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Chaotic phenomena in Josephson circuits coupled quantum cellular neural networks*

Wang Sen(王 森), Cai Li(蔡 理), Li Qin(李 芹), and Wu Gang(吴 刚)

The Science Institute, Air Force Engineering University, Xi' an 710051, *China*

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In this paper the nonlinear dynamical behaviour of a quantum cellular neural network (QCNN) by coupling Josephson circuits was investigated and it was shown that the QCNN using only two of them can cause the onset of chaotic oscillation. The theoretical analysis and simulation for the two Josephson-circuits-coupled QCNN have been done by using the amplitude and phase as state variables. The complex chaotic behaviours can be observed and then proved by calculating Lyapunov exponents. The study provides valuable information about QCNNs for future application in high-parallel signal processing and novel chaotic generators.

Keywords: quantum cellular neural network , Josephson junction, chaos, Lyapunov exponent
PACC: 0545, 7450

1. Introduction

A wide interest has been devoted to Josephson junctions, with particular attention toward quantum computing as a massive parallel computation method which promises to be revolutionary in future developments of computer science.[1−9] The advantage of using this approach is that it is based on the nanoscale.[2]

The cellular neural network (CNN)[10] is capable of high-speed parallel signal processing and its local interconnection feature makes it suitable for very large scale integration implementation. The potential application of CNNs using quantum cellular automata[11−13] has been investigated; and based on Schrödinger equation, a new quantum CNN (QCNN) architecture where cells are composed of interacting superconducting circuits of Josephson junctions has been proposed.[14] In this architecture the quantum dynamics of the Josephson circuits is formulated as the state dynamics of CNN cells. One of their peculiarities lies in the extremely high-packing densities, and the other lies in the further degree of freedom possessed by each cell due to quantum interaction between Josephson circuits. That fact allows obtaining complex dynamics even in a network with only two cells. This paper studies the complex dynamics of QCNN by suitable variation of coupling parameters and initial conditions. Because QCNNs may provide a technological improvement suggesting nano-metric realizations for CNN, the study we will discuss could be very important for high-parallel signal processing such as image processing and pattern recognition as well as for novel chaotic generators for control, synchronization and secure communication.

2. The Josephson circuit

The Josephson circuit in principle consists of a loop with three Josephson junctions in series that encloses a magnetic flux driven by an external magnet[1] as shown in Fig.1.

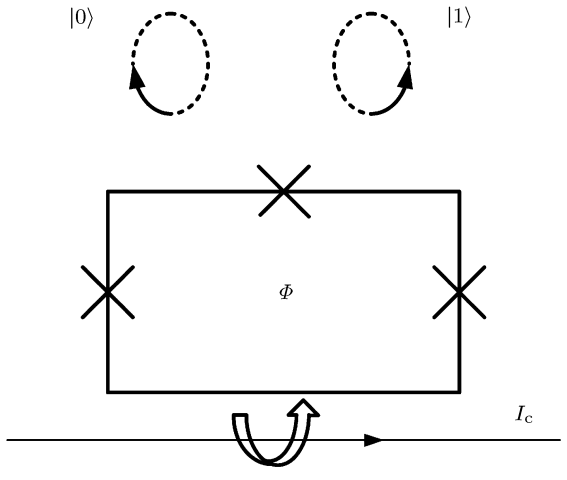


Fig.1. Josephson circuit.

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Each junction is indicated with a cross and superconductors are coupled with Josephson junctions. The ideal Josephson junction has a current-phase relation $I_i = I_0 \sin\varphi_i$ with $I_0 = (4\pi e/h)EE_{\rm J}$; I_0 is the critical current of the Josephson junction, h is Planck's constant, $E_{\rm J}$ is the Josephson energy and φ_i is the gauge-invariant phase of junction i. Two of the junctions in the loop have equal Josephson energy $E_{\rm J}$ and the third has $aE_{\rm J}$. If the applied magnetic flux is $f\Phi_0$ with $f \in [1/2 - f_{\rm c}, 1/2 + f_{\rm c}]$, where $f_{\rm c} \leq 1/4$, this circuit has two stable states $|0\rangle$ and $|1\rangle$ with opposite circulating persistent currents.

