Vol 16 No 9, September 2007
1009-1963/2007/16(09)/2683-06

Chinese Physics

© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

Evidence of intermittent fluctuation of target fragments in ^{84}Kr–AgBr interactions at 1.7 A GeV*

Zhang Dong-Hai(张东海)†, Li Xue-Qin(李雪琴), Jia Hui-Ming(贾会明),
He Chun-Le(何春乐), Liu Fang(刘 芳), Zhao Hui-Hua(赵惠华),
Li Zhen-Yu(李振宇), and Li Jun-Sheng(李俊生)

Institute of Modern Physics, Shanxi Normal University, Linfen 041004, *China*

(Received 2 December 2006; revised manuscript received 3 February 2007)

Intermittency and fractal behaviour have been studied of emission spectra of target associated fragments from ^{84}Kr–AgBr interactions at 1.7 A GeV in emission angle space and azimuthal angle space separately. The intermittent behaviour is observed in the two spaces separately. From the intermittency exponent, the anomalous fractal dimension d_q is calculated and the variation of d_q with the order q is investigated. It is found that the anomalous dimensions are found to increase with the order of moments q, thereby indicating the relation of multifractality to production mechanism of target associated fragments.

Keywords: relativistic heavy-ion collision, intermittency, nuclear emulsion
PACC: 2570N, 2587, 2940R

1. Introduction

In recent years the subject of intermittency has gained significant attention in the theoretical and experimental investigation of multiparticle production. It is the phenomenon of power law behaviour of scaled factorial moments (SFMs) with decreasing bin size and is capable of extracting non-statistical fluctuations after eliminating the statistical part. The concept of intermittency was first introduced by Bialas and Peschanski[1] to resolve unusual fluctuations in a cosmic ray event[2] (JACCE) of heavy ion interaction through the analysis of the distribution of produced particles with the help of SFMs in the rapidity phase space. The success in intermittency analysis of resolving non-statistical fluctuations in the cosmic ray JACCE event encouraged many other groups to study it more elaborately with the accelerator data including a wide variety of projectiles, targets and incident energies. The detailed investigation can be found in a review paper.[3]

It is interesting to note that, both theoretically and experimentally, emphasis is placed on the study of pions, while little attention has been paid to the analysis of the target fragments. The study of the target fragments may also provide important information for a good understanding of forces and dynamics involved during and immediately after nuclear impact. One of the well-known models explaining the origin of target fragments is the evaporation model.[4] According to this model, shower and grey tracks are emitted from the nucleus very soon after the instant of impact. This leaves the nucleus in a highly excited state. Indeed, the excitation energy may sometimes be comparable with the total binding energy of the nucleus. The emission of heavily charged particles from this state, however, takes place relatively slowly. In order to escape from the residual nucleus, a particle must await a favourable statistical fluctuation; the random collisions between the nucleons within the nucleus can lead the particle to approach the nuclear boundary closely and to travel in an outward direction and with a kinetic energy greater than the binding energy of the nucleus. After the evaporation, this particle in a relatively long period on nuclear time scale, say 10^{-17} s, will commonly elapse before a second particle is also placed in conditions favourable for escape and so on. The process continues until the excitation energy of the residual nucleus is so small that transition to the ground state is likely to be affected by the

*Project supported by the National Natural Science Foundation of China (Grant No 10475054), the Foundation for Key Program of Ministry of Education of China (Grant No 205026), the Natural Science Foundation of Shanxi Province, China (Grant No 20021007) and Shanxi Provincial Foundation for Returned Scholars of China (Grant No 20031046).
†E-mail: zhangdh@dns.sxnu.edu.cn

