Vol 16 No 9, September 2007
1009-1963/2007/16(09)/2656-05

Chinese Physics

© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

Tunnelling cooling by a normal–vacuum–superconductor junction

Zhang Xin(张 鑫)[a)b)†] and Zhang Dian-Lin(张殿琳)[b)]

[a)] *Department of Physics, Northwest University, Xi'an* 710069, *China*

[b)] *Institute of Physics, Chinese Academy of Sciences, Beijing* 100080, *China*

(Received 7 March 2007; revised manuscript received 12 March 2007)

The possibility of cooling a system from liquid helium temperature, 4.2 K, using a tunnel junction refrigerator is analysed. Calculations show that the device can be used over a wide temperature range from 4 K down to well below 1 mK with necessary cooling power. However, several serious difficulties must be overcome before the method can be used in low temperature laboratories.

Keywords: normal–vacuum–superconductor junction, tunnelling refrigeration, low temperature
PACC: 0720M

1. Introduction

Thanks to the great progress in cryogenics, a variety of commercial products are currently available, allowing different samples to be cooled to any temperature from room temperature down to milliKelvin (mK) range. However, while a simple cryocooler can cover the temperature range from 300 K to around 4 K, elaborate setups are usually needed to reach the ultralow temperatures below 1 K. To be able to reach this temperature range with a simple setup has been an attractive goal of the cryogenics community for a long time.

In 1961 Parmenter noticed that electron tunnelling between two superconductors may cool one of the superconductors and warm the other.[1] The possibility of using this phenomenon for refrigeration was analysed by Melton *et al* in detail.[2] Nahum *et al*[3] first realized refrigeration by a normal metal–insulator–superconductor (NIS) tunnel junction. The effect was then improved with a double junction (SINIS) structure.[4] However, these works aimed at microrefrigerator due to the small tunnelling area and the thermal backflow through the insulating layer of the device, both of which seriously limited the cooling power available.

Mahan and Wood[5] discussed the possibility of thermionic refrigeration. Several device designs based on this idea have been put forward.[6−8] The difficulties inherent to thermionic refrigeration (high bias for overcoming the energy barrier, unacceptably large heat load on anode etc) make the devices unrealistic for cryogenics. Hishinuma *et al*[9] discussed in detail the refrigeration efficiency by combined tunnelling and thermionic emission in vacuum. The result is encouraging. For work function of ∼0.8 eV, they showed that temperatures down to 150 K can be reached. Since the density of states in both cathode and anode is continuous across the Fermi level for normal metals and the tunnelling currents by states below the Fermi level is detrimental for refrigeration, this normal metal–vacuum–normal metal (NVN) structure cannot be expected to reach very low temperatures.

In the present work, we investigate the possibility of using a normal metal–vacuum–superconductor (NVS) structure for refrigeration. We show that, starting from the temperature of 4.2 K and taking reasonable parameters, the cold end of a single stage refrigerator can reach as low as ∼60 mK. Even temperatures well below 1 mK are possible with a two-stage cascade refrigerator.

2. Model description

Our device model is similar to that described by Hishinuma *et al.*[9] The only change is the right-hand-side electrode, the normal metal of which is replaced by a superconductor (Fig.1). The benefit of this change is obvious: By biasing the Fermi level of the left metal to coincide with the upper edge of

†Corresponding author. E-mail: xzhang@aphy.iphy.ac.cn

the superconductor energy gap, the tunnelling current has only contribution from the states above the Fermi level, therefore yielding the most efficient cooling effect. Since the energy gap of superconductors is usually in the order of meV, such a low bias level will greatly lighten the heat load of the hot end of the refrigerator. For such a low bias, the thermionic current can be neglected. Therefore, in our device we need only discuss the cooling effect by electron tunnelling.