3. QCNN using Josephson circuits

The Josephson circuits can be used to implement QCNN architecture. In this architecture the quantum dynamics of the Josephson circuits is formulated as the state dynamics of CNN cells.[14]

Assuming that the coupling between cells is only via classical degrees of freedom, i.e. there are no quantum entanglements among cells, the dynamics of this Josephson network can be described by a set of Schrödinger equations for each cell as follows:

$$\mathrm{i}\hbar\frac{\mathrm{d}|\Psi_k\rangle}{\mathrm{d}t} = H_k|\Psi_k\rangle. \tag{1}$$

The coupling of the Josephson circuits was modelled as the interaction of flux generated by the superconducting currents through mutual inductance. The Hamiltonian of the cell k becomes

$$H_k = -I_{\rm p}\Big((f_0 - 1/2)\Phi_0 + \sum_{i\in r} M_{ik}I_i\Big)\sigma_z - \lambda\sigma_x, \tag{2}$$

where $I_{\rm p}$ is the amplitude of the persistent currents, f_0 the initial magnetic fluctuation, I_i the persistent current of a neighbour circuit, M_{ik} the mutual inductance and σ_z, σ_x the Pauli matrices.[2]

The quantum state of a cell, indexed by k, can be described as

$$|\Psi_k\rangle = \alpha_k|0\rangle + \beta_k|1\rangle \tag{3}$$

with

$$|\alpha_k|^2 + |\beta_k|^2 = 1. \tag{4}$$

In order to transform the quantum mechanical description of an array into a CNN-style description, we let

$$\alpha_k = A_k, \qquad \beta_k = B_k\mathrm{e}^{\mathrm{i}\varphi_k}, \tag{5}$$

where A_k, B_k are amplitude variables and φ_k a phase variable.

Taking the Eqs.(2), (3), (4) and (5) together, we find that the Schrödinger equation (1) can be written as a set of equations for the amplitude variable A_k (or B_k, essentially equivalent to A_k) and the phase variable φ_k respectively:

$$\begin{aligned}
\hbar\frac{\mathrm{d}A_k}{\mathrm{d}t} &= -\lambda\sqrt{1-A_k^2}\sin\varphi_k,\\
\hbar\frac{\mathrm{d}\varphi_k}{\mathrm{d}t} &= -I_{\rm p}\Big((f_0 - 1/2)\Phi_0 + \sum_{i\in r} M_{ik}I_i\Big)\\
&\quad + \lambda\frac{A_k}{\sqrt{1-A_k^2}}\cos\varphi_k.
\end{aligned} \tag{6}$$

Equation (6) constitutes the QCNN state equations and the dynamics is characterized by two variables A_k and φ_k.

Let us consider a two-Josephson-circuit-coupled QCNN, then the model can be described generally by the following equations:

$$\begin{aligned}
\frac{\mathrm{d}A_1}{\mathrm{d}t} &= -b_1\sqrt{1-A_1^2}\sin\varphi_1,\\
\frac{\mathrm{d}\varphi_1}{\mathrm{d}t} &= -c - w_{21}I_2 + b_1\frac{A_1}{\sqrt{1-A_1^2}}\cos\varphi_1,\\
\frac{\mathrm{d}A_2}{\mathrm{d}t} &= -b_2\sqrt{1-A_2^2}\sin\varphi_2,\\
\frac{\mathrm{d}\varphi_1}{\mathrm{d}t} &= -c - w_{12}I_1 + b_2\frac{A_2}{\sqrt{1-A_2^2}}\cos\varphi_2,
\end{aligned} \tag{7}$$

where b_1 and b_2 are proportional to the intracell tunnelling matrix element (for identical Josephson circuits $b_1 = b_2$), with w_{21} and w_{12} proportional to the mutual inductance M_{21} and M_{12}, $c = I_{\rm p}(f_0 - 1/2)\Phi_0/\hbar$.

