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Controlling spiral wave with target wave in oscillatory systems*

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Spiral waves have been controlled by generating target waves with a localized inhomogeneity in the oscillatory medium. The competition between the spiral waves and target waves is discussed. The effect of the localized inhomogeneity size has also been studied.

Keywords: target wave, spiral wave, localized inhomogeneity, oscillatory system
PACC: 0547, 0565, 8220K

1. Introduction

Spiral wave patterns are ubiquitous in many systems such as the Rayleigh–Benard convection,[1] the Belousov–Zhabotinsky (BZ) reaction,[2] the aggregating slime-mould cells,[3] nonlinear optics,[4] gas discharge,[5,6] and the cardiac muscles.[7] A spiral wave is organized by a topological defect and self-sustained. It is found that spiral wave in cardiac muscles plays an essential role in heart diseases such as arrhythmia and fibrillation, the latter being the leading cause of death in the industrialized world. Therefore, effective control methods are needed to control and eliminate the spiral waves. Up to date, several methods have been proposed to suppress spiral waves.[8−15] An effective method for eliminating spiral wave is to apply external periodic force resonant with the spiral wave frequency.[8] A gradient force produced by external field also can make the spiral core drift.[9] Wang *et al* suggested that weak spatial perturbation can be used to suppress the spiral waves.[10,11] Yuan *et al* eliminated the spiral waves by using travelling wave impulses.[12] Mode-modulation method with the intrinsic system mode has also been proposed to killing the spiral waves.[13] Recently, Zhang *et al*[15] have succeeded in suppressing the spiral waves and spatiotemporal chaos by generating target waves in the excitable media. In Ref.[16], target waves generated by localized inhomogeneity are applied to suppress the spatiotemporal chaos in the oscillatory medium. In this article, we propose another method to control the spiral wave by generating target waves in the oscillatory medium. The competition between the spiral waves and target waves is also studied.

2. Model analysis

As a theoretical model to study pattern formation in oscillatory systems,[17] we choose the complex Ginzburg–Landau equation (CGLE):

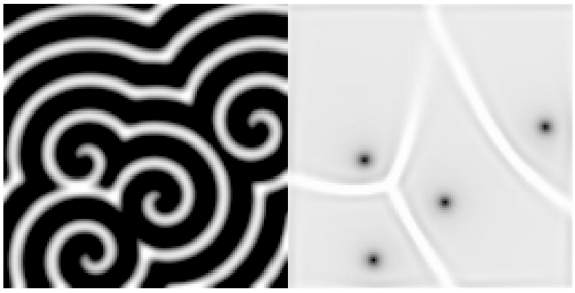
$$\partial_t A = (1+\mathrm{i}\delta)A - (1+\mathrm{i}\alpha)|A|^2 A + (1+\mathrm{i}\beta)\nabla^2 A, \quad (1)$$

here $A(r,t)$ is the complex amplitude, α, β and δ are real numbers. This equation provides a universal description for extended media in which the homogeneous state is oscillatory and near a Hopf bifurcation. δ denotes the linear frequency shift, and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ denotes the two-dimensional Laplacian operator. The numerical simulations are performed for system size 128×128 with fixed $\alpha = 0.34$ and $\beta = -1.45$. No-flux boundary conditions are imposed.

It is well known that both spiral wave solutions and inwardly rotating spiral wave (anti-spiral wave) solutions can be observed separately in an appreciable region of (α,β) parameter space.[18] In the following discussion, all the spiral waves are inwardly rotating waves. A random initial condition rapidly leads to the formation of spiral wave domains distributed through-

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out the system, as shown in Fig.1. In this state, spiral waves propagate inwardly, and their domains are separated by narrow walls with each domain having a vortex in its interior, easily seen in the plot of $|A|$ in Fig.1(b).



(a) (b)

Fig.1. (a) spiral pattern in the homogeneous CGLE and (b) the corresponding amplitude distribution to (a). $t = 1000$ t.u.