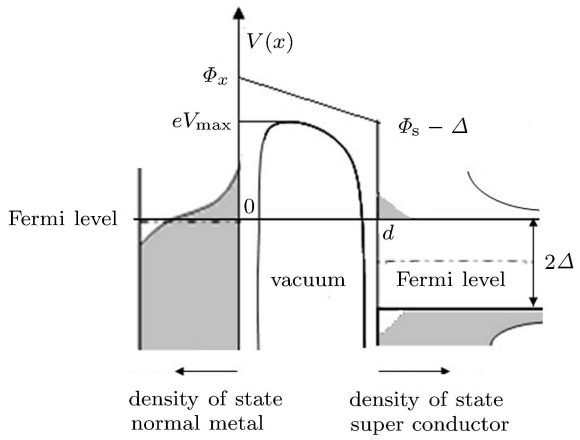


Fig.1. Sketch of the potential barrier between electrodes.

The physical picture of the device is simple. We may divide the electrons into 'hot' and 'cold' ones. All electrons above the Fermi level are 'hot' because extracting these electrons from the normal metal decreases its average thermal energy. In the configuration shown in Fig.1, it is easy to see that only the 'hot' electrons can tunnel through the vacuum barrier; and the tunnelling electrons from the normal metal to the superconductor are substantially more than that for the opposite direction at the starting state when both sides are in thermal equilibrium, i.e. the temperature of the left normal metal $T_{\rm n}$ equals the temperature of the right superconductor $T_{\rm s}$. As the tunnelling proceeds, the normal metal loses more and more 'hot' electrons, and hence becomes cooler and cooler until a final temperature is reached. At this temperature, because of the decrease of both the 'hot' electron number and the average thermal energy $k_{\rm B}T_{\rm n}$ in the normal metal, the heat flows in both directions are balanced.

In the following we will give a brief estimation of the cooling power P as a function of superconductor's energy gap 2Δ, barrier's width d, work functions of the materials, $\Phi_{\rm n}$ and $\Phi_{\rm s}$, and temperatures $T_{\rm n}$, $T_{\rm s}$. We will calculate the heat flow from normal metal to superconductor, $P_{\rm n\to s}$, and for the counterflow, $P_{\rm s\to n}$, separately. The net cooling power is

$$P = P_{\rm n\to s} - P_{\rm s\to n}\,. \tag{1}$$

First we calculate the number of tunnelling electrons in each direction. Assuming a planar junction whose junction area is perpendicular to the x-axis, with $N_{\rm n}(E_x)\mathrm{d}E_x$ being the number of electrons incident in unit time on unit area of the junction from the normal metal side, and whose kinetic energy along x direction is in the range between E_x and $E_x+\mathrm{d}E_x$, then using a free-electron approximation for the normal metal, we obtain

$$\begin{aligned} N_{\rm n}(E_x)\mathrm{d}E_x &= \frac{2}{(2\pi\hbar)^3}\sqrt{\frac{2E_x}{m}}\iint\limits_{P_yP_z} f(E,T_{\rm n})\mathrm{d}P_y\mathrm{d}P_z\mathrm{d}P_x \\ &= \frac{2}{(2\pi\hbar)^3}\iint\limits_{P_yP_z} f(E,T_{\rm n})\mathrm{d}P_y\mathrm{d}P_z\mathrm{d}E_x, \end{aligned} \tag{2}$$

where $2/(2\pi\hbar)^3$ is the free electron density of states (DOS) in momentum space $(P_xP_yP_z)$, and $f(E,T_{\rm n})$ the Fermi distribution function. Similarly, the electron incident rate from the superconductor side is

$$\begin{aligned} N_{\rm s}(E_x)\mathrm{d}E_x = \frac{2}{(2\pi\hbar)^3}\iint\limits_{P_yP_z} & n_{\rm s}(E+\Delta) \\ & \times f(E+\Delta,T_{\rm s})\mathrm{d}P_y\mathrm{d}P_z\mathrm{d}E_x, \end{aligned} \tag{3}$$

where $n_{\rm s}(E+\Delta)$ is the normalized DOS of the superconductor. In the BCS approximation

$$n_{\rm s}(E+\Delta) = \begin{cases} 0 & E<0, \\ \dfrac{|E+\Delta|}{\sqrt{(E+\Delta)^2-\Delta^2}} & E>0. \end{cases} \tag{4}$$

