents, a_q, with increasing order of moment, are also depicted in the figure. After the correction to nonflat single-particle pseudorapidity distribution has been made, we observe that (in Fig.3) the characteristics of intermittency exist, though the effect turns weak. In any case, the existence of intermittency or the self-similarity in produced charged pions in the data set is obvious. In terms of speculation by Bialas and Hwa,[25] the dependence of values of anomalous dimensions d_q on the order of moment, q, as shown in

Table 3, indicates the cascading process of hadronization.

With our data of the ^{16}O+AgBr interaction at 3.7 A GeV, the intermittency analysis is performed using the azimuthal angle space around the beam axis as the variable for produced charged pions in $\triangle\phi = 2\pi$. Here, we have used the vertical analysis (Eqs.(6) and (7)) to eliminate extra size dependence factor of phase space.

Figure 4 shows the azimuthal angle distribution for the interactions of ^{16}O with different emulsion targets, i.e. CNO, AgBr and emulsion nuclei, respectively. Compared with the pseudorapidity distribution (Fig.1), the distributions become flat.

From the plots of $\ln < F_q >_{\rm V}$ vs $-\ln(\delta\phi)$ in Fig.5, the validity of the power law in ϕ space is obvious, for the general features of the plots in Fig.5 are similar to those in Figs.2 and 3. The intermittency effect appears to be stronger in ϕ-space than in η-space as can be understood in terms of the slope parameters given in Table 2.

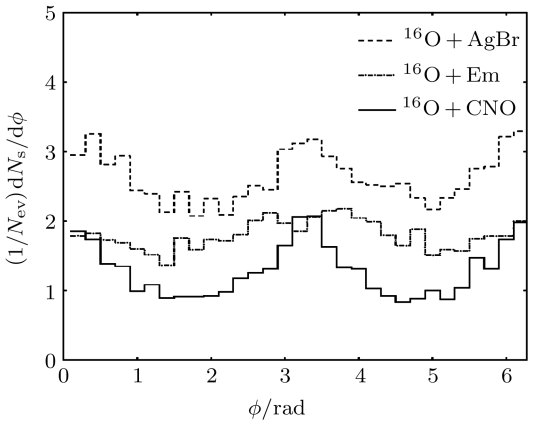


Fig.4. Azimuthal angle distribution of the produced charged pions in ^{16}O+AgBr, CNO, and emulsion nuclei interactions.

