Vol 16 No 9, September 2007
1009-1963/2007/16(09)/2676-07

Chinese Physics

© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

Scaling of anisotropic flows and nuclear equation of state in intermediate energy heavy ion collisions*

Yan Ting-Zhi(颜廷志)[a)b)], Ma Yu-Gang(马余刚)[a)†], Cai Xiang-Zhou(蔡翔舟)[a)], Fang De-Qing(方德清)[a)], Guo Wei(郭 威)[a)b)], Ma Chun-Wang(马春旺)[a)b)], Shen Wen-Qing(沈文庆)[a)], Tian Wen-Dong(田文栋)[a)], and Wang Kun(王 鲲)[a)]

[a)] *Shanghai Institute of Applied Physics, Chinese Academy of Sciences, Shanghai* 201800, *China*

[b)] *Graduate School of the Chinese Academy of Sciences, Beijing* 100039, *China*

(Received 26 December 2006; revised manuscript received 4 July 2007)

Elliptic flow (v_2) and hexadecupole flow (v_4) of light clusters have been studied in detail for 25 MeV/nucleon ^{86}Kr + ^{124}Sn at large impact parameters by using a quantum molecular dynamics model with different potential parameters. Four sets of parameters including soft or hard equation of state (EOS) with or without symmetry energy term are used. Both number-of-nucleon (A) scaling of the elliptic flow versus transverse momentum ($p_{\rm t}$) and the scaling of v_4/A^2 versus $(p_{\rm t}/A)^2$ have been demonstrated for the light clusters in all above calculation conditions. It is also found that the ratio of $v_4/{v_2}^2$ maintains a constant of 1/2 which is independent of $p_{\rm t}$ for all the light fragments. Comparisons among different combinations of the EOS and the symmetry potential term show that the above scaling behaviours are sound and independent of the details of potential, while the strengths of flows are sensitive to the EOS and the symmetry potential term.

Keywords: anisotropic flow, number-of-nucleon scaling, EOS, symmetry energy
PACC: 2410, 2570, 2587

Anisotropic flows are very useful to explore heavy-ion collision dynamics as they result from the transition of the original space-time asymmetry into a momentum space anisotropy for the non-central collision.[1−14] Different mechanisms of the final momentum anisotropy, i.e. flow have been proposed. The studies of the dependences of the directed flow (v_1) and the elliptic flow (v_2) on beam energies, mass number, isospin and impact parameter have been carried out, and much interesting physics on the properties and origin of the collective motion at either nucleonic or partonic levels has been demonstrated. Very recently, we have carried out a calculation by using a quantum molecular dynamics model with hard equation of state and symmetry energy term and found that there is a nucleon-number scaling for the elliptic flow of light particles with mass number A up to 4.[15] In the present work, we present more details about the nucleon number dependences of the anisotropic flows v_2 and v_4 for ^{86}Kr + ^{124}Sn collisions at 25 MeV/nucleon and large impact parameters ($b = 7-10$ fm) with different EOSs and symmetry energy interactions. The scaling behaviours look robust since they do not depend on the parameters used in the model, and the sensitivities of v_2 and v_4 to EOS and symmetry potential are discussed.

Anisotropic flows are defined as the different n-th harmonic coefficients v_n of the Fourier expansion for the particle invariant azimuthal distribution

$$\frac{{\rm d}N}{{\rm d}\phi} \propto 1 + 2\sum_{n=1}^{\infty} v_n \cos(n\phi), \tag{1}$$

where ϕ is the azimuthal angle between the transverse momentum of the particle and the reaction plane. Note that the z-axis is defined as the direction along the beam and the impact parameter axis is labelled as x-axis. The first harmonic coefficient v_1 represents directed flow, $v_1 = \langle \cos\phi \rangle = \langle p_x/p_{\rm t} \rangle$, where $p_{\rm t} = \sqrt{p_x^2 + p_y^2}$ is transverse momentum. While the v_2, which measures the eccentricity of the particle distribution in the momentum space, represents elliptic flow, and it is expressed as

$$v_2 = \langle \cos(2\phi) \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_{\rm t}^2} \right\rangle, \tag{2}$$

*Project supported by the National Natural Science Foundation of China (Grant Nos 10535010 and 10610285), the Shanghai Development Foundation for Science and Technology of China (Grant Nos 06JC14082 and 05XD14021), and Chinese Academy of Sciences project (Grant No KJCX3.SYW.N2).