The number of transmitted electrons from the normal metal to the superconductor in the energy range of E_x and $E_x+\mathrm{d}E_x$ is

$$\begin{aligned} & n_{\rm n\to s}(E_x,T_{\rm n},T_{\rm s})\mathrm{d}E_x \\ & = N_{\rm n}(E_x)D_{\rm n\to s}n_{\rm s}(E+\Delta) \\ & \quad \times [1-f(E+\Delta,T_{\rm s})]\mathrm{d}E_x\,, \end{aligned} \tag{5}$$

where $D_{\rm n\to s}$ is the transmission coefficient, $n_{\rm s}(E+\Delta)[1-f(E+\Delta,T_{\rm s})]$ is a factor showing how many unoccupied states in the superconductor are needed to ensure the tunnelling electrons not being bounced back. The counter-flow electrons are

$$\begin{aligned} & n_{\rm s\to n}(E_x,T_{\rm s},T_{\rm n})\mathrm{d}E_x \\ & = N_{\rm s}(E_x)D_{\rm s\to n}[1-f(E,T_{\rm n})]\mathrm{d}E_x. \end{aligned} \tag{6}$$

Then we have

$$P_{\mathrm{s}\to\mathrm{n}} = \int_0^{eV_{\max}} E\, n_{\mathrm{n}\to\mathrm{s}}(E_x, T_{\mathrm{n}}, T_{\mathrm{s}})\mathrm{d}E_x \qquad (7)$$

and

$$P_{\mathrm{s}\to\mathrm{n}} = \int_0^{eV_{\max}} E\, n_{\mathrm{s}\to\mathrm{n}}(E_x, T_{\mathrm{s}}, T_{\mathrm{n}})\mathrm{d}E_x, \qquad (8)$$

where E is the thermal energy of the tunnelling electron, which can be approximated by $E = kT + E_x$, and $eV_{\max}$ is the maximum number of barriers. The net energy flow becomes

$$P = \int_0^{eV_{\max}} [(k_{\mathrm{B}}T_{\mathrm{n}} + E_x)n_{\mathrm{n}\to\mathrm{s}}(E_x, T_{\mathrm{n}}, T_{\mathrm{s}}) - (k_{\mathrm{B}}T_{\mathrm{s}} + E_x)n_{\mathrm{s}\to\mathrm{n}}(E_x, T_{\mathrm{s}}, T_{\mathrm{n}})]\mathrm{d}E_x\,. \qquad (9)$$

The transmission coefficients in WKB approximation read[10]

$$D_{\mathrm{n}\to\mathrm{s}} = \exp\Big\{-\frac{2}{\hbar}\int_0^d \sqrt{2m[eV(x) - E]}\mathrm{d}x\Big\}, \qquad (10)$$

$$D_{\mathrm{s}\to\mathrm{n}} = \exp\Big\{-\frac{2}{\hbar}\int_d^0 \sqrt{2m[eV(d-x) - E]} \times \mathrm{d}(d-x)\Big\}. \qquad (11)$$

The potential barrier $V(x)$ in the vacuum gap takes the form[11]

$$eV(x) = \Phi_{\mathrm{n}} + (\Phi_{\mathrm{s}} - \Phi_{\mathrm{n}} - \Delta)\frac{x}{\mathrm{d}} - \frac{e^2}{4\pi\varepsilon_0}\Big[\frac{1}{4x} + \frac{1}{2}\sum_{n=1}^{\infty}\Big(\frac{nd}{n^2\mathrm{d}^2 - x^2} - \frac{1}{nd}\Big)\Big], \qquad (12)$$

where Φ_{n} is the work function of the normal metal, Φ_{s} is the work function of the superconductor, x denotes the position of the tunnelling electron with respect to the left electrode, d is the width of the vacuum gap, and ε_0 is the vacuum permittivity. The third term is the image charge corrections near the two electrodes (Fig.1).[12]

Combining formulas (2)–(12), we can calculate the net cooling power of the NVS tunnelling junction when other heating effects and heat back flow are ignored.