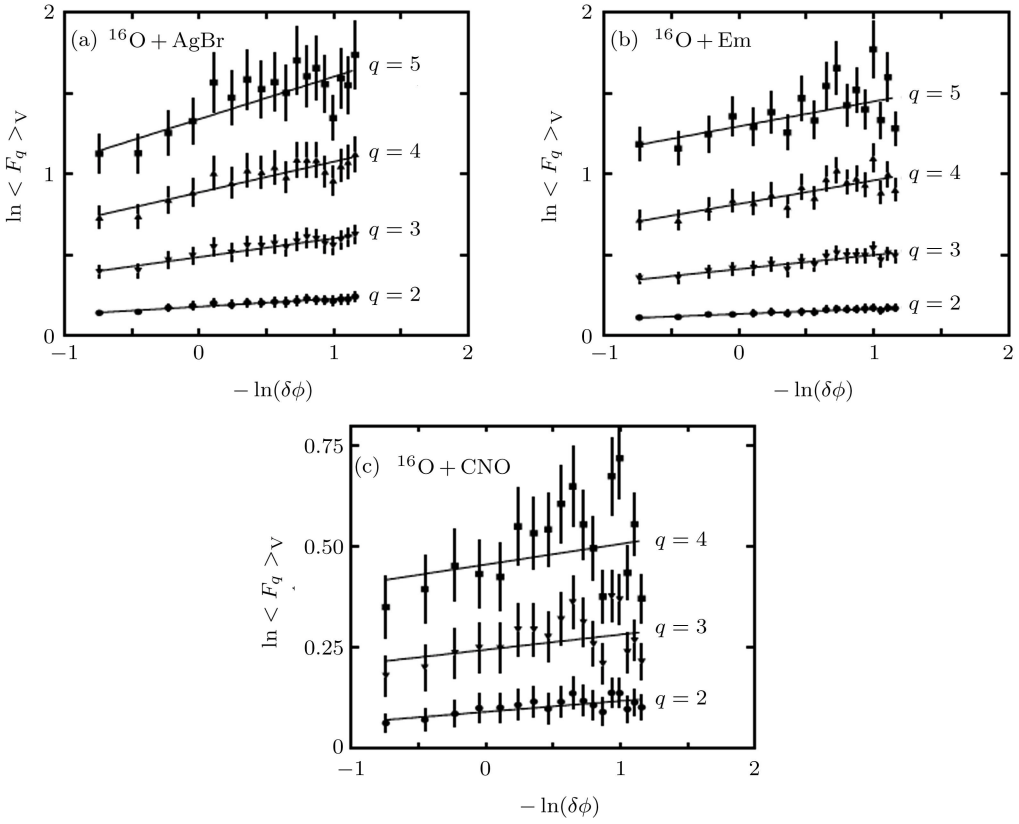


Fig.5. $\ln < F_q >_{\rm V}$ vs $-\ln(\delta\phi)$ for produced charged pions in ^{16}O+AgBr (a), CNO (c), and emulsion (b) nuclei interactions.

Ochs and Wosiek[28,29] have suggested that the intermittency effects should be more evident if the fluctuations are analysed simultaneously in the two-dimensional space $(\eta - \phi)$. For a quantitative analysis of the normalized multiplicity moments in the two-dimensional space, the ranges of η and ϕ are considered to be $\triangle\eta = 4.0$ and $\triangle\phi = 2\pi$. The $\triangle\eta\triangle\phi$ space has been divided into $M_{\eta\phi} = M_\eta M_\phi$ intervals and

$\delta\eta\delta\phi = \triangle\eta\triangle\phi/M_\eta M_\phi$ with $M_\eta = M_\phi$.

In the two-dimensional analysis, the reduced scaled factorial moments for the intervals $\delta\eta\delta\phi$ are calculated and similar to those given in Eqs.(2), (4) and (5). Figure 6 shows $\ln < F_q >^{\rm corr}$ as a function of $-\ln(\delta\eta\delta\phi)$ for the considered data for $q = 2-4$. As can be seen in Fig.6 and Table 2, the values of the slope parameter is found to be bigger in two-dimensional analysis than in one-dimensional analysis.

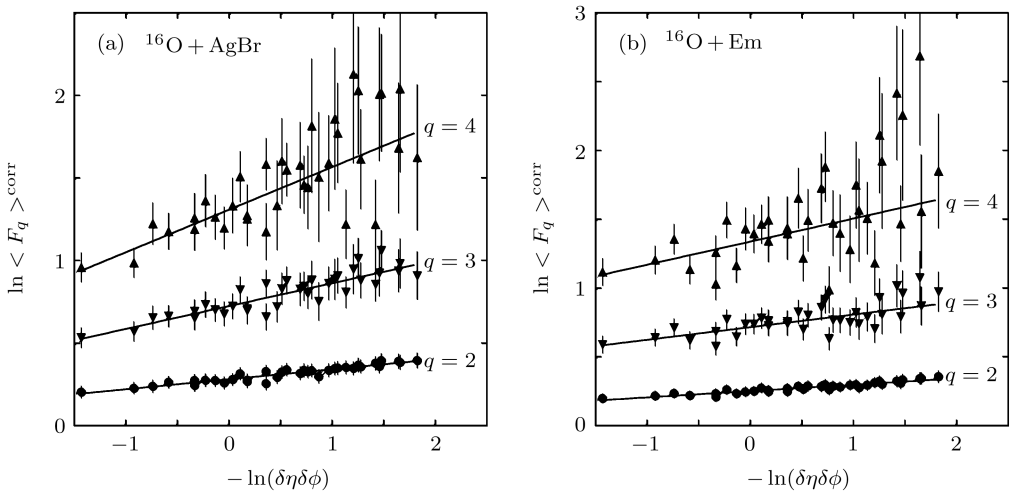


Fig.6. $\ln < F_q >^{\rm corr}$ vs $-\ln(\delta\eta\delta\phi)$ for produced charged pions in ^{16}O+AgBr (a) and ^{16}O+emulsion interactions (b).