3. Simulation results

In order to suppress the spiral waves, target pattern has been utilized. Since target wave pattern is unstable in the homogeneous CGLE, a change of δ within a small region, which has a square profile with half-width R, is designed. We set

$$\delta(x,y) = \begin{cases} 0 & |x| > R, \quad |y| > R \\ \delta & |x| \le R, \quad |y| \le R \end{cases}. \tag{2}$$

Figure 2 gives the target wave pattern formation from random initial condition with $\delta = 0.28$ and $R = 5$. Concentric waves are automatically created around the localized inhomogeneity at $t = 400$ t.u., see Fig.2(b). Although the concentric waves propagate inwardly in this case, the target wave pattern is the dominant pattern, whose domain walls move outward, increasing the domain size at the expense of the adjacent competing domains. i.e. the target wave pattern gradually invades into the region where the spiral wave pattern dominates, as seen in Fig.2(c). Finally, the whole system is dominated by one large target wave pattern (Fig.2(d)). It can be seen that the target wave has constant $|A|$ in time. Similar results have been observed by Hendrey and co-workers in Ref.[19], where they called it stationary target pattern. In contrast to spiral wave, the value of $|A|$ is not zero in the target wave centre. Far from the centre of the pattern, the curvature effect is small and a perfectly target wave solution can be considered as a plane wave solution, which is described by

$$A(r,t) = \sqrt{1-k^2}\exp(\mathrm{i}kr - \mathrm{i}\omega_{\mathrm{T}}t). \tag{3}$$

Substituting this solution into Eq.(1) yields the dispersion relation

$$\omega_{\mathrm{T}} = \alpha + (\beta - \alpha)k^2. \tag{4}$$



$t = 200$ $t = 400$ $t = 600$ $t = 800$
(a) (b) (c) (d)

Fig.2. The process of competition between the spirals and the target waves. $\delta = 0.28$.

This dispersion relationship is approximately valid to the target wave solutions in our system. The corresponding numerical simulation results are given in Fig.3.

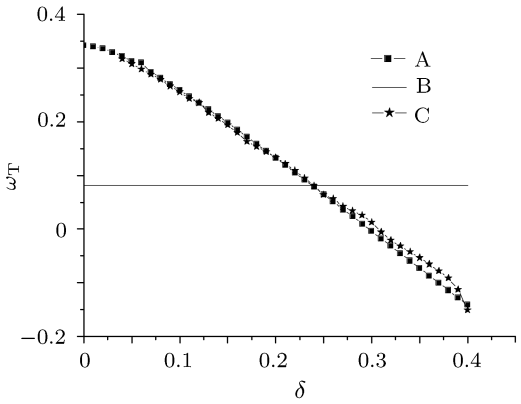


Fig.3. Target wave frequency ω_{T} as a function of the parameter δ. The spiral waves' frequency is also plotted where $\omega_{\mathrm{s}} = 0.0814$. A denotes the simulation result of the target wave frequency ω_{T}; B stands for the spiral waves frequency ($\omega_{\mathrm{s}} = 0.0814$); C denotes the target wave frequency ω_{T} obtained from the dispersion relationship of Eq.(4).

How can target waves suppress the spiral waves and become the dominant pattern? In order to understand the competition between the spiral and the target wave patterns in our system, we numerically determine the frequency of target wave as the parameter δ is varied. Since the target wave solution has no

θ dependence, the Laplacian operator can be given by $\nabla^2 = \partial^2/\partial r^2 + (1/r)(\partial/\partial r)$, and then the 2D CGLE can be reduced to the 1D radial CGLE.[19] Figure 3 shows the frequency of the target wave as a function of the parameter δ. It is shown that the frequency of the target wave is apparently linearly related to δ. For $\delta < 0.24$ and $\delta > 0.35$, the frequency of target wave is larger than that of spiral wave, and the spiral wave pattern dominates. An example is shown in Fig.4. For $0.24 < \delta < 0.35$, the target frequency changes its sign, and the propagating direction changes from inward to outward. However, as long as the absolute value of the target frequency is lower than that of spiral wave, the target wave pattern dominates. The process in Fig.2 corresponds to this case. For $1+\alpha\beta > 0$, all the target wave are stable as long as they satisfy the relation

$$|k| < |k_c| = \sqrt{\frac{1+\alpha\beta}{3+\alpha\beta+2\alpha^2}}.$$

For $\alpha = 0.34$ and $\beta = -1.45$, the critical wave number is $k_c = 0.43$.