3. Result and discussion

Before proceeding with the detailed calculation, we may make a rough estimation of how far we can go with the NVS tunnelling refrigeration. Assuming $\Phi_{\mathrm{n}} = \Phi_{\mathrm{s}} = \Phi$ and neglecting the image correction, since the work function of a metal is in the range of eV, and Δ for conventional superconductors is often less than 1–2 meV, it is pretty safe in our case to replace Eq.(12) by a square barrier. Then

$$D = D_{\mathrm{n}\to\mathrm{s}} = D_{\mathrm{s}\to\mathrm{n}} \cong \exp\Big[-2d\sqrt{\frac{2m}{\hbar^2}(\Phi - E)}\,\Big] \cong \exp\Big[-2d\sqrt{\frac{2m}{\hbar^2}\,\Phi}\,\Big]. \qquad (13)$$

Arbitrarily, taking $d = 2$ nm, $\Phi = 1$ eV, we have $D \approx 1.3 \times 10^{-9}$. The integral in expression (9) can be simplified in the following way. Suppose at the beginning of refrigeration when the entire junction is in equilibrium at temperature 4 K, all the states in the normal metal within an energy interval of $k_{\mathrm{B}}T_{\mathrm{n}}$ above the Fermi level are occupied and all the states above the energy gap of the superconductor are empty. Each tunnelling electron carries the same thermal energy $k_{\mathrm{B}}T_{\mathrm{n}}$. Then no electron counter flow exists and Eq.(9) gives the heat transfer is about $P \approx 2 \times 10^6 D \approx 2$ mW/cm^2. The cooling power increases to about 4 W/cm^2 if the work function can be reduced to, say, 0.4 eV.

Since the average kinetic energy carried by an electron is about $k_{\mathrm{B}}T_{\mathrm{n}}$, and the input energy is about Δ, the cooling efficiency is in the order of $k_{\mathrm{B}}T_{\mathrm{n}}/\Delta$. Taking $T_{\mathrm{n}} = T_{\mathrm{s}} = 4$ K, and $\Delta = 2$ meV, we have the efficiency $\eta \approx 17\%$.

Now let us look at the influences of different parameters on the cooling effect of the NVS tunnelling device. Figure 2 shows how the cooling power and efficiency change with the ratio $\Delta/k_{\mathrm{B}}T_{\mathrm{s}}$. We see that with the increase of the ratio, the cooling power P increases, while the efficiency η decreases. The results are what expected, because, as we mentioned above, the tunnelling electrons increase with bias $eV_{\mathrm{bias}} = \Delta$, but η scales with $k_{\mathrm{B}}T_{\mathrm{n}}/\Delta$, and is weakly dependent on the number of tunnelling electrons.

Figure 3 shows how the cooling power changes in the process of refrigeration for different gaps. P decreases with $T_{\mathrm{n}}/T_{\mathrm{s}}$, as expected. The apparent decrease of P with Δ is caused by the fact that we always assume $\Delta = eV_{\mathrm{bias}}$, and the decrease of bias reduces the tunnelling current. However, for $\Delta = 0.1$ meV, the superconducting energy gap is already substantially lower than $k_{\mathrm{B}}T_{\mathrm{s}}$. The term $P_{\mathrm{s}\to\mathrm{n}}$ can no longer be neglected in this case, and the tunnelling of 'cold'

electrons may contribute a backflow of heat, and hence decreases the cooling effect.

