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Effect of strong magnetic field on electron capture of iron group nuclei in crusts of neutron stars*

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In this paper electron capture on iron group nuclei in crusts of neutron stars in a strong magnetic field is investigated. The results show that the magnetic fields have only a slight effect on electron capture rates in a range of $10^8 - 10^{13}$G on surfaces of most neutron stars, whereas for some magnetars the magnetic fields range from 10^{13} to 10^{18} G. The electron capture rates of most iron group nuclei are greatly decreased, reduced by even four orders of magnitude due to the strong magnetic field.

Keywords: strong magnetic field, electron capture, neutron star
PACC: 2340, 8760D, 9760J

1. Introduction

According to the stellar evolution and nucleosynthesis theory, the nuclear burning will soon come into the stage of the oxygen burning, when the temperature increases up to 2.5×10^9 K or more. The staple is the mid-mass elements (Si∼Ca). When the temperature increases up to 3.5×10^9 K or more, the nuclear burning would enter into the stage of the silicon core burning. At this time, the course of nucleosynthesis will push the element speedily to an iron group nucleus. As is well know, the poignant supernova explosions will lead up because of the unstable nuclear burning and iron nucleus collapse. Some researches show that the electron capture (EC) plays an important role in this process. Thus it is very important to investigate the electron capture, especially, on iron group nuclei, which is of benefit to finding out the characters of the higher density matters, testing the interaction model of the nucleus and investigating the late evolution of stellar and r-process nucleosynthesis.

The EC rates of many an iron group nucleus are investigated by Aufderheide *et al*[1,2] under the supernova-like explosion conditions. In the same environment, Dean *et al*[3] calculated the EC rates of the iron group nuclei too, but they discussed the EC in the case of no magnetic field. Dai *et al* (1993)[4] discussed the influence of a strong magnetic field (SMF) on EC at a zero temperature in neutron stars. And in 1996, the EC rates at non-zero temperatures in SMF were analysed by Luo and Peng.[5] In their work, they focused only on the ground state transitions of simple nuclei, and paid no attention to the Gamow–Teller (G–T) transition, as well as lost sight of the EC rates of a number of iron group nuclei.

The strengths of the surface magnetic fields of most neutron stars range from 10^8 to 10^{13} G[6] according to some researches and observation data. And some magnetars are observed to have magnetic fields ranging from 10^{13} to 10^{18} G,[7−10] the densities from 10^4 to 10^{11} g/cm^3, and the temperatures from 10^5 to 10^9 K. Under a strong magnetic field and higher density and temperature, the Fermi energy of the electron gas may go up to 10 Mev. The G–T transition must be considered due to the large effect on EC. Therefore it is necessary to discuss the EC rates of the iron group nuclei including the G–T transition. In this paper we focus on the iron group nuclei and investigate their EC rates. The remainder of the article is organized as follows: in the next section, we analyse the EC rates of the stars in SMF. In Section 3 we discuss the effect of SMFs on EC rates of most iron group nuclei. And some conclusions are given in Section 4.

2. The EC in a SMF

A strong magnetic field B is considered to be along the z-axis. The Dirac equation can be solved

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exactly. The positive energy levels are given by

$$\varepsilon_n = (p_z^2 + 1 + 2n\Theta)^{1/2}, \quad (n = 0, 1, 2, 3, \cdots), \quad (1)$$

where $\Theta = h_{\rm e}B/m_{\rm e}^2c = 0.0225B_{12}$, $B = 10^{12}B_{12}$ and p_z is the electron momentum along the field. Note that in this paper all of the energies and the moments are respectively in units of $m_{\rm e}c^2$ and $m_{\rm e}c$, where $m_{\rm e}$ is the electron mass and c is the speed of the light. The electron chemical potential can be found from the expression for the lepton number density