†Corresponding author. E-mail: ygma@sinap.ac.cn

and v_4 represents the 4-th momentum anisotropy, namely hexadecupole flow, and it is given by

$$v_4 = \left\langle \frac{p_x^4 - 6p_x^2p_y^2 + p_y^4}{p_t^4} \right\rangle. \quad (3)$$

The intermediate energy heavy-ion collision dynamics is complex, for both mean field and nucleon–nucleon collisions are playing the competition roles. Furthermore, the isospin-dependent role should be also incorporated for asymmetric reaction systems. Isospin-dependent quantum molecular dynamics model (IDQMD) has been affiliated with isospin degrees of freedom under mean field and nucleon–nucleon collisions.[16−23] The IDQMD model can explicitly represent the many-body state of the system and mainly contains correlation effects to all orders and all fluctuations, and can well describe the time evolution of the colliding system. When the spatial distance (Δr) is less than 3.5 fm and the momentum difference (Δp) is smaller than 300 MeV/c between two nucleons, two nucleons can coalesce into a cluster.[16] With this simple coalescence mechanism, which has been extensively applied in transport theory, the clusters with different sizes can be recognized.

In the model the nuclear mean-field potential is parameterized as

$$U(\rho, \tau_z) = \alpha\Big(\frac{\rho}{\rho_0}\Big) + \beta\Big(\frac{\rho}{\rho_0}\Big)^\gamma + \frac{1}{2}(1-\tau_z)V_C + C_{sym}\frac{(\rho_n - \rho_p)}{\rho_0}\tau_z + U^{Yuk}, \quad (4)$$

where ρ_0 is the normal nuclear matter density (0.16 fm^{-3}); ρ_n, ρ_p and ρ are the neutron, proton and total densities, respectively; τ_z is z-th component of the isospin degree of freedom, which equals 1 or -1 for neutrons or protons, respectively; the coefficients α, β and γ are parameters for nuclear equation of state; C_{sym} is the symmetry energy strength due to the difference in density between neutrons and protons in a nuclear medium, which is important for asymmetrical nuclear matters;[24−26] $C_{sym} = 32$ MeV is used to consider symmetry energy effect or isospin-dependent potential; $C_{sym} = 0$ for no symmetry energy effect or isospin-independent potential; V_C is the Coulomb potential; U^{Yuk} is the Yukawa (surface) potential. In the present work, we take $\alpha = 124$ MeV, $\beta = 70.5$ MeV and $\gamma = 2$, which corresponds to the so-called hard EOS with an incompressibility of $K = 380$ MeV, $\alpha = -356$ MeV, $\beta = 303$ MeV and $\gamma = 7/6$, which corresponds to the so-called soft EOS with an incompressibility of $K = 200$ MeV. Four combinations with different potential parameters, i.e. parameters of hard or soft EOS with or without the symmetry energy effect (i.e. $C_{sym} = 32$ or 0 MeV), for the collision system of ^{86}Kr + ^{124}Sn at 25 MeV/nucleon with impact parameters ranging from 7 to 10 fm are performed. The physical results are extracted at the time of 200 fm/c when the system has been in the freeze-out stage.

Figures 1(a), 1(b), 1(e) and 1(f) show the transverse momentum dependences of elliptic flows for mid-rapidity light fragments under four different calculation conditions: (a) for soft EOS with the symmetry potential (soft_iso); (b) for hard EOS with the symmetry potential (hard_iso); (e) for soft EOS without the symmetry potential (soft_niso) and (f) for hard EOS without the symmetry potential (hard_niso). In all cases, elliptic flow is positively valued and it increases with p_t increasing, which is apparently similar to RHIC's results.[12,13] Of course, their mechanisms are quite different. In an intermediate energy domain, collective rotation is one of the main mechanisms to induce the positive elliptic flow.[2,3,27−29] However, at RHIC energies the mechanism is the strong pressure which is built by the early initial almond anisotropy of the geometrical overlap zone between two colliding nuclei, which drives the positive elliptic flow.[12] The corresponding nucleon-number scaled elliptic flows are plotted in Figs.1(c), 1(d), 1(g) and 1(h) as a function of transverse momentum per nucleon. From these subfigures, it seems that the number-of-nucleon scaling for elliptic flow exists for light fragments at low values of p_t/A ($p_t/A < 0.2$ GeV/c). This behaviour is apparently similar to the number-of-constituent quark scaling of elliptic flow versus transverse momentum per constituent quark (p_t/n) for different mesons and baryons, which was observed at RHIC.[12] Since all calculations show similar scaling behaviours, this scaling behaviour is robust, and it is independent of the details of EOS and symmetry potential.