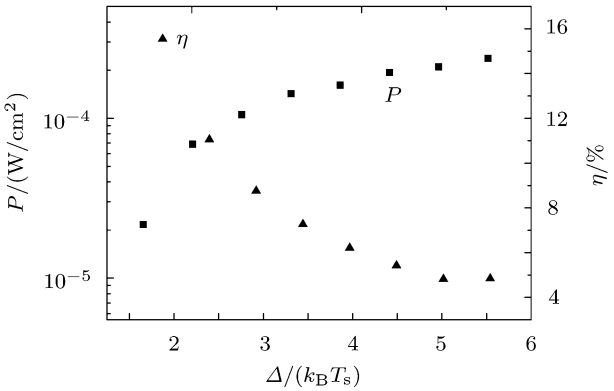


Fig.2. Cooling power P and cooling efficiency η vs $\Delta/(k_\mathrm{B}T_\mathrm{s})$ with $T_\mathrm{n} = 1.0$ K, $T_\mathrm{s} = 4.2$ K, $eV_\mathrm{bias} = \Delta$, $d = 4$ nm, $\Phi_\mathrm{n} = \Phi_\mathrm{s} = \Phi = 0.8$ eV.

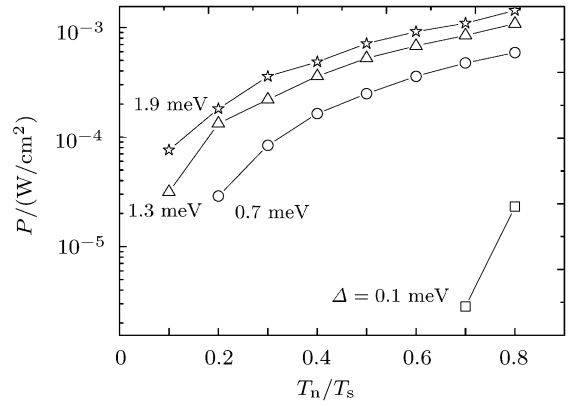


Fig.3. Cooling power P vs $T_\mathrm{n}/T_\mathrm{s}$ for $T_\mathrm{s} = 4.2$ K, $eV_\mathrm{bias} = \Delta$, $d = 4$ nm, $\Phi_\mathrm{n} = \Phi_\mathrm{s} = \Phi = 0.8$ eV.

The exponential decrease of tunnelling current with barrier width is well known. Figure 4 gives an example of how the cooling power changes with d for two ratios of $T_\mathrm{n}/T_\mathrm{s}$.

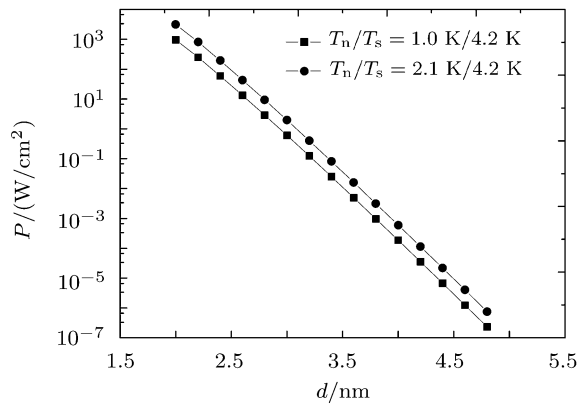


Fig.4. Cooling power vs the width of the vacuum barrier for $T_\mathrm{n}/T_\mathrm{s} = 1/2$, $5/21$, $eV_\mathrm{bias} = \Delta = 1.55$ meV, $\Phi = 0.8$ eV.

The work function Φ is equally important. Figure 5 shows the cooling power as a function of d for different work functions. We see that for practical purpose, finding materials with very low work function is of great importance.

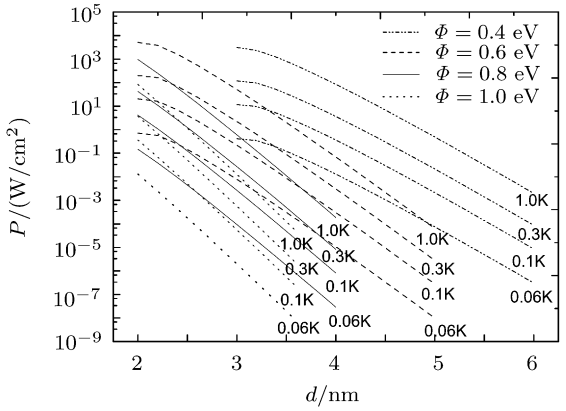


Fig.5. Cooling power P vs d for $\Phi = 0.4$, 0.6, 0.8, 1.0 eV and $T_\mathrm{s} = 4.2$ K, $eV_\mathrm{bias} = \Delta = 1.55$ meV at different temperatures of cold end of the junction.