emission of gamma rays. The target on fragmentation, produces highly ionizing particles responsible for heavy tracks in the nuclear emulsion. Conveniently the heavy tracks are subdivided into two groups: grey and black. The grey tracks with range ≥ 3 mm and velocity $0.3 \leq \beta \leq 0.7$ are due to fast target protons with energies ranging from 30 to 400 MeV. According to the evaporation model, these particles are in a low energy part of the internucleon cascade and they leave the nucleus during or shortly after the passage of the incident projectile nucleus. Since they are produced immediately after collisions, they are believed to carry relevant information about the dynamics of emission process. The black tracks with range < 3 mm and velocity $\beta \leq 0.3$ are due to the particles emitted long after the passage of the incident nucleus and are attributed to the evaporation from highly excited nuclei in a thermodynamically equilibrium state. In the rest system of the target nucleus in this model, the directions of emission of the target evaporation particles are distributed isotropically. The evaporation model is based on the assumption that statistical equilibrium has been established in the decaying system and lifetime is much longer than the time taken to distribute the energy among the nucleons within the nucleus. The evaporation model predicts an isotropic emission of target black fragments in the laboratory system, i.e. the rest system of the target nucleus. However different experimental data indicate the existence of a non-equilibrium process involved in the emission of black particles.[5−9]

In this paper, we investigate the variation of factorial moments with the bin size in target fragmentation of ^{84}Kr–AgBr interactions at 1.7 A GeV in emission angle space and azimuthal angle space separately.

2. Experimental details

Stacks of Ilford G-5 nuclear emulsion plates horizontally exposed to a ^{84}Kr beam at 1.7 A GeV at Bevalac Berkeley are used in this work. The scanning of plates is performed with the help of the XSJ-1 and XSJ-2 microscopes. Plates are scanned by using the optics: $100\times$ oil immersion objective and $16\times$ ocular lenses. The tracks are picked up at a distance of 5 mm from the edge of the plates and are carefully followed until they either interacted with emulsion nuclei or escaped from the plates. Interactions which are within 30 μm from the top or bottom surface of the emulsion plates are not considered for final analysis. All the primary tracks are followed back to ensure that the events chosen do not include interactions from the secondary tracks of other interactions. When they are observed to do so the corresponding events are removed from the sample. Details of scanning and classification of events can be found in our previous papers.[10−13]

All the tracks of the charged secondaries in these events are classified in accordance with the standard emulsion terminology. The relativistic shower tracks with ionization less than $1.4\,I_0$ (I_0 is the minimum ionization) are mainly the produced pions. The target fragments with ionization greater than $1.4\,I_0$ produce either black or grey tracks. The projectile fragments are a different class of tracks with constant ionization, long range and small emission angle.

To ensure that the targets in the emulsion are silver or bromine nuclei, we have chosen only the events with at least eight heavy ionizing tracks of particles ($N_{\rm h} \geq 8$).

3. Method of analysis

The intermittency phenomenon is described as the power-law behaviour of normalized SFMs with decreasing phase-space bin size: $< F_q > \propto (\delta x)^{-a_q}$ as $\delta x \to 0$. $< F_q >$ is the qth order normalized SFM defined[1] for a sample of events with $< N >$ as the mean multiplicity for a partition of an overall phase space, interval $\Delta X/M$,

$$< F_q > = < M^{q-1} \sum_{j=1}^{M} \frac{n_i(n_j-1)\cdots(n_j-q+1)}{< N >^q} >, \tag{1}$$

where M is the number of bins, n_j is the multiplicity in the jth bin, and angular brackets denote an average over the total sample.

Recently it has been recommended[14] that the error introduced due to the nonuniform shape of the particle distribution should be corrected by dividing the factor

$$R_q = \frac{1}{M} \sum_{j=1}^{M} M^q \frac{< n_j^q >}{< N >^q}, \tag{2}$$

which is unity for a flat distribution. Now we have the corrected or reduced scaled factorial moment (RSFM)

$$< F_q^R > = \frac{< F_q >}{R_q}, \tag{3}$$

which is a more appropriate expression for the analysis of intermittency. For given q and M values, RSFMs are calculated for all the events and then averaged over the whole sample of events to obtain $< F_q^R >$. If the non-statistical fluctuations are self-similar in nature, in the limit of small bin size, RSFM is given by

$$< F_q^R > \propto M^{a_q}, \tag{4}$$

i.e.