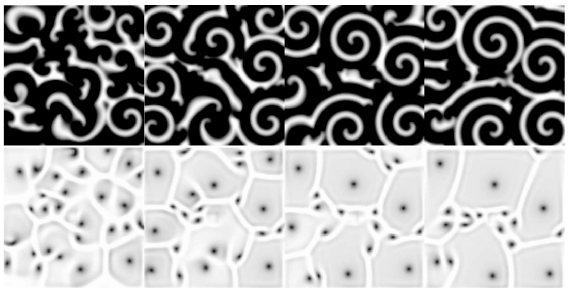


Fig.4. Time evolution of dominant spiral waves. $\delta = 0.20$.

A simple theoretical explanation of target pattern in oscillatory systems is that their wave sources are founded by localized inhomogeneity which modifies the properties of the medium such that the oscillation frequency is locally increased.[3,16] It has been also proved in the light-sensitive BZ reaction.[20] The size of the localized inhomogeneity should be large enough to show the pacemaker. It is found that the target wave pattern can be generated only when $R \geq 5$. In order to explore this issue in more detail, we display in Fig.5 the frequency of the target wave pattern as a function of the half-width R with different frequency shift δ. In the case of $\delta = 0.26$, there is a minimum value of the frequency when R equals 5. For $R < 5$, the frequency is sharply increased as R decreases, and the system exhibits spiral waves without any target pattern. For $R > 5$, the frequency is gradually increased as R increases. In addition, the wave number of the target wave pattern keeps constant in this case. For $R \to \infty$, the system becomes homogeneous, and its frequency gradually becomes

$$\omega_T = -\delta + \alpha. \tag{5}$$

Therefore, the frequency is gradually decreased to -0.02 in the case of $\delta = 0.36$ while it is gradually increased to 0.08 in the case of $\delta = 0.26$ as R increases, as shown in Fig.5. To ensure this result, the simulation in Fig.6 gives the typical examples for a spiral wave dominated pattern and a target wave dominated pattern with different sizes of the localized inhomogeneity with fixed parameter $\delta = 0.36$. In Fig.6(a) $R = 5$, the frequency of the target wave ($\omega_T = 0.0868$) is larger than that of the spiral wave ($\omega_s = 0.0814$), and the spiral wave pattern dominates. In Fig.6(b), $R = 8$, the frequency of target wave ($\omega_T = 0.0459$) is lower than that of spiral wave, and the target wave pattern dominates.

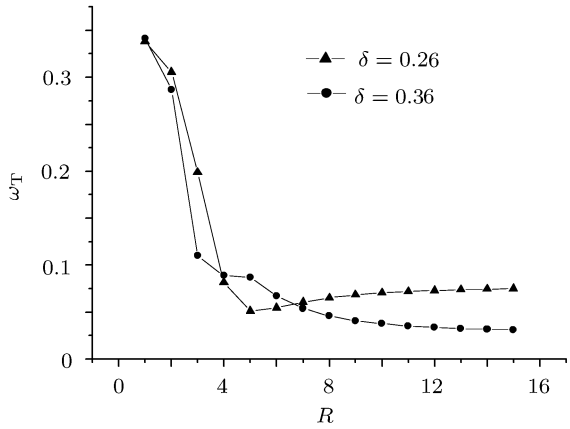


Fig.5. Target wave frequency as a function of the size R of the local inhomogeneity.

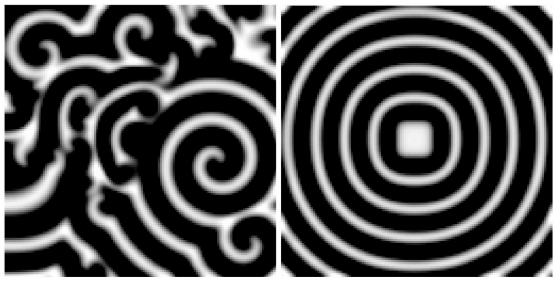


Fig.6. The spiral waves and target waves with different sizes of local inhomogeneities.

Spiral wave can no longer be made to disappear through boundaries with a periodic boundary condition.[13] The method in this paper is also available in controlling and suppressing the spiral waves with periodic boundary conditions.

4. Conclusion

In summary, we have presented an effective and operative method for controlling and suppressing the spiral waves by generating target waves in the oscillatory media. The competition between the spiral waves and target waves has also been studied. In the case of $\beta < \alpha$, the target waves suppress the spirals for $\omega_{\mathrm{T}} < \omega_{\mathrm{s}}$. The local inhomogeneity plays the role of pacemaker. It is found that the target wave pattern can be generated only when $R \geq 5$.

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