The anomalous fractal dimensions of intermittency d_q in η-space, ϕ-space and (η,ϕ)-space, calculated using Eq.(10), are presented in Table 3. It can be seen that the value of d_q increases with the increase of the rank of the moments. The dependences of the anomalous fractal dimensions on the rank of the moments in η-, ϕ-, and (η,ϕ)-spaces for ^{16}O+AgBr interactions are shown in Fig.7. The analyses of our experimental data and those for ^{16}O at 2.1 A GeV[27], 60 A GeV and 200 A GeV[3,8], have shown that the magnitude of the anomalous fractal dimension increases approximately linearly with the increase of rank of the moments for η-, ϕ- and (η,ϕ)-spaces at various energies and for different experimental conditions and selection criteria (see Table 2). The results are in agreement with those obtained in the cascade and branching model calculations.[2]

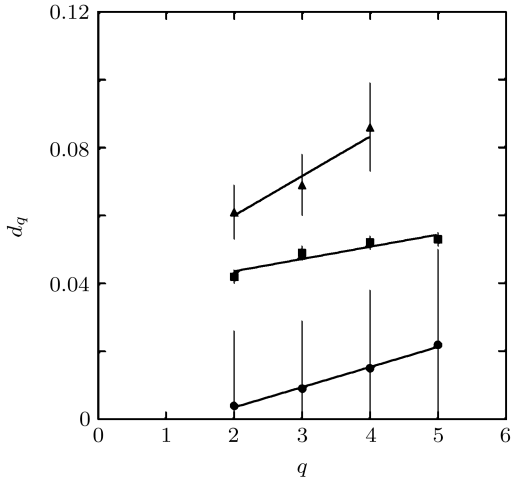


Fig.7. Relation between the anomalous fractal dimension d_q and the rank of moment q for the produced charged pions in η-space(dots), ϕ-space(square) and (η,ϕ)-space(triangle) in ^{16}O+AgBr interactions at 3.7 A GeV.

Table 3. Anomalous dimensions d_q calculated from the intermittency exponents a_q for produced pions in ^{16}O+AgBr interactions at 3.7 A GeV.

q	d_q in η-space	d_q in ϕ-space	d_q in (η,ϕ)-space
2	0.004 ± 0.022	0.042 ± 0.002	0.061 ± 0.008
3	0.009 ± 0.020	0.049 ± 0.002	0.069 ± 0.009
4	0.015 ± 0.023	0.052 ± 0.002	0.086 ± 0.013
5	0.022 ± 0.028	0.053 ± 0.002	

The variations of G_q moments with width of the pseudorapidity bin (a) and azimuthal angle bin (b) for ^{16}O-emulsion interactions are presented in Fig.8. In the case of (a) or (b) the moments with positive values of q show linearity over a wide range of $\delta\eta$ or $\delta\phi$ whereas the moments with negative values of q are saturated as $\delta\eta$ or $\delta\phi$ decreases. This saturation effect can be due to the availability of a smaller number of particles as the bin size is reduced. The lines in Fig.8 are the least-square fitting to the data for very

few number of points because of this saturation effect. The linear rise of the multifractal moments in pseudorapidity and azimuthal angle phase space is a manifestation of the self-similarity in the production mechanism of the investigated reactions. The values of slope $\tau(q)$ in the linear region of the plots $< \ln G_q >$ versus $-\ln(\delta\eta)$ and $-\ln(\delta\phi)$ for each data set are calculated by using the least-squares fitting to the data and are plotted as a function of q in Fig.9. For both the data set $\tau(q) = 0$ at $q = 1$ and negative values of q, $\tau(q)$ increases significantly while it turns flat with increasing positive values of q.

