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Adaptive tracking control for a class of uncertain chaotic systems*

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The paper is concerned with adaptive tracking problem for a class of chaotic system with time-varying uncertainty, but bounded by norm polynomial. Based on adaptive technique, it proposes a novel controller to asymptotically track the arbitrary desired bounded trajectory. Simulation on the Rossler chaotic system is performed and the result verifies the effectiveness of the proposed method.

Keywords: adaptive track, Rossler system, uncertain chaotic system
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1. Introduction

Since the control of chaotic system was studied by Ott *et al*,[1] it has been of broad interest in the past decades.[2] Recently, more and more researches have been diverted to robust control of chaotic system.[3−9] In 2003, Li *et al*[5] proposed a controller scheme to asymptotically track the arbitrary desired trajectory for a class of parameter uncertain chaotic system. However, the external disturbance was excluded from the scheme. Furthermore, according to Ref.[5], the Assumption 2 in Ref.[5] is strict, because it is very difficult to assume that a time varying nonlinear system is asymptotically stable, for example, the controller in Ref.[5] can not track a fixed point $[0 \quad 0 \quad 0]^{\mathrm{T}}$ for the Rossler chaotic system. Even if the Assumption 2 is held, it is also very difficult to find out the concrete Lyapunov function satisfying the Assumption 2. In order to overcome these deficiencies, a novel adaptive controller is proposed to asymptotically track the arbitrary desired trajectory for a class of uncertain chaotic system.

In this paper, based on the Lyapunov stability theory and adaptive technique, a novel adaptive controller is proposed for a class of chaotic systems with time-varying uncertainty which includes not only parameter uncertainty but also both external disturbance and structure disturbance. With the resulting controller, it guarantees that the system trajectory will asymptotically track arbitrary desired bounded trajectory. Compared with the controller of Ref.[5], there are two advantages: (1) it is robust against external disturbance and a class of structure disturbance as well as parameter uncertainty; (2) it simplifies not only the procedure of controller design but also extends the admissible desired trajectory because the Assumption 2 in Ref.[5] is removed.

2. Problem description

Consider the following uncertain chaotic system:

$$\dot{x} = F(x,t) + \Delta f(x,t) + u(t), \tag{1}$$

where x and $u \in R^n$ are the system state and control input respectively, $F(x,t)$ is a nonlinear function satisfying

$$\|F(x,t)\| \leq \sum_{i=0}^{r} \beta_i \|x\|^i - \|x\|, \tag{2}$$

and $\Delta f(x,t)$ is a time-varying uncertainty satisfying

$$\|\Delta f(x,t)\| \leq \sum_{i=0}^{p} d_i \|x\|^i, \tag{3}$$

where $\beta_i (i = 0, \cdots r)$ and $d_i (i = 0, 1, \cdots p)$ are known non-minus constants.

Remark 1 Lots of chaotic systems, such as, Lorenz system, Chen system, Lu system and Rossler system, do not satisfy the Lipschitz condition, but satisfy the Eq.(2). Especially, some systems, such as Duffing oscillator and Rossler system, require $\beta_0 \neq 0$.

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In Eq.(3), the uncertainty includes external disturbance ($d_0 \neq 0$) as well as both parameter disturbance and structure disturbance. Here, we write $\|F(x,t)\| \leq \sum_{i=0}^{r} \beta_i \|x\|^i - \|x\|$ but not $\|F(x,t)\| \leq \sum_{i=0}^{r} \beta_i \|x\|^i$ just for convenience of the proof and expression of Theorem 1.

Let x_d be an arbitrary desired bounded smooth trajectory. Defining $e = x - x_\mathrm{d}$ we get the error dynamical system

$$\dot{e} = F(e + x_\mathrm{d}, t) + \Delta f(e + x_\mathrm{d}, t) - \dot{x}_\mathrm{d} + u(t). \quad (4)$$

Our goal is to find out a proper $u(t)$ such that the system (1) will asymptotically track x_d, i.e., $\lim_{t\to\infty} \|e\| = 0$.

3. Adaptive control law design

Theorem 1 Suppose system (4) with nonlinear item and uncertainty satisfying Eqs.(2) and (3) respectively, the following feedback controller is proposed:

$$u(t) = \dot{x}_\mathrm{d} + x_\mathrm{d} + \bar{\theta}\frac{\partial V(e)}{\partial e} - \bar{d}\,\|e\|^{-1} e, \quad (5)$$

where $V(e)$ is positive definite function defined with

$$V(e) = \sum_{i=1}^{h} V_i, \quad V_i = \frac{1}{i}(e^\mathrm{T} e)^i, \quad (6)$$

where $h = \max\{r, p\}$ and $\bar{d}$ is a constant satisfying

$$\sum_{i=0}^{h} (d_i + \beta_i) \|x_\mathrm{d}\|^i \leq \bar{d}, \quad (7)$$

$\bar{\theta}$ is an adaptive parameter defined with

$$\dot{\bar{\theta}} = -l \left\| \frac{\partial V(e)}{\partial e} \right\|^2, \quad (8)$$

where l is a positive scalar which can be selected arbitrarily, the controller (5) will render the closed-loop system (1) asymptotically stable at $e = 0$, implying that x asymptotically converges to x_d.