$$n_{\rm e} = \rho/\mu_{\rm e} = \frac{\Theta}{2\pi^2\lambda_{\rm e}^3}\sum_{n=0}^{\infty} q_{n0}\int_0^{\infty}(f_{-{\rm e}} - f_{+{\rm e}}){\rm d}p_z, \quad (2)$$

where ρ is the density in g/cm^3; $\mu_{\rm e}$ is the average molecular weight; $\lambda_{\rm e} = h/m_{\rm e}c$ is the Compton wavelength; $q_{n0} = 2 - \delta_{n0}$ is the electron degenerate number;

$$f_{-{\rm e}} = \Big[1 + \exp\Big(\frac{\varepsilon_n - U_{\rm F} - 1}{kT}\Big)\Big]^{-1}$$

and

$$f_{+{\rm e}} = \Big[1 + \exp\Big(\frac{\varepsilon_n + U_{\rm F} + 1}{kT}\Big)\Big]^{-1}$$

are the electron and positron distribution functions respectively; k is the Boltzmann constant; T is the electron temperature; $U_{\rm F}$ is the electron chemical potential.

The capture rate for the kth nucleus (Z, A) in thermal equilibrium at temperature T is given by a sum over the initial parent states numbered with i and the final daughter states numbered with f[11,12]

$$\lambda_k = \sum_i \frac{(2J_i + 1){\rm e}^{-E_i/kT}}{G(Z, A, T)}\sum_f \lambda_{if}, \quad (3)$$

where J_i and E_i are the spin and excitation energies of the parent states; $G(Z, A, T)$ is the nuclear partition function; λ_{if} is the EC rate from one of the initial states to all possible final states, i.e. $\lambda_{if} = (\ln 2/(ft)_{if})f_{if}$ with the relation

$$\frac{1}{(ft)_{if}} = \frac{1}{(ft)_{if}^{\rm F}} + \frac{1}{(ft)_{if}^{\rm GT}}.$$

The ft-values and the corresponding G–T or Fermi transition matrix elements are related by the following expressions:

$$\frac{1}{(ft)_{if}^{\rm GT}} = \frac{10^{3.596}}{|M_{\rm GT}|_{if}^2}, \quad \text{and} \quad \frac{1}{(ft)_{if}^{\rm F}} = \frac{10^{3.79}}{|M_{\rm F}|_{if}^2}. \quad (4)$$

We can find the phase space factor in SMF from Refs.[5,13], and it is defined as

$$f_{if}^{\rm B} = \frac{\Theta}{2}\sum_{n=0}^{\infty}\theta_n, \quad (5)$$

where

$$\theta_n = q_{n0}\int_{q_n}^{\infty}(Q_{if} + \varepsilon_n)^2\frac{F(z, \varepsilon_n)}{1 + \exp\Big(\frac{\varepsilon_n - U_{\rm F} - 1}{kT}\Big)}{\rm d}p_z,$$

with $Q_{if} = Q_{00} + E_i - E_f$ is the EC threshold energy; $Q_{00} = M_{\rm p}c^2 - M_{\rm d}c^2$, with $M_{\rm p}$ and $M_{\rm d}$ being the masses of the parent nucleus and the daughter nucleus respectively; E_i and E_f are the excitation energies of the ith state and fth state of the nucleus respectively; ε_n is the total rest mass and kinetic energies; $F(Z, \varepsilon_n)$ is the Coulomb wave correction which is the ratio of the square of the electron wavefunction distorted by the Coulomb scattering potential to the square of wavefunction of the free electron as shown below:[1]

$$F(Z, \varepsilon_n) = 2(1+\gamma)\Big(\frac{2p_{\rm e}R}{\hbar}\Big)^{-2(1-\gamma)}{\rm e}^{\pi\nu}\frac{|\Gamma(\gamma + {\rm i}\nu)|^2}{|\Gamma(2\gamma + 1)|^2} \quad (6)$$

with $\nu = Z\alpha(E_{\rm e}/pc)$; $\gamma = [1 - (Z\alpha)^2]^{1/2}$, R being the nuclear radius, Γ being the gamma function, Z being the nuclear charge of the parent state, α being the fine-structure constant; q_n is defined as

$$q_n = \begin{cases}(Q_{if}^2 - 1 - 2n\Theta)^{1/2}, & (Q_{if} < -(1 + 2n\Theta)^{1/2}),\\ 0, & \text{otherwise.}\end{cases} \quad (7)$$