$$\ln < F_q^R > = a_q \ln M + c. \tag{5}$$

This property is called 'intermittency'. a_q measures the strength of intermittency, and it is called the intermittency exponent. The values of intermittency exponent a_q are obtained by performing best fits according to Eq.(5). The anomalous fractal dimension d_q of fractals and multifractals are related to the intermittency exponent as[15]

$$d_q = a_q/(q-1). \tag{6}$$

The self-similar nature of the multiplicity fluctuation directly implies a relation between intermittency and fractality. Different methods of studying the fractal structures in multiparticle data have been proposed. Hwa and others[16–18] proposed an approach, in terms of G_q moment, to revealing the multifractal nature of the particle spectra. But for low multiplicity events, the fractal moment G_q are dominated by the statistical noise. Hwa and Pan[15] suggested the modified form of multifractal multiplicity moments of order q defined for a single event as the expression

$$G_q = \sum_{j=1}^{M} (n_j/N)^q \theta(n_j - q), \tag{7}$$

where $\theta(n_j - q)$ is a step function which is 1 for $n_j \geq q$ and 0 for $n_j < q$. This step function has been added to the old definition of fractal moments[17] in order to suppress the effect of statistical noise.

Chiu and Hwa[17] proposed a self-similar particle production process, characterized by a power-law dependence of $< G_q >$. This can be expressed as

$$< G_q > \propto M^{\tau_q}, \tag{8}$$

where τ_q, the fractal indices, can be obtained from the slope

$$\tau_q = \frac{\delta < \ln G_q >}{\delta \ln(M)} \tag{9}$$

on a double logarithmic plot, after averaging over all events in the sample. For an ensemble of events, the average is taken as

$$< G_q > = \frac{1}{N_{\rm ev}} \sum_{N_{\rm ev}} G_q, \tag{10}$$

where $N_{\rm ev}$ is the total number of events in the ensemble.

Once τ_q is determined from $< G_q >$, one can employ the theory of multifractals to calculate the generalized dimension D_q which is related to the mass exponents τ_q by an expression[19]

$$D_q = \frac{\tau_q}{q-1}, \tag{11}$$

where, D_0, D_1, and D_2 are known as the fractal, information and correlation dimensions, respectively.

4. Experimental results

Figures 1(a) and 1(b) represent the nature of variation of RSFMs of the order 2, 3 and 4 on the bin size in logarithmic scale for ^{84}Kr–AgBr interactions in $\cos\theta$-, and ϕ-spaces separately. The plots clearly

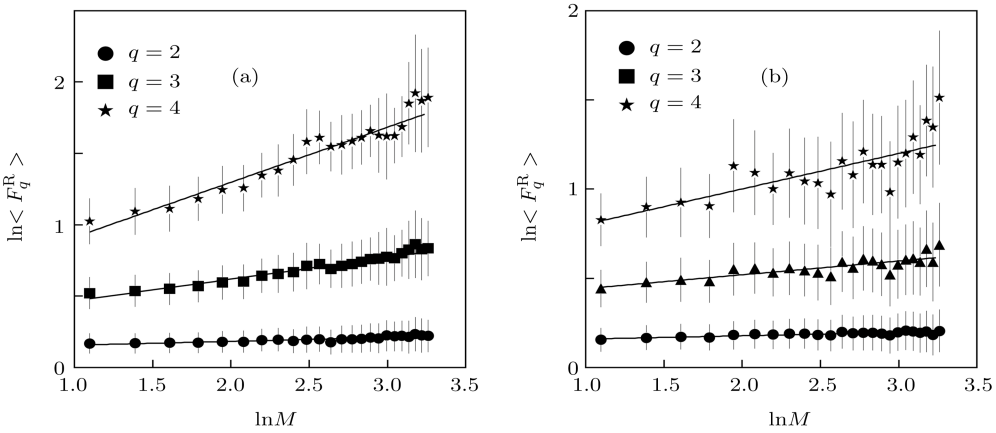


Fig.1. $\ln < F_q^R >$ against $\ln M$ in emission angle space (a) and azimuthal angle space (b) in case of ^{84}Kr–AgBr interactions.