Proof If $F(x,t)$ is rewritten as

$$F(x,t) = -x + f(x,t), \quad (9)$$

then we have

$$\|f(x,t)\| = \|F(x,t) + x\| \leq \|F(x,t)\| + \|x\|.$$

From Eq.(2), we obtain

$$\|f(x,t)\| \leq \sum_{i=0}^{r} \beta_i \|x\|^i.$$

Choose the Lyapunov function as

$$W = V(e) + \frac{1}{2l}\tilde{\theta}^2, \quad (10)$$

where $\tilde{\theta} = \theta + \bar{\theta}$. Taking the time derivative of W, we get

$$\begin{aligned}
\dot{W} &= \frac{1}{l}\tilde{\theta}\dot{\tilde{\theta}} + \dot{V}(e) \\
&= \frac{1}{l}\tilde{\theta}\dot{\tilde{\theta}} + \sum_{i=1}^{h} (e^\mathrm{T} e)^{i-1}[2e^\mathrm{T}(-(e + x_\mathrm{d}) + f(e + x_\mathrm{d}, t) + \Delta f(e + x_\mathrm{d}, t) - \dot{x}_\mathrm{d} + u(t))] \\
&= \frac{1}{l}\tilde{\theta}\dot{\tilde{\theta}} + \sum_{i=1}^{h} (e^\mathrm{T} e)^{i-1} \left[2e^\mathrm{T}\left(-e + \bar{\theta}\frac{\partial V(e)}{\partial e} - \bar{d}\,\|e\|^{-1} e + f(e + x_\mathrm{d}, t) + \Delta f(e + x_\mathrm{d}, t)\right)\right] \\
&= \frac{1}{l}\tilde{\theta}\dot{\tilde{\theta}} - 2\sum_{i=1}^{h} \|e\|^{2i} + \bar{\theta}\left\|\frac{\partial V(e)}{\partial e}\right\|^2 - \bar{d}\left\|\frac{\partial V(e)}{\partial e}\right\| + \frac{\partial V(e)}{\partial e}^\mathrm{T} (f(e + x_\mathrm{d}, t) + \Delta f(e + x_\mathrm{d}, t)). \quad (11)
\end{aligned}$$

Using the well-known Cauchy inequality, we have

$$\begin{aligned}
\frac{\partial V(e)}{\partial e}^\mathrm{T} \Delta f(e + x_\mathrm{d}, t) &\leq \left\|\frac{\partial V(e)}{\partial e}\right\| \|\Delta f(e + x_\mathrm{d}, t)\| \\
&\leq \sum_{i=0}^{p} \left\|\frac{\partial V(e)}{\partial e}\right\| d_i \|e + x_\mathrm{d}\|^i \leq \sum_{i=0}^{p} \left\|\frac{\partial V(e)}{\partial e}\right\| d_i (\|x_\mathrm{d}\| + \|e\|)^i \\
&= \left\|\frac{\partial V(e)}{\partial e}\right\| \left(\sum_{i=0}^{p} d_i \|x_\mathrm{d}\|^i + \sum_{i=1}^{p}\sum_{k=0}^{i-1} C_i^k d_i \|x_\mathrm{d}\|^k \|e\|^{i-k}\right)
\end{aligned}$$

$$= \left\|\frac{\partial V(e)}{\partial e}\right\| \left(\sum_{i=0}^{p} d_i \|x_{\mathrm{d}}\|^i + \sum_{i=1}^{p} \lambda_i \|e\|^i\right)$$
$$\leq \sum_{i=1}^{p} \left(\frac{\lambda_i^2}{4\tau_i}\left\|\frac{\partial V(e)}{\partial e}\right\|^2 + \tau_i \|e\|^{2i}\right) + \sum_{i=0}^{p} d_i \|x_{\mathrm{d}}\|^i \left\|\frac{\partial V(e)}{\partial e}\right\|, \quad (12)$$

where $\lambda_i = \sum_{j\geq i}^{p} C_j^{j-i} d_j \|x_{\mathrm{d}}\|^{j-i}$ and τ_i, $(i = 1, \cdots p)$ are positive scalars which can be selected arbitrarily. Since $\|x_{\mathrm{d}}\|$ is bounded, λ_i is bounded. Similarly, we have