To quantitatively investigate the difference of the flows calculated under different conditions, we compare the values of v_2/A simulated under four conditions (see Fig.2). The subfigures show that the differences among different simulations are big for neutrons and protons but a little small for the fragments of $A = 2$, 3 and 4. The reason is that the emitted protons and neutrons can feel the role of mean field (EOS) directly, while the light fragments have a weak sensitivity since they are indirected products by the coalescence mechanism in the present model. Approximately, at the same value of p_t/A, the elliptic flow is

larger for soft EOS than for hard EOS, and it is larger for EOS without the symmetry potential than with the symmetry potential. As the symmetry potential basically is positively valued for the studied reaction system (more neutrons than protons), the symmetry potential will make the whole EOS stiffer. In this case, we can say that the stiffer the EOS, the smaller the flow. In other words, we can say the strength of elliptic flow per nucleon is sensitive to the EOS and symmetry potential.

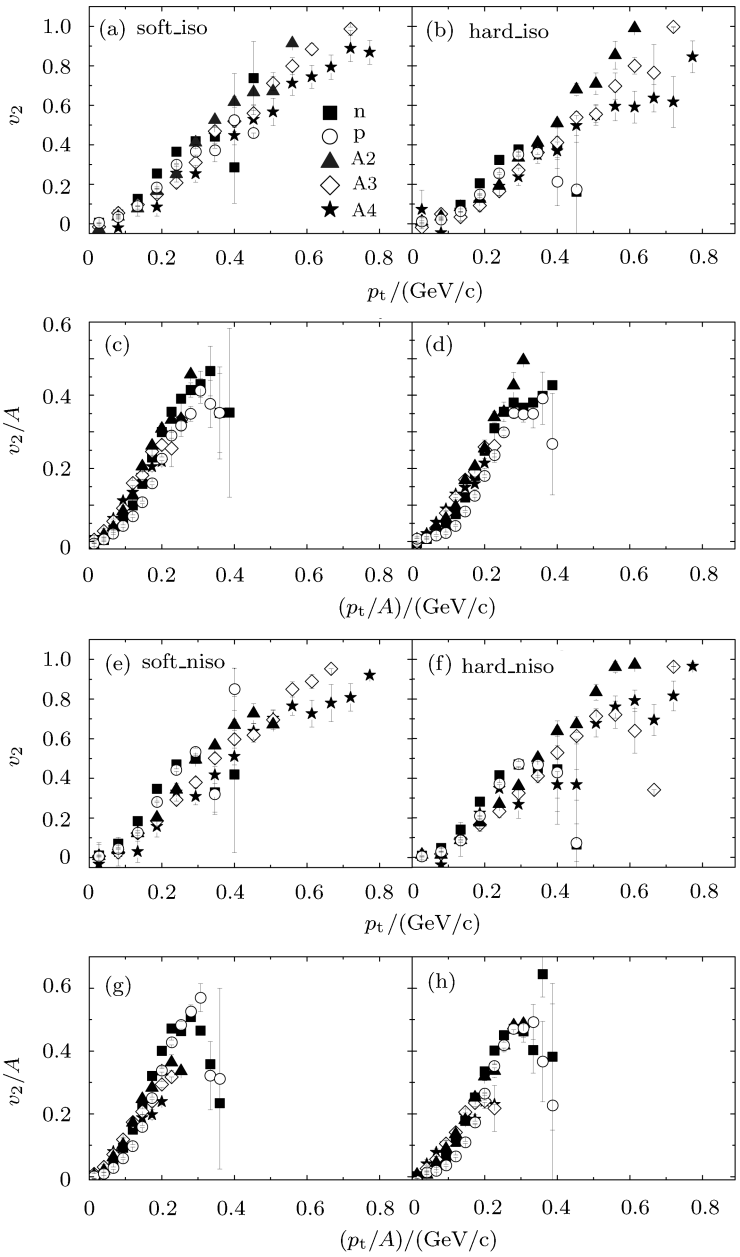


Fig.1. Elliptic flow as a function of transverse momentum (p_t) for the simulation under different parameters of EOS with or without the symmetry energy term. (a) for soft EOS with the symmetry potential (soft_iso); (b) for hard EOS with the symmetry potential (hard_iso); (e) for soft EOS without the symmetry potential (soft_niso); and (f) for hard EOS without the symmetry potential (hard_niso). Squares are for neutrons, circles for protons, triangles for fragments of $A = 2$, diamonds for $A = 3$, and stars for $A = 4$. Subfigures (c), (d), (g) and (h) are for nucleon-number normalized elliptic flow as a function of transverse momentum per nucleon, corresponding to the cases of (a), (b), (e) and (f), respectively.