indicate a power-law rise of the RSFM averages with decreasing bin size. For each plot the linear fit has been performed and the value of slope is presented in Table 1. In both cases the data slope increases with order of moment. The variation of d_q with q is shown in Fig.2 for $\cos\theta$-, and ϕ-spaces, separately. From the plot it is seen that d_q is dependent on the order q for both spaces. More specifically, the value of d_q increases linearly with the order q for all the data sets. For the data in $\cos\theta$-space, $d_q = (0.051 \pm 0.018)q - (0.076 \pm 0.059)$, with $\chi^2 = 0.004$. For the data in ϕ-space, $d_q = (0.024 \pm 0.019)q - (0.032 \pm 0.060)$, with $\chi^2 = 0.010$. These suggest that some cascade-type mechanism may be responsible for the emission of target fragments.

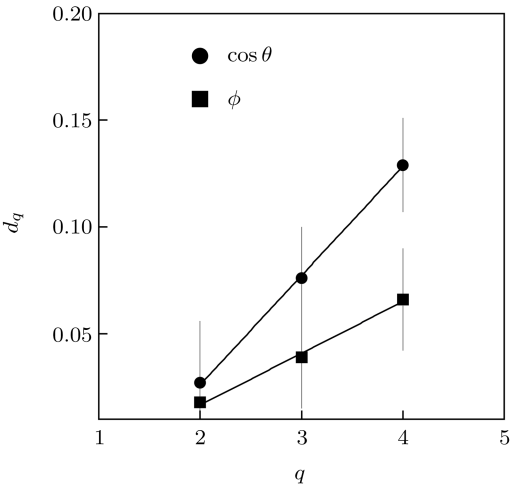


Fig.2. Variation of d_q with the order of moments in $\cos\theta$-space, and ϕ-space for ^{84}Kr–AgBr interactions.

Table 1. Values of the intermittency exponents and anomalous dimensions for target fragments from the ^{84}Kr–AgBr interactions at 1.7 A GeV in both $\cos\theta$- and ϕ-spaces.

q	$\cos\theta$-space		ϕ-space	
	a_q	d_q	a_q	d_q
2	0.027 ± 0.029	0.027 ± 0.029	0.018 ± 0.029	0.018 ± 0.029
3	0.152 ± 0.047	0.076 ± 0.024	0.077 ± 0.048	0.039 ± 0.024
4	0.386 ± 0.067	0.129 ± 0.022	0.199 ± 0.073	0.066 ± 0.024

The values of $< G_q >$ in $\cos\theta$- and ϕ-spaces are calculated using Eqs.(7) and (10) for order $q = 2$, 3, and 4. Figures 3(a) and 3(b) show the variation of $\ln < G_q >$ with $\ln(M)$ for order $q = 2$, 3, and 4 for ^{84}Kr–AgBr interactions at 1.7 A GeV. A linear dependence of $\ln < G_q >$ on $\ln(M)$ is observed in both phase spaces, indicating the self-similarity in particle emission processes. The exponent τ_q of the power-law behaviour is obtained by using the least square fitting of the data points, which is listed in Table 2. From the results in Fig.3 it can be seen that most of experimental points calculated using the modified from