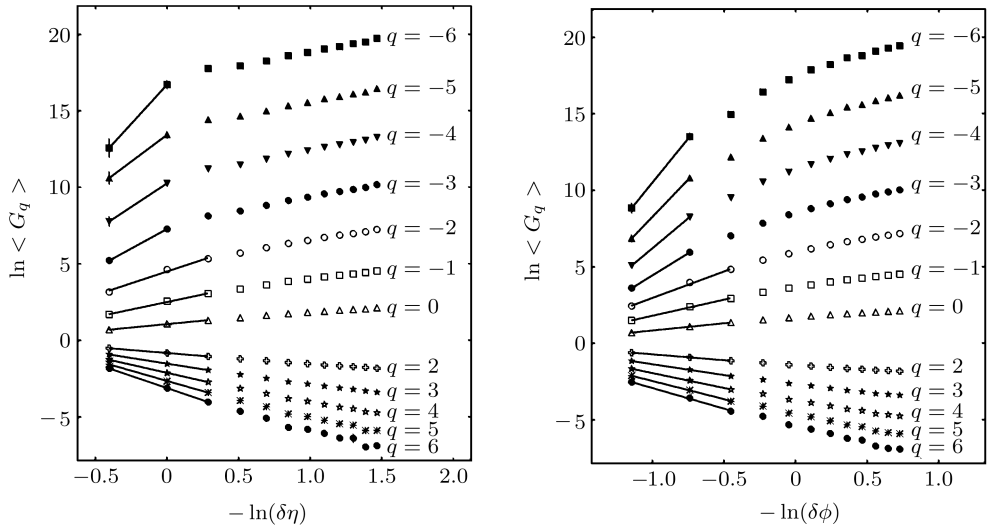


Fig.8. $< \ln G_q >$ versus $-\ln(\delta\eta)$ in the range $\triangle\eta = 3$ (a) and $-\ln(\delta\phi)$ in the range $\triangle\phi = 2\pi$ (b) for ^{16}O-emulsion interactions at 3.7 A GeV.

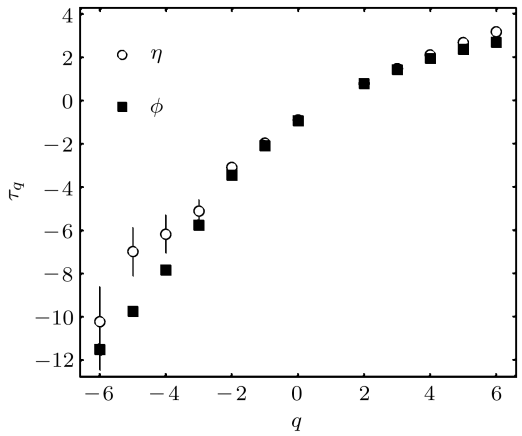


Fig.9. $\tau(q)$ versus q in ϕ- and η-phase spaces for ^{16}O-emulsion interactions at 3.7 A GeV.

The generalized dimensions D_q can be calculated from Eq.(15). In Fig.10, we plot the variation of D_q as a function of q, in η- and ϕ-spaces, respectively. For both cases, D_q decreases with increasing q, and it is always positive for each order of q. For $q \leq 0$, the values of D_q in ϕ-phase space are always greater than the corresponding values in η-phase space, but for $q > 0$, the values of D_q in ϕ-phase space are always less than the corresponding values in η-phase space. From Figs.9 and 10 we can see that the curves for values of D_q and τ_q in ϕ-phase space for $q \leq 0$ turn flatter than those in η-phase space, which is mainly caused by edge effects due to the non-uniformity of pseudorapidity distribution in a given pseudorapidity interval $\triangle\eta = 3.5 - 0.5 = 3.0$.

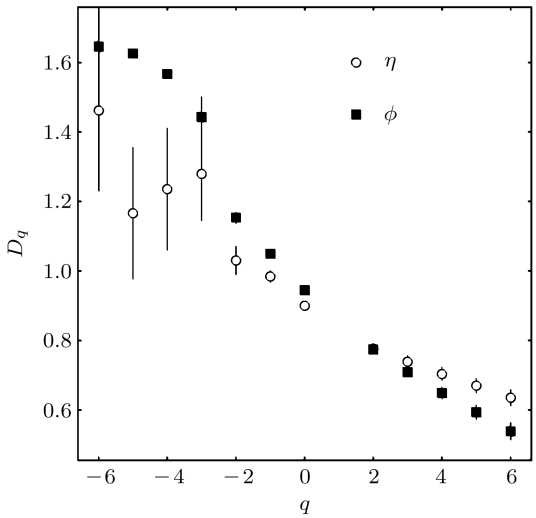


Fig.10. D_q versus q in ϕ- and η-phase spaces for ^{16}O-emulsion interactions at 3.7 A GeV.