From the above calculations we see that with a single stage of NVS device it is possible to cool a system down to milliKelvin. It can be shown that even the temperature well below 1 mK can be reached. However, several technical difficulties should be overcome before a useful device is realized.

1) Large area homogeneous vacuum gap with the gap width of only a few nm can be obtained.

2) Materials with very low work function can be found, which may ease the difficulty mentioned in 1). It would be interesting to note a recently work in which a method is suggested to arbitrarily adjust the work function of a metal.[13]

3) An intrinsic difficulty in the NVS device is the low bias across the junction. This means that if we expect a useful cooling power, say 200 mW at 4.2 K, we need an input current as large as 500 A! One way to solve the problem is by using a large amount of devices connected in series. In such case how to ensure the bias in each device equal to its superconducting gap is again a challenging problem. However, since the currents in all the junctions should be equal, we expect that this confinement will automatically adjust the bias across each junction not far from its Δ.

4. Conclusion

We have analysed the possibility of using a normal metal–vacuum–superconductor junction for electronic refrigeration. Calculations show that the device can

be used to reach a wide temperature range from 4 K down to well below 1 mK. An additional advantage of the method over the traditional methods is that the electron subsystem is directly cooled in the method, which is very important when the electronic properties are interested. However, several serious difficulties must be overcome before the method can be used in low temperature laboratories.

Acknowledgments

Zhang Xin is grateful to Wang Yun-Ping, Su Shao-Kui, Zuo Jun-Wei, Song Xiao-Hui, Li Yan-Rong and Jin Yi-Rong for valuable discussion and important comments.

References

[1] Parmenter R H 1961 *Phys. Rev. Lett.* **7** 274
[2] Melton R G, Paterson J L and Kaplan S B 1980 *Phys. Rev.* B **21** 1858
[3] Nahum M, Eiles T M and Martinis J M 1994 *Appl. Phys. Lett.* **65** 3123
[4] Leivo M M and Pekola J P 1996 *Appl. Phys. Lett.* **68** 1996
[5] Mahan G D 1994 *J. Appl. Phys.* **76** 4362
Mahan G D and Wood L M 1998 *Phys. Rev. Lett.* **80** 4016
[6] Shakouri A and Bowers J E 1997 *Appl. Phys. Lett.* **71** 1234
Shakouri A and Bowers J E 1999 *Appl. Phys. Lett.* **74** 88
[7] Korotkov A N and Likharev K K 1999 *Appl. Phys. Lett.* **75** 2491
[8] Chao K A, Larsson M and Malshukov A G 2005 *Appl. Phys. Lett.* **87** 022103
[9] Hishinuma Y, Geballe T H, Moyzhes B Y and Kenny T W 2001 *Appl. Phys. Lett.* **78** 2572
Hishinuma Y, Geballe T H, Moyzhes B Y and Kenny T W 2002 *Appl. Phys. Lett.* **81** 4242
Hishinuma Y, Geballe T H, Moyzhes B Y and Kenny T W 2003 *J. Appl. Phys.* **94** 4690
[10] Magera G G and Wallden L 1980 *Phys. Rev.* B **22** 5967
[11] Sommer A H 1980 *Photoemissive Materials* (New York: Krieger)
[12] Fomenko V S 1966 *Handbook of Thermionic Properties* (New York: Plenum)
[13] Avto Tavkhelidze, a_Amiran Bibilashvili, Larissa Jangidz, Alex Shimkunas, Philip Mauge Gertrude F. Rempfer, Luis Almaraz, Todd Dixon, Martin E. Kordesch, Nechama Katan and Hans Walitzki 2006 *J. Vac. Sci. Technol.* B **24** 1413