4. Chaotic behaviour of the two-Josephson-circuit-coupled QCNN

MATLAB is employed to simulate the system, and the chaotic dynamics can be obtained in many sets of initial values when $b_1 = 0.13$, $b_2 = 0.3$, $w_{12} = w_{21} = 0.3$ and $c = 0$. Figure 2 shows the time waveforms of the variables A_2 and φ_1 and Fig.3 the state trajectories in $A_1 - \varphi_1$ and $A_2 - \varphi_2$ planes.

From Figs.2 and 3, we can see that the time waveforms present the characteristics of aperiodic oscillation and the state trajectories possess two chaotic attractors. Both of them demonstrate that this system possesses chaotic dynamical behaviour.

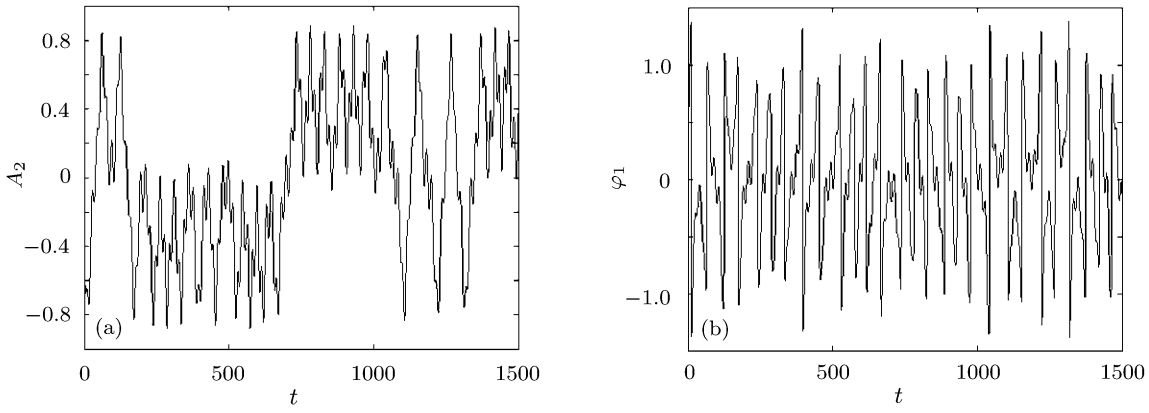


Fig.2. The time waveforms: (a) the time waveform of variable A_2; (b) the time waveform of variable φ_1.

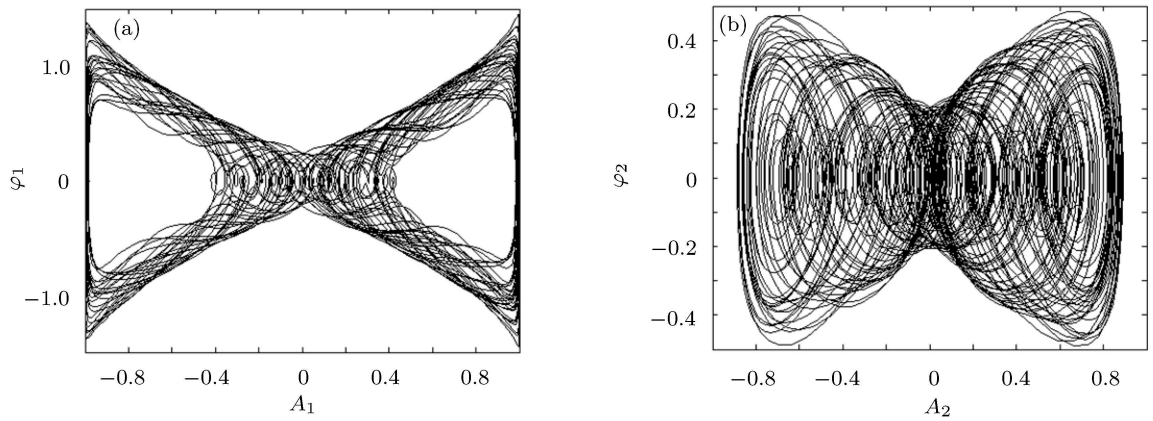


Fig.3. The state trajectories: (a) the state trajectory in the $A_1 - \varphi_1$ plane; (b) the state trajectory in the $A_2 - \varphi_2$ plane.