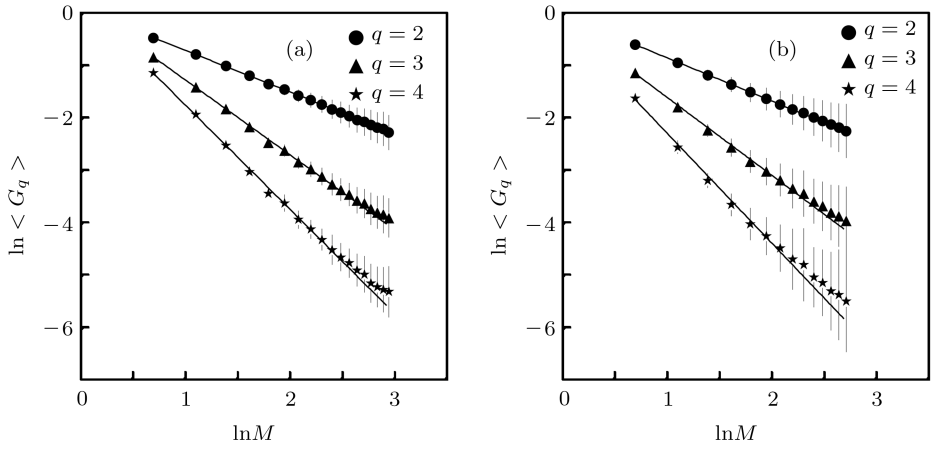


Fig.3. $\ln < G_q >$ against $\ln M$ in emission angle space (a) and azimuthal angle space (b) in case of ^{84}Kr–AgBr interactions.

of multifractal multiplicity moments of order q can be well represented by a linear fit, but for $q = 4$ and the smaller values of bin size (bigger values of M) the experimental points are higher than the corresponding fitted values. This can be explained by introducing a step function which can not only suppress the effect of statistical noise but also eliminate the contribution of lower multiplicity ($n_j < 4$, for $q = 4$) in a given bin. For multiplicity $n_j = 1$ and 2 in a given bin most of these experimental points can be explained by using a statistical noise, but for $n_j = 3$ most of them are not due to the statistical noise. So the modified form of multifractal multiplicity moments suggested by Hwa and Pan[15] is not effective in the multifractal moment analysis for higher orders q. The generalized dimension D_q, calculated by using expression (11), is also presented in Table 2. From the results of fractal indices τ_q (or mass exponents) and generalized dimensions D_q we can concluded that the τ_q increases with the order of q, but generalized dimension D_q decreases with the increase of q for target fragment emission in ^{84}Kr–AgBr interactions at 1.7 A GeV. This tendency is the same as that of the results of G_q analysis for target fragment emission in high energy lepton-nucleus,[20] hadron-nucleus,[21] and nucleus-nucleus[5−9,22,23] interactions. The variation of D_q with q is shown in Fig.4 for $\cos\theta$- and ϕ-spaces separately. From the plots it is seen that the value of D_q decreases with the increase of the order of moments q.

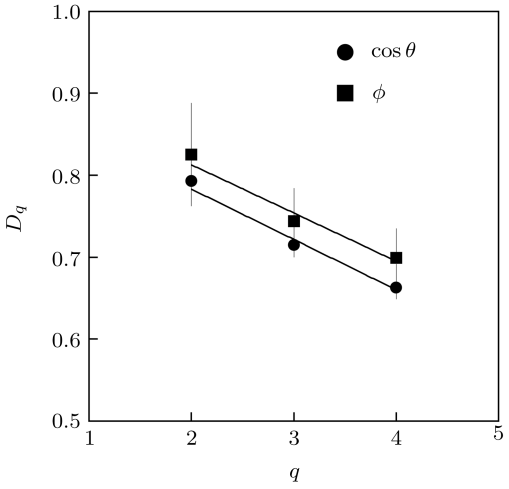


Fig.4. Variation of D_q with the order of moments in $\cos\theta$-space and ϕ-space for ^{84}Kr–AgBr interactions.

Table 2. Values of the fractal indices and generalized dimensions for target fragments from the ^{84}Kr–AgBr interactions at 1.7 A GeV in both $\cos\theta$- and ϕ-spaces.

q	$\cos\theta$-space		ϕ-space	
	τ_q	D_q	τ_q	D_q
2	0.793 ± 0.026	0.793 ± 0.026	0.825 ± 0.063	0.825 ± 0.063
3	1.429 ± 0.030	0.715 ± 0.015	1.487 ± 0.079	0.744 ± 0.040
4	1.990 ± 0.042	0.663 ± 0.014	2.096 ± 0.108	0.699 ± 0.036

5. Conclusion

From the present study of the 1.7 A GeV ^{84}Kr–AgBr interactions, the effects of the intermittency and multifractality have been observed in the selected interactions. The dependences of the RSFMs and the G_q moments on the bin size of $\cos\theta$- and ϕ-spaces, are found to obey a characteristic power law. The anomalous fractal dimensions of the intermittency are found to increase with the increase of the rank of the moments.

Acknowledgment

We are thankful to Professor I. Otterlund of the Lund University in Sweden for supplying the emulsion plates.

References

[1] Bialas A and Peschanski R 1986 *Nucl. Phys.* B **273** 703
Bialas A and Peschanski R 1988 *Nucl. Phys.* B **308** 857

[2] Burnett T H, Dake S, Fuki M, Gregory J C, Hayashi T, Holynski R, Iwai J, Jones W V, Jurak A, Lord J J, Miyamura O, Oda H, Ogata T, Parnell T A, Saito T, Tabuki T, Takahashi Y, Tominaga T, Wilczyuska B, Wilkes R J, Wolter W and Wosiek B (JACEE Collaboration) 1983 *Phys. Rev. Lett.* **50** 2062

[3] DeWolf E A, Dremin I M and Kittel W 1996 *Phys. Rep.* **270** 1

[4] Powell C F, Fowell P H and Perkins D H 1959 *The Study of Elementary Barthides by the Photographic Method* (Oxford: Pergamon) pp450-464 and references therein
[5] Ghosh D, Deb A, Bhattacharyya S, Das J G R and Mukherjee S 2003 *Int. J. Mod. Phys.* **12** 407
[6] Ghosh D, Deb A, Bhattacharyya S and Ghosh J 2004 *Int. J. Mod. Phys. Lett.* E **13** 737
[7] Ghosh D, Deb A, Sahoo S R, Patra K K, Haldar P K and Ghosh J 2002 *Czech. J. Phys.* **52** 789
[8] Ghosh D, Ghosh P, Ghosh A and Roy J 1994 *J. Phys.* G **20** 1077
[9] Sarkar S, Goswami T D, Ghosh D and Deb A 2003 *Czech. J. Phys.* **53** 133
[10] Zhang D H, Liu F, He C L, Zhao H H, Jia H M, Li X Q, Li Z Y and Li J S 2006 *Chin. Phys.* **15** 2564
[11] Zhang D H, Zhao H H, Liu F, He C L, Jia H M, Li X Q, Li Z Y and Li J S 2006 *Chin. Phys.* **15** 1987
[12] Song F, Zhang D H and Li J S 2005 *Chin. Phys.* **14** 942
[13] Zhang D H, Li Z Y, Li H L and Li J S 2005 *Chin. Phys.* **14** 2451
[14] Fialkowski K, Wosick B and Wosick J 1989 *Acta Phys. Pol.* B **20** 639
[15] Hwa R C and Pan J 1992 *Phys. Rev.* D **45** 1476
[16] Hwa R C, 1990. *Phys. Rev.* D **41** 1456
[17] Chiu C B and Hwa R C 1991 *Phys. Rev.* D **43** 100
[18] Florkowski W and Hwa R C 1991 *Phys. Rev.* D **43** 1548
[19] Grassberger P and Procaccia I 1984 *Physica* D **13** 34
[20] Ghosh D, Deb A, Lahiri M B, Ghosh P, Ahmed S I and Halder P K 2004 *Phys. Rev.* C **70** 054903
[21] Ghosh D, Ghosh P, Deb A, Halder D, Das S, Hossain A, Dey A and Roy J 1992 *Phys. Rev.* D **46** 3712
[22] Ghosh D, Deb A, Chattopadhyay R, Sarkar S, Jafry A K, Lahiri M, Das S, Purkait K, Biswas B, Roychoudhury J and Das S 1998 *Phys. Rev.* C **58** 3553
[23] Ghosh D, Chattopadhyay R, Sarkar S, Jafry A K, Lahiri M, Das S, Purkait K, Roychoudhury J, Biswas B and Deb A 1997 *Z. Phys.* C **73** 269