3. The EC rates of iron group nuclei in SMF

Figures 1 and 2 show the EC rates of the nuclei $^{53-57,59,61}$Fe and $^{55-60}$Co as a function of magnetic field at a lepton number density $\rho/\mu_{\rm e} = 3.16 \times 10^8$ mol/cm^3 and temperature $T = 7 \times 10^9$ K. It can be seen from Figs.1 and 2 that when $B < 10^{14}$ G the magnetic field has only a slight effect on EC rate. However, when $B > 10^{14}$ G, the influence of SMF is increased. The EC rates of some nuclei in a SMF may be reduced by almost one order of magnitude (see Fig.1). But for $^{55-60}$Co the reductions of EC rates are within one order of magnitude (see Fig.2). Figures 3, 4 and 5 show the EC rates of the nuclei $^{56-63}$Ni, $^{55-61}$Mn and $^{53-60}$Cr as a function of magnetic field under the same condition. It can be seen from the three figures that when $B < 10^{14}$ G the magnetic field has only a mirror effect on EC rate for most nuclei. But the EC rates of most nuclei can be influenced greatly when $B > 10^{14}$ G. Figure 3 shows that for some nuclei (such as ^{56}Ni, ^{57}Ni and ^{61}Ni), the EC rates may be reduced by almost more than one order of magnitude in SMF. However, from Figs.4 and 5, we can see that

for $^{55-61}$Mn and $^{53-60}$Cr, the reductions in EC rate for most nuclei are within one order of magnitude.

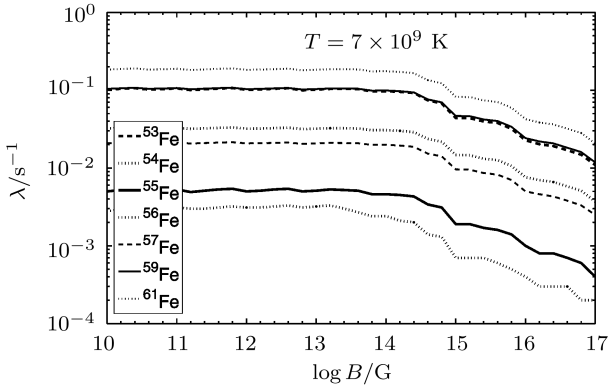


Fig.1. The EC rates as a function of magnetic field at a lepton number density $\rho/\mu_e = 3.16 \times 10^8$ mol/cm^3 and temperature $T = 7 \times 10^9$ K.

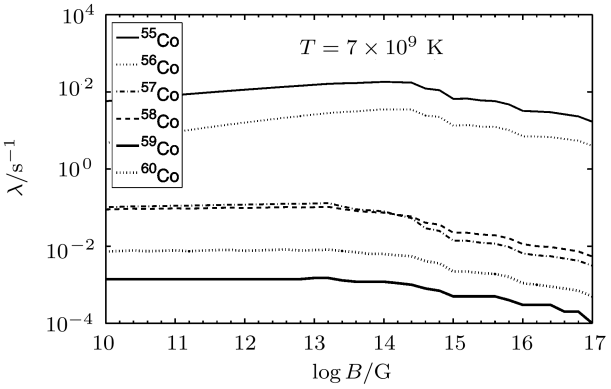


Fig.2. The EC rates as a function of magnetic field at a lepton number density $\rho/\mu_e = 3.16 \times 10^8$ mol/cm^3 and temperature $T = 7 \times 10^9$ K.