In order to have a quantitative measure of the chaotic behaviour, Lyapunov exponent is employed. Lyapunov exponents are defined as[15]

$$\lambda_i = \lim_{t\to\infty} \frac{1}{t} \ln |m_i(t)|, \qquad i = 1, \cdots, n, \tag{8}$$

where $m_1(t) \cdots m_n(t)$ are the eigenvalues of $\Phi_t(x_0)$, which is a state variable of the variational equation of state equation (7). The variational equation is

$$\begin{aligned} &\dot{\Phi}_t(x_0, t_0) = D_x f(\phi_t(x_0, t_0), t)\, \Phi_t(x_0, t_0), \\ &\Phi_{t0}(x_0, t_0) = I, \end{aligned} \tag{9}$$

where $D_x f(\phi_t(x_0, t_0), t)$ is the Jacobi matrix of Eq.(7).

Through repeatedly numerically integrating the variational equation (9), the four Lyapunov exponents of Eq.(7) can be acquired as follows: $\lambda_1 = 0.043205$, $\lambda_2 = 0.00054504 \approx 0$, $\lambda_3 = -0.024793$ and $\lambda_4 = -0.018964$.

The initial matrix is identity matrix, and the integral time T is 10. From the four Lyapunov exponents we can see that the system possesses chaotic behaviour because one of the four Lyapunov exponents is positive.

From the above analysis we can see that the state equation of QCNN using Josephson circuits is analogous to the state equation of CNN and the QCNN cell dynamics is characterized by two variables A_k and φ_k. Cells interact with their neighbours through the mutually inductive couplings. This QCNN architecture presents a novel computing paradigm other than quantum computing and classical computing based on binary logic. And it must be pointed out that the two stable states with opposite circulating persistent currents are necessary in quantum computing and classical computing, but it is likely that chaotic behaviour occurs in the arrays of Josephson circuits when the parameters of Josephson circuits are chosen improperly, which will lead to the Josephson circuits running into an unstable dynamical state. It will make the quantum computing and classical computing produce error. Therefore the parameters of Josephson circuits should be chosen properly to avoid causing chaotic oscillation when QCNNs are used in quantum computing and classical computing.

5. Conclusions

QCNNs have very important aspect for future evolution in the field of CNN and, at the same time, in high-parallel signal processing. For one thing, QCNNs may provide a technological improvement suggesting nano-metric realizations for CNN, for another a still developed hardware like CNN circuit could be adopted as a useful platform for high-parallel signal processing.

This paper studied the complicated dynamical behaviour of QCNN and found that the QCNN using only two Josephson circuits can generate chaotic oscillation. Chaotic behaviour has been observed in nonidentical Josephson-circuits-coupled QCNNs and at the same time Lyapunov exponents have been proposed in order to have a quantitative measure of the chaotic behaviour. This fact suggests that large scale QCNNs could easily engender chaotic oscillation when they employ nonidentical Josephson circuits. Thus on the one hand we should employ identical Josephson circuits to avoid causing chaotic oscillation when QCNNs are used in high-parallel signal processing such as image processing and pattern recognition because chaotic oscillation will lead to QCNNs running into an uncertainly dynamical state; on the other hand we can employ nonidentical Josephson circuits to realize novel ultra-small chaotic generators for control, synchronization and secure communication, and moreover these chaotic generators which employ nonidentical Josephson circuits possess a very simple structure.

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