Table 4. Values of the generalized dimensions D_q in ϕ- and η-phase spaces for ^{16}O-emulsion interactions at different energies.

q	D_q in η-space			D_q in ϕ-space		
	3.7 A GeV	60 A GeV	200 A GeV	3.7 A GeV	60 A GeV	200 A GeV
2	0.752 ± 0.007	0.755 ± 0.012	0.790 ± 0.010	0.726 ± 0.005	0.813 ± 0.010	0.845 ± 0.007
3	0.714 ± 0.009	0.723 ± 0.012	0.759 ± 0.010	0.662 ± 0.007	0.778 ± 0.011	0.814 ± 0.008
4	0.679 ± 0.009	0.699 ± 0.012	0.735 ± 0.010	0.604 ± 0.009	0.751 ± 0.011	0.790 ± 0.008
5	0.646 ± 0.011	0.680 ± 0.012	0.717 ± 0.010	0.552 ± 0.011	0.730 ± 0.011	0.771 ± 0.008
6	0.613 ± 0.013	0.666 ± 0.012	0.703 ± 0.010	0.502 ± 0.013	0.714 ± 0.012	0.756 ± 0.008

Table 4 gives the generalized dimensions D_q in η- and ϕ-phase spaces for ^{16}O-emulsion interactions at 3.7 A GeV. For comparison the corresponding results for ^{16}O-emulsion interactions at 60 and 200 A GeV[30] are also presented in the table. It can be concluded that the values of D_q for $q > 0$ in both phase spaces decrease with the increasing beam energy for the same projectile.

It should be pointed out that the singely charged particle pseudorapidity distribution is non-uniform. This nonuniformity of the distribution influences the scaling behaviour of factorial and multifractal moments. Bialas and Gazdzicki[31] have proposed a new variable $X(\eta)$ which drastically reduces the distortion produced in the study of multiplicity fluctuations due to nonuniformity of charged particle distribution. A lot of experiments have been carried out to investigate the nonstatistical fluctuation in pseudorapidity phase space and emission angle space using the methods of factorial moments and multifractal moments in terms of new variable $X(\eta)$ for pionization[32−36] and target fragmentation[37−40] in high energy nucleus–nucleus interactions. It is found that the existence of dynamical fluctuations not only in the pionization but also in the target fragmentation using the factorial moments and the multifractal moments reveals power law behaviour. The power dependence obtained by using two methods gives an evidence of self-similar structure in multiparticle production and target fragmentation. The reduction of the generalized fractal dimensions, D_q, with q clearly indicates the presence of multifractality. These results, therefore, suggest that the existence of the fractal structure observed in high energy nucleus–nucleus interactions using new scaled variable $X(\eta)$ is consistent with the predictions of other results.

5. Conclusion

From the present study of the 3.7 A GeV^{16}O interactions with AgBr emulsion nuclei, it may be concluded that the effect of the intermittency has been observed in the selected interactions. The dependences of the factorial moments, the reduced factorial moments and multifractal moments on the bin size of considered variables, the pseudorapidity, the azimuthal angle and both of them, have been found to obey a characteristic power law. The anomalous fractal dimensions of the intermittency have been found to increase with the increase of the rank of the moments. This result agrees with that calculated from the cascade and branching theoretical model.

Multifractal analysis, though it is found to be quite effective in describing fluctuations of the phase space distribution under different types of interaction, is still too early a stage to draw conclusions on the method of studying the fractality in multiparticle production. In fact, fractality in multiparticle production is still an open question and needs more and more phenomenological analyses in order to enrich the knowledge of this research subject in hopes of understanding hadronization.

Acknowledgment

We are thankful to Professor Otterlund I of Lund University in Sweden for supplying the emulsion plates.

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