So far, there have been little studied the higher order flows, such as v_4, both experimentally and theoretically in this energy domain. Here we try to explore the behaviour of v_4. First we draw v_4/A as a function of p_t/A to mimic the behaviours of elliptic flows (see subfigures (a), (b), (e) and (f) of Fig.3) calculated under four different conditions. These subfigures show that v_4/A is positively valued and increases with p_t/A, but there seems to exist no simple scaling behaviour as v_2 shows. Considering that RHIC experimental data have demonstrated that a scaling relation among hadron anisotropic flows holds, i.e. $v_n(p_t) \sim v_2^{n/2}(p_t)$,[30] we plot v_4/A^2 as a function of $(p_t/A)^2$ in Figs.3(c), 3(d), 3(g) and 3(h) for the calculation conditions corresponding to those for Figs.3(a), 3(b), 3(e) and 3(f) respectively. Now the points for different light fragments nearly merge together at low values of $(p_t/A)^2$, which means that a certain scaling law holds between two variables. All the calculation cases show that there exists a scaling behaviour for v_4/A^2 versus $(p_t/A)^2$, and this behaviour is robust regardless of the parameters which we used for EOS.

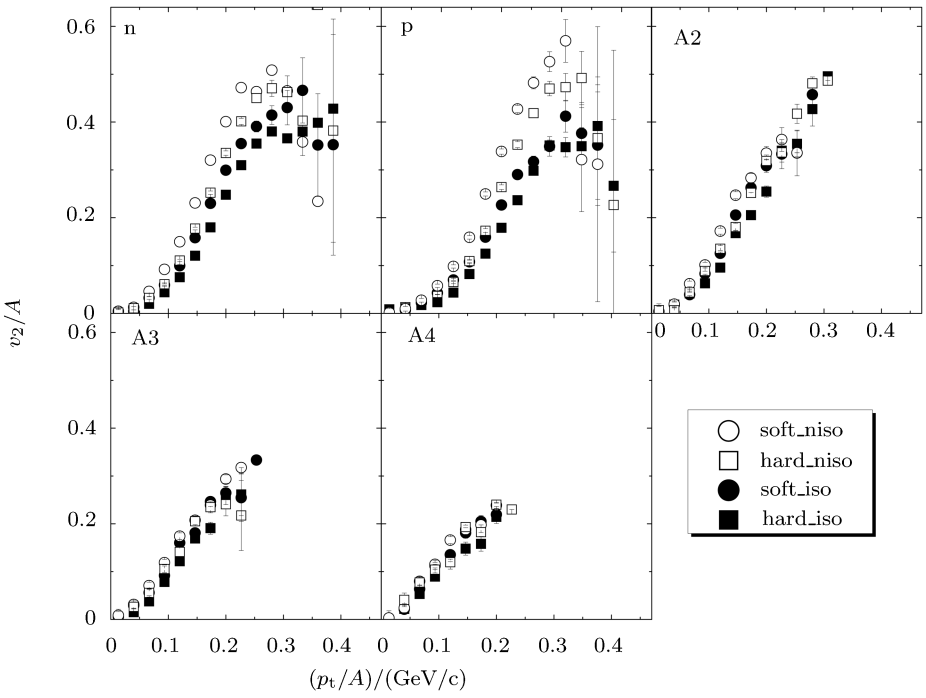


Fig.2. The values of v_2/A versus p_t/A simulated under different simulation conditions. The meanings of the symbols are indicated in a frame on right bottom corner.