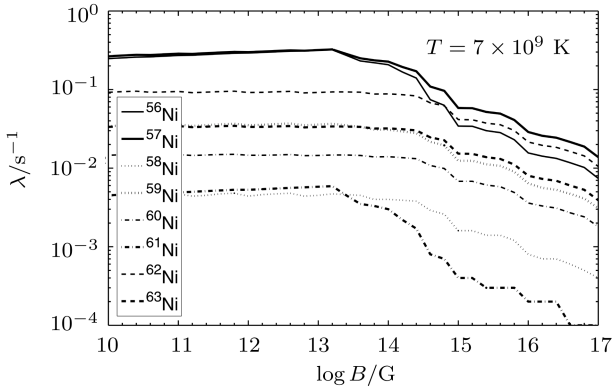


Fig.3. The EC rates as a function of magnetic field at a lepton number density $\rho/\mu_e = 3.16 \times 10^8$ mol/cm^3 and temperature $T = 7 \times 10^9$ K.

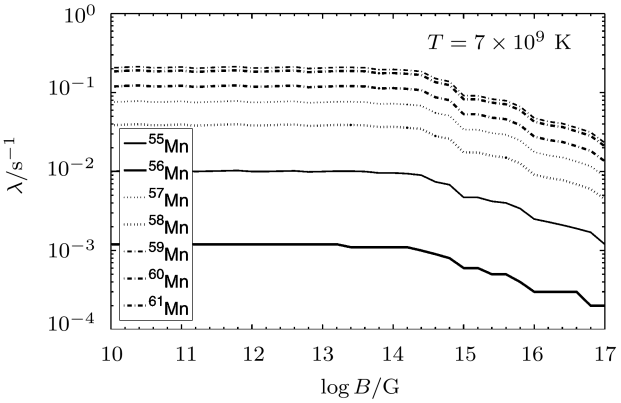


Fig.4. The EC rates as a function of magnetic field at a lepton number density $\rho/\mu_e = 3.16 \times 10^8$ mol/cm^3 and temperature $T = 7 \times 10^9$ K.

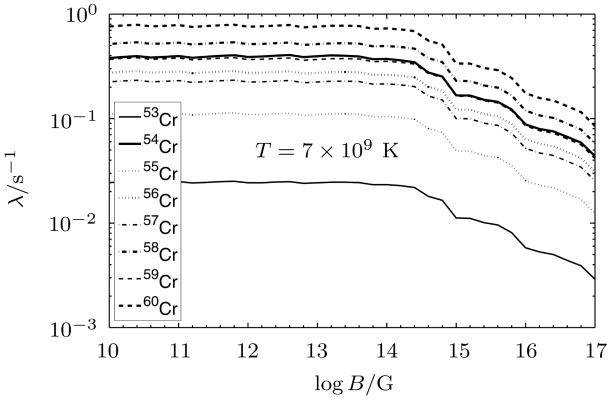


Fig.5. The EC rates as a function of magnetic field at a lepton number density $\rho/\mu_e = 3.16 \times 10^8$ mol/cm^3 and temperature $T = 7 \times 10^9$ K.

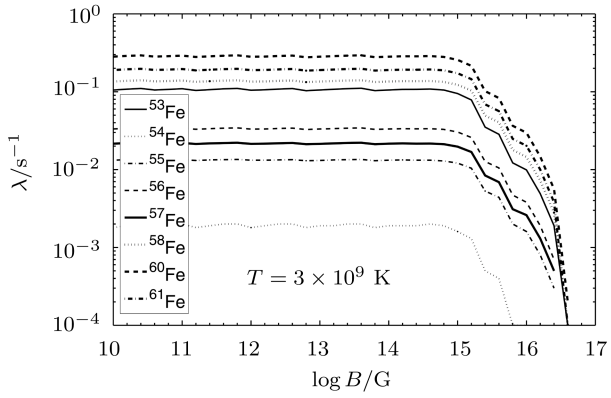


Fig.6. The EC rates as a function of magnetic field at a lepton number density $\rho/\mu_e = 3.16 \times 10^8$ mol/cm^3 and temperature $T = 3 \times 10^9$ K.