$$\frac{\partial V(e)}{\partial e}^{\mathrm{T}} f(e + x_{\mathrm{d}}, t) \leq \sum_{i=1}^{r} \left(\frac{\alpha_i^2}{4\gamma_i}\left\|\frac{\partial V(e)}{\partial e}\right\|^2 + \gamma_i \|e\|^{2i}\right) + \sum_{i=0}^{r} \beta_i \|x_{\mathrm{d}}\|^i \left\|\frac{\partial V(e)}{\partial e}\right\|, \quad (13)$$

where $\alpha_i = \sum_{j\geq i}^{r} C_j^{j-i} \beta_j \|x_{\mathrm{d}}\|^{j-i}$ and γ_i, $(i = 1, \cdots r)$ are positive scalars which can be selected arbitrarily. Since $\|x_{\mathrm{d}}\|$ is bounded, γ_i is bounded. Without the loss of generality, let $h = r \geq p$ and $\tau_i = 0, d_i = 0$ when $p < i \leq h$. Substituting Eqs.(12) and (13) into Eq.(11), and letting $\theta = \sum_{i=1}^{h}\left(\frac{\lambda_i^2}{4\tau_i} + \frac{\alpha_i^2}{4r_i}\right)$, $\tilde{d} = \left(\sum_{i=0}^{h}(d_i + \beta_i)\|x_{\mathrm{d}}\|^i\right) - \bar{d}$ we get $\dot{\tilde{\theta}} = -l\left\|\frac{\partial V(e)}{\partial e}\right\|^2$ and

$$\dot{W} \leq \frac{1}{l}\tilde{\theta}\dot{\tilde{\theta}} - 2\sum_{i=1}^{h}\|e\|^{2i} + \tilde{\theta}\left\|\frac{\partial V(e)}{\partial e}\right\|^2 + \tilde{d}\left\|\frac{\partial V(e)}{\partial e}\right\| + \sum_{i=1}^{h}(\gamma_i + \tau_i)\|e\|^{2i} \leq \sum_{i=1}^{h}(\gamma_i + \tau_i - 2)\|e\|^{2i}. \quad (14)$$

Note that parameters γ_i and τ_i can be selected small enough to satisfy $\gamma_i + \tau_i - 2 = F_i < 0$. Thus, from Eq.(14), we have

$$\dot{W} \leq \sum_{i=1}^{h} F_i \|e\|^{2i},$$

from which we conclude that the closed-loop system is asymptotically stable at $e = 0$.

Remark 2 In fact, problem of chaos equilibrium point stabilization can be viewed as a special case $\dot{x}_d \equiv 0$. Furthermore, when $x_{\mathrm{d}} \equiv 0$, the controller can be simplified to $u(t) = \bar{\theta}\frac{\partial V(e)}{\partial e} - \bar{d}\|e\|^{-1} e$ where $\beta_0 + d_0 \leq \bar{d}$.

Remark 3 Since $\|u(t)\| \leq \|x_{\mathrm{d}}\| + \|\dot{x}_{\mathrm{d}}\| + \|\bar{\theta}\|\left\|\frac{\partial V(e)}{\partial e}\right\| + \bar{d}$, obviously, $u(t)$ is bounded.

4. Simulation result

In this section, a Rossler chaotic system[5] is treated as an example to illustrate the effectiveness of the proposed controller.

Consider the Rossler chaotic system as

$$\begin{aligned}\dot{x}_1 &= -x_2 - x_3,\\ \dot{x}_2 &= x_1 + ax_2,\\ \dot{x}_3 &= x_1x_3 + b - cx_3.\end{aligned} \quad (15)$$

This system exhibits the chaos when $a = b = 0.2$, $c = 5.7$.

According to Eq.(9), we have

$$f(x) = \begin{bmatrix} x_1 - x_2 - x_3 \\ x_1 + 1.2x_2 \\ 0.2 + x_3(x_1 - 4.7)\end{bmatrix}. \quad (16)$$

Obviously, $\|f(x)\| \leq 0.2 + 5.7\|x\| + 0.5\|x\|^2$. Let the time-varying uncertainty be

$$\Delta f(x,t) = \begin{bmatrix} 0.2x_1\sin(t) + 0.3\cos(3t) \\ 0.4x_3\cos(2t) + 0.5\sin(0.5t) \\ \sqrt{x_2^2 + 1 + \sin(t)} + 0.8\sin(2t)\end{bmatrix}, \quad (17)$$

then

$$\|\Delta f(x,t)\| \leq \left\| \begin{array}{c} 0.2x_1\sin(t) \\ 0.4x_3\cos(2t) \\ \sqrt{x_2^2+1+\sin(t)} \end{array} \right\| + \left\| \begin{array}{c} 0.3\cos(3t) \\ 0.5\sin(0.5t) \\ 0.8\sin(2t) \end{array} \right\| \leq \sqrt{\|x\|^2+2}+1 \leq 2.5+\|x\|\,. \tag{18}$$