Since in the above scaling behaviour assumed is $v_n(p_t) \sim v_2^{n/2}(p_t)$, we plot v_4/v_2^2 as a function of p_t in Fig.4 for the four simulations. The subfigures show that the ratios of v_4/v_2^2 for different fragments with mass number A up to 4 are around a constant of 1/2 in all simulation cases. Because v_2/A can be scaled with p_t/A, v_4/A^2 should be scaled by $(p_t/A)^2$, which is exactly what we see in Fig.4. One point is worth mentioning compared with the RHIC studies where the data show $v_4/v_2^2 \sim 1.2$,[30] and $v_4/v_2^2 \sim 1/2$ for the light nuclear fragments in this nucleonic level coalescence mechanism rather than the value of 3/4 for mesons or 2/3 for baryons in quark coalescence model.[31] Coincidentally, the predicted value of the ratio of v_4/v_2^2 for hadrons is also 1/2 if the matter produced in ultra-relativistic heavy ion collisions reaches a thermal equilibrium and its subsequent evolution follows the laws of ideal fluid dynamics.[32] It is interesting to note that the same ratio is predicted in two different models at very different energies, which is, of course, worth to be further investigated in the near future. One possible interpretation is that the big nucleon–nucleon cross sections in low energy HIC make the system reach a thermal equilibrium and may induce the fluid-like behaviour of nuclear medium before the light fragments are coalesced by nucleons. In this case, the value of v_4/v_2^2 of light fragments could be $\sim 1/2$ as shown in Ref.[32].

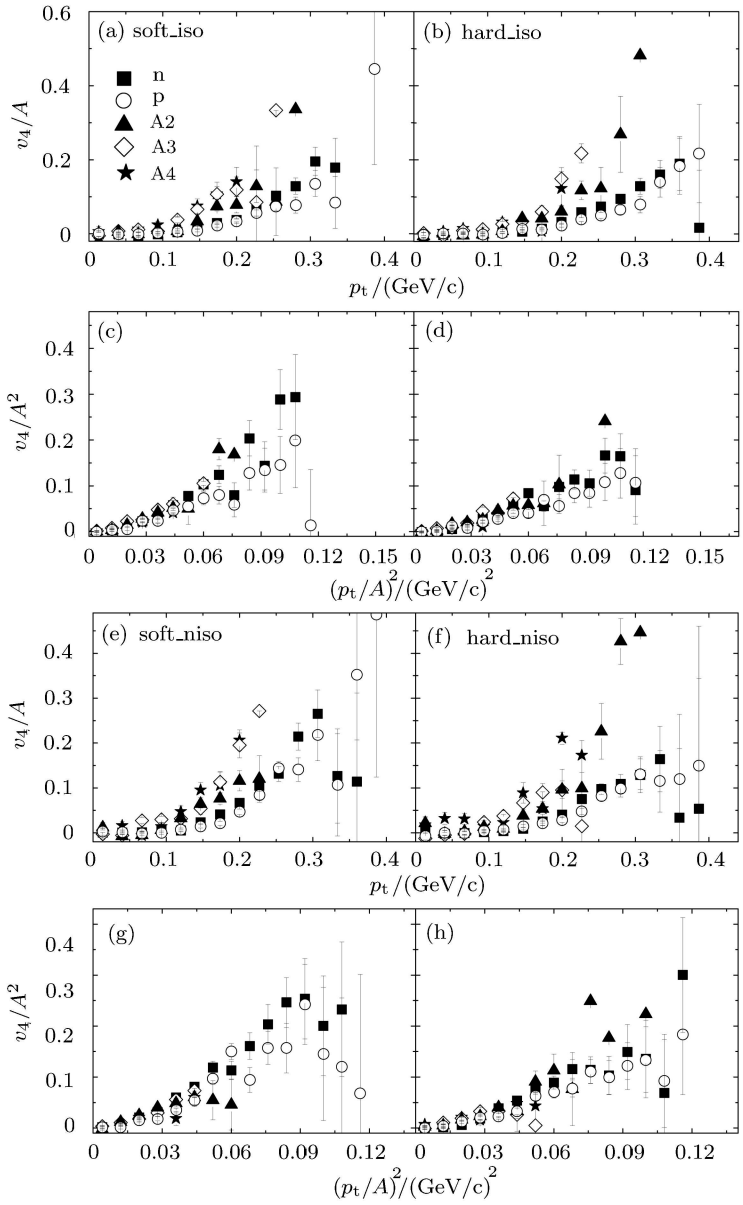


Fig.3. The same as Fig.1 but for v_4/A versus p_t [(a), (b), (e) and (f)] and v_4/A^2 versus $(p_t/A)^2$ [(c), (d), (g) and (h)].

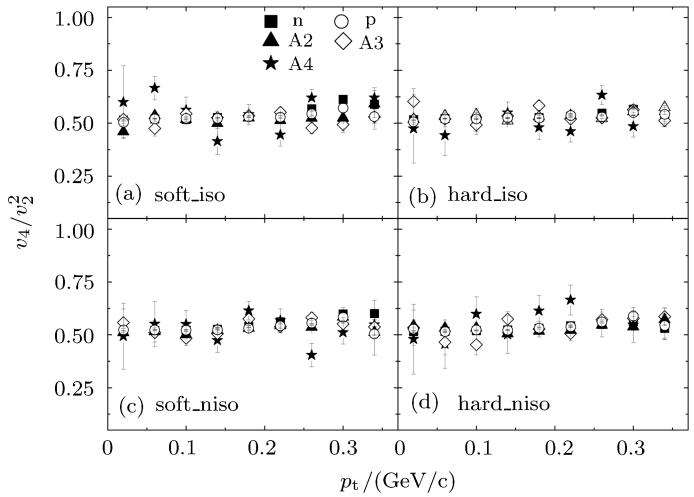


Fig.4. The ratios of v_4/v_2^2 for neutrons (squares), proton (circles), the fragments of $A = 2$ (triangles), $A = 3$ (diamonds) and $A = 4$ (stars) versus p_t for four simulations under different conditions.

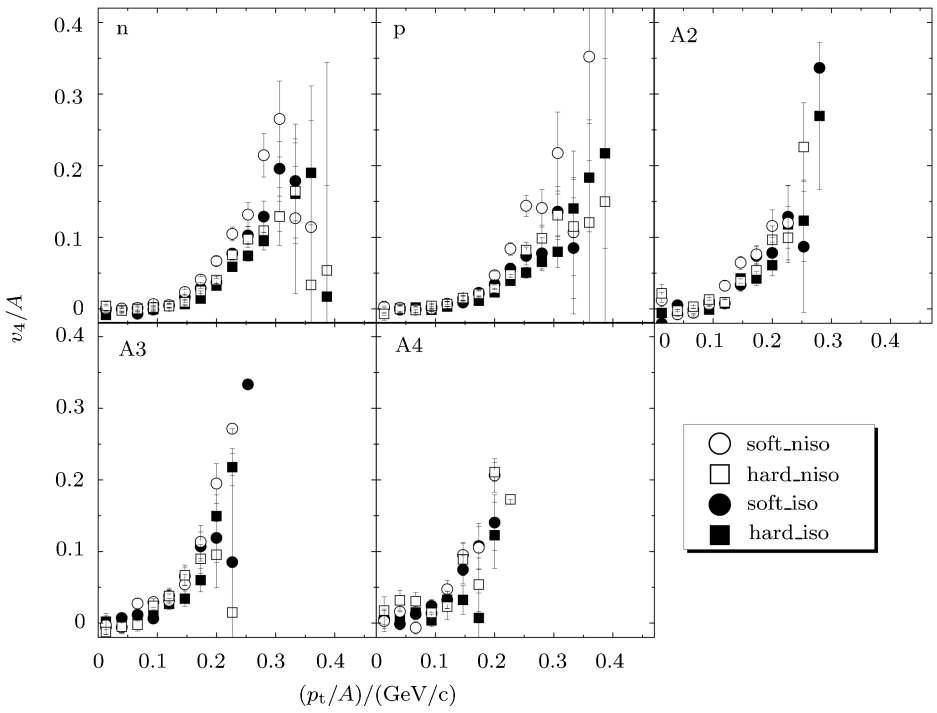


Fig.5. The values of v_4/A versus p_t/A. The meanings of the symbols are indicated in a frame on right bottom corner.

The values of v_4/A versus p_t/A with different simulation parameters are also presented for light fragments (see Fig.5). The subfigures are similar to those in Fig.2, and the effects of EOS and symmetry potential on v_4/A are also similar to their effects on v_2. However, comparing with the sensitivity of v_2 to the EOS and the symmetry potential, v_4/A is not so salient.

In summary, we have investigated the behaviour of anisotropic flows as a function of transverse momentum for light fragments through the simulation of 25 MeV/nucleon ^{86}Kr+^{124}Sn collisions in peripheral collisions by using the IDQMD model under the potential parameters of hard or soft EOS with or without the symmetry energy term. It is found that for all simulations for the four types, v_2 and v_4 of light fragments are positively valued and increase with p_t/A. The curves of v_2 per nucleon (v_2/A) versus p_t/A for all light particles all approach the same curve. Similarly, the values of v_4/A^2 merge together as a function of $(p_t/A)^2$ for all light particles. Furthermore, it is found that v_4 can be well scaled by v_2^2, and the value of $v_4/v_2^2 \sim 1/2$ independent of transverse momentum. The above scaling behaviour can be seen as a consequence of the nucleonic coalescence, and it illustrates that the number-of-nucleon scaling for elliptic flow exists in intermediate energy heavy ion collisions. In addition, the values of v_2/A and v_4/A are compared under different simulation conditions, and it is shown that the values of the v_2 are sensitive to the EOS and symmetry potential, especially for neutrons and protons.

References

[1] Ollitrault J 1992 *Phys. Rev.* D **46** 229

[2] Ma Y G, Shen W Q, Feng J and Ma Y Q 1993 *Phys. Rev.* C **48** 1492
Ma Y G, Shen W Q, Feng J and Ma Y Q 1993 *Z. Phys.* A **344** 469
Ma Y G, Shen W Q and Zhu Z Y 1995 *Phys. Rev.* C **51** 1029
Ma Y G and Shen W Q 1995 *Phys. Rev.* C **51** 3256

[3] Shen W Q, Péter J, Bizard G, Broua R, Cussola D, Louvela M, Patrya J P, Régimbarta R, Steckmeyera J C, Sullivana J P, Tamaina B, Cremab E, Doubreb H, Hagelb K, Jing G M, Péghaireb A, Saint-Laurentb F, Cassagnouc Y, Legrainc R, Lebrund C, Rosatoe E, MacGrathf R, Jeongh S C, Leeh S M, Nagashimah Y, Nakagawah T, Ogiharah M, Kasagii J and Motobayashib T 1993 *Nucl. Phys.* A **551** 333

[4] Sorge H 1997 *Phys. Lett.* B **402** 251
Sorge H 1997 *Phys. Rev. Lett.* **78** 2309
Sorge H 1999 *Phys. Rev. Lett.* **82** 2048

[5] Danielewicz P, Lacey R A, Gossiaux P B, Pinkenburg C, Chung P, Alexander J M and McGrath1et R L 1998 *Phys. Rev. Lett.* **81** 2438

[6] Teaney D and Shuryak E V 1999 *Phys. Rev. Lett.* **83** 4951

[7] Kolb P F, Sollfrank J and Heinz U 2000 *Phys. Rev.* C **62** 054909

[8] Zheng Y M, Ko C M, Li B A and Zhang B 1999 *Phys. Rev. Lett.* **83** 2534

[9] Li Z X and Zhang Y X in AIP Conference Proceeding CP865: *Nuclear Physics Trends*: 6th China–Japan Joint Nuclear Physics Symposium (Melville, NY: American Institute of Physics), eds Ma Y G and Ozawa A p276

[10] Perslam D and Gale C 2002 *Phys. Rev.* C **65** 064611

[11] Lukasik J, Auger G, Begemann-Blaich M L, Bellaize N, Bittiger R, Bocage F, Borderie B, Bougault R, Bouriquet B, Charvet J L, Chbihi A, Dayras R, Durand D, Frankland J D, Galichet E, Gourio D, Guinet D, Hudan S, Lautesse P, Lavaud F, Le Févre A, Legrain R, Lopez O, Lynen U, Müller W F J, Nalpas L, Orth H, Plagnol E, Rosato E, Saija A, Schwarz C, Sfienti C, Tamain B, Trautmann W, Trzciński A, Turzó K, Vient E, Vigilante M, Volant S and Zwiegliński B 2004 *Phys. Lett.* B **608** 223

[12] Adams J *et al* (STAR Collaboration) 2004 *Phys. Rev. Lett.* **92** 052302
Adams J *et al* (STAR Collaboration) 2005 *Phys. Rev.* C **72** 014904
Adams J *et al* (STAR Collaboration) 2005 *Phys. Rev. Lett.* **95** 122301

[13] Chen J H, Ma Y G, Ma G L, Cai X Z, He Z J, Huang H Z, Long J L, Shen W Q, Zhong C and Zuo J X 2006 *Phys. Rev.* C **74** 064902

[14] Ma Y G 2006 *J. Phys.* G **32** S373

[15] Yan T Z, Ma Y G, Cai X Z, Chen J G, Fang D Q, Guo W, Ma C W, Ma E J, Shen W Q, Tian W D and Wang K 2006 *Phys. Lett.* B **638** 50

[16] Aichelin J 1991 *Phys. Rep.* **202** 233

[17] Ma Y G and Shen W Q 1995 *Phys. Rev.* C **51** 710

[18] Zhang F S, Chen L W, Zhao Y M and Zhu Z Y 1999 *Phys. Rev.* C **60** 064604

[19] Liu J Y, Guo W J, Ren Z Z, Xing Y Z, Zuo W and Lee X G 2006 *Chin. Phys.* **15** 1738

[20] Zhang H Y, Ma Y G, Su Q M, Shen W Q, Cai X Z, Fang D Q, Hu P Y and Han D D 2001 *Acta Phys. Sin.* **50** 193 (in Chinese)

[21] Wei Y B, Ma Y G, Shen W Q, Ma G L, Wang K, Cai X Z, Zhong C, Guo W and Chen J G 2004 *Phys. Lett.* B **586** 225
Wei Y B, Ma Y G, Shen W Q, Ma G L, Wang K, Cai X Z, Zhong C, Guo W, Chen J G, Fang D Q, Tian W D and Zhou X F 2004 *J. Phys.* G **30** 2019

[22] Ma Y G, Zhang H Y and Shen W Q 2002 *Prog. Phys.* **22** 99 (in Chinese)

[23] Ma Y G, Wei Y B, Shen W Q, Cai X Z, Chen J G. Chen J H, Fang D Q, Guo W, Ma C W, Ma G L, Su Q M, Tian W D, Wang K, Yan T Z, Zhong C and Zuo J X 2006 *Phys. Rev.* C **73** 014604

[24] Ma Y G 2000 *Acta Phys. Sin.* **49** 654 (in Chinese)
Ma Y G 1999 *Acta Phys. Sin.* **48** 1839 (in Chinese)
Ma Y G, Su Q M, Shen W Q, Han D D, Wang J S, Cai X Z, Fang D Q and Zhang H Y 2000 *Phys. Rev.* C **60** 024607

[25] Zhong C, Ma Y G, Fang D Q, Cai X Z,Chen J G, Shen W Q, Tian W D, Wang K, Wei Y B, Chen J H, Guo W, Ma C W, Ma G L, Su Q M, Yan T Z and Zuo J X 2006 *Chin. Phys.* **15** 1481

[26] Yong G C, Lee B A and Zuo W 2005 *Chin. Phys.* **14** 1549

[27] Sullivan J P and Péter J 1992 *Nucl. Phys.* A **540** 275

[28] Lacey R, Elmaani A, Lauret J, Li T, Bauer W, Craig D, Cronqvist M, Gualtieri E, Hannuschke S, Reposeur T, Vander Molen A, Westfall G D, Wilson W K, Winfield J S, Yee J, Yennello S, Nadasen A, Tickle R S and Norbeck E 1993 *Phys. Rev. Lett.* **70** 1224

[29] He Z Y, Angelique J C, Auger A, Bizard G, Brou R, Buta A, Cabot C, Crema E, Cussol D, Dai G X, Masri Y E, Eudes P, Gonin M, Hagel K, Jin G M, Kerambrum A, Lebrun C, Ma Y G, Péghaire A, Péter J, Popescu R, Régimbart R, Rosato E, Saint-Laurent F, Shen W Q, Steckmeyer J C, Tamain B, Vient E, Wada R and Zhang F S 1996 *Nucl. Phys.* A **598** 248

[30] Adams J *et al* (STAR Collaboration) 2004 *Phys. Rev. Lett.* **92** 062301
Adams J *et al* (STAR Collaboration) 2005 *Phys. Rev.* C **72** 014904

[31] Kolb P 2002 *Phys. Rev.* C **68** 031902(R)

[32] Borghini N and Ollitrault J Y 2006 *Phys. Lett.* B **642** 227