Figure 6 shows the influence of SME on EC rate of nuclei $^{53-58,60,61}$Fe at a lepton number density $\rho/\mu_e = 3.16 \times 10^8$ mol/cm^3 and temperature $T = 3 \times 10^9$ K. It can be seen from this figure that when $B < 10^{15}$ G,

the magnetic field has only a slight effect on EC rate for most nuclei, while the EC rates of most nuclei are influenced greatly when $B > 10^{15}$ G. And the EC rates of some nuclei (such as ^{53}Fe, ^{54}Fe, ^{60}Fe and ^{61}Fe) in a SMF can be reduced by even more than three orders of magnitude.

The EC rates of nuclei $^{55-60}$Co, $^{56-63}$Ni as a function of magnetic field are shown in Figs.7 and 8 under the same condition as that in Fig.6. From the two figures it can be seen that when $B < 10^{14}$ G the magnetic field has only a mirror effect on EC rate for most nuclei. The EC rates of most nuclei can be influenced greatly when $B > 10^{14}$ G. Figure 7 shows that the EC rates of ^{55}Co in a SMF can be reduced by almost three orders of magnitude. From Fig.8, however, for ^{56}Ni and ^{61}Ni, the EC rates are affected distinctly by SMF when $B > 10^{13}$ G. For some nuclei (such as ^{56}Ni, ^{57}Ni and ^{62}Ni) the EC rates are reduced by even three orders of magnitude in a range of $B > 10^{14}$ G. The influence on EC rates for nuclei $^{55-61}$Mn and $^{53-60}$Cr can be seen from Figs.9 and 10. The two figures show that the magnetic field has only a mirror effect on EC rate for most of the nuclei when $B < 10^{15}$ G, but the influence is great when $B > 10^{15}$ G. For some nuclei (such as ^{59}Mn, ^{60}Mn, ^{61}Mn and ^{62}Mn) in Fig.9, the EC rates in SMF may be reduced quickly by more than three orders of magnitude. However Fig.10 shows that for ^{58}Cr and ^{60}Cr, the EC rates decrease greatly and can be reduced by even four orders of magnitude due to SMF.

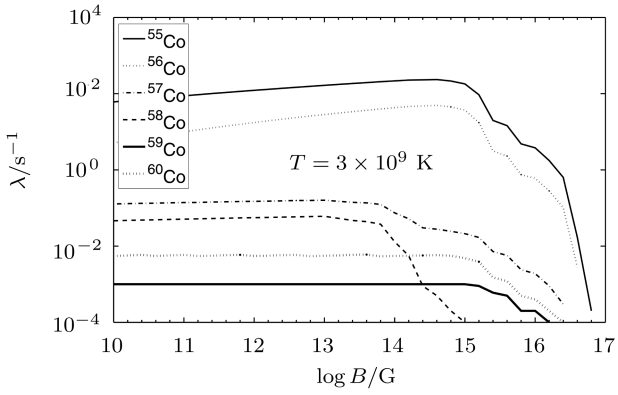


Fig.7. The EC rates as a function of magnetic field at a lepton number density $\rho/\mu_e = 3.16 \times 10^8$ mol/cm^3 and temperature $T = 3 \times 10^9$ K.

From Figs.1–10, we can see that at the same lepton numbers density, the SMF has different effects on EC rate of the iron group nuclei at different temperatures. The lower the temperature is, the larger the influence on EC rate will be when $B > 10^{14}$ G, because the electron energy is so low at lower temperatures that the SMF can strongly affect the EC rate. Due to the electron gas quantum domino effect, the curves have some gurgitations in a degressive process which can be seen from all the figures.

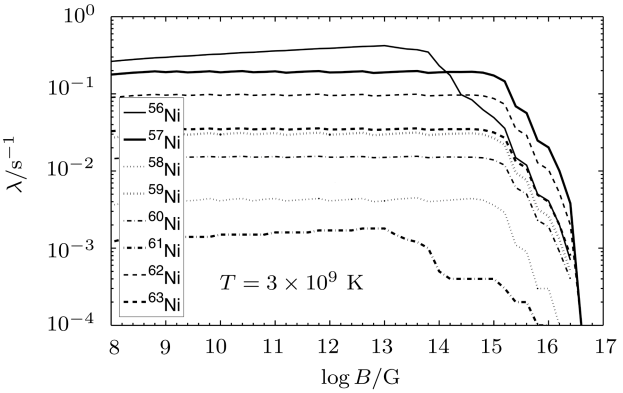


Fig.8. The EC rates as a function of magnetic field at a lepton number density $\rho/\mu_e = 3.16 \times 10^8$ mol/cm^3 and temperature $T = 3 \times 10^9$ K.

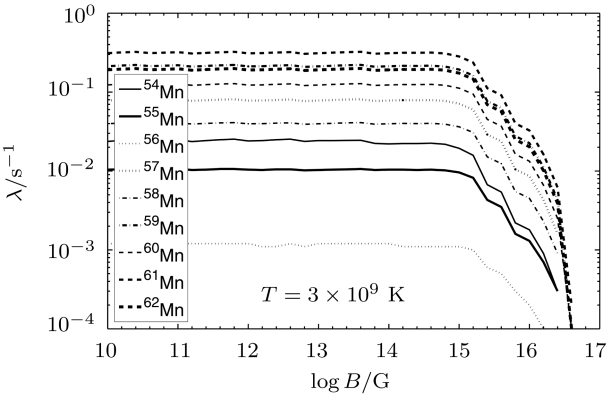


Fig.9. The EC rates as a function of magnetic field at a lepton number density $\rho/\mu_e = 3.16 \times 10^8$ mol/cm^3 and temperature $T = 3 \times 10^9$ K.

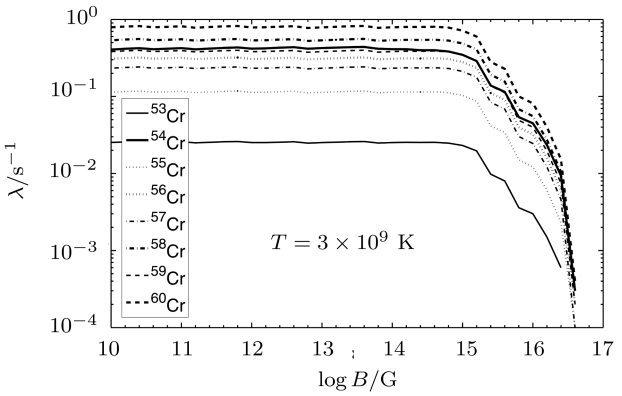


Fig.10. The EC rates as a function of magnetic field at a lepton number density $\rho/\mu_e = 3.16 \times 10^8$ mol/cm^3 and temperature $T = 3 \times 10^9$ K.

4. Conclusion

The EC rates of iron group nuclei in a SMF are investigated. We can draw a conclusion that the magnetic field has only a slight effect on EC rate for iron group nuclei in a range of $10^8 - 10^{13}$ G on surfaces of most neutron stars, whereas for some magnetars, the magnetic fields are in a range of $10^{13} - 10^{18}$ G, the EC rates of most iron group nuclei are diminished greatly and may be reduced by even four orders of magnitude due to SMF. The above conclusion maybe has a significant influence on the further research of nuclear astrophysics, especially the late evolution of neutron stars and r-process nucleosynthesis in the neutron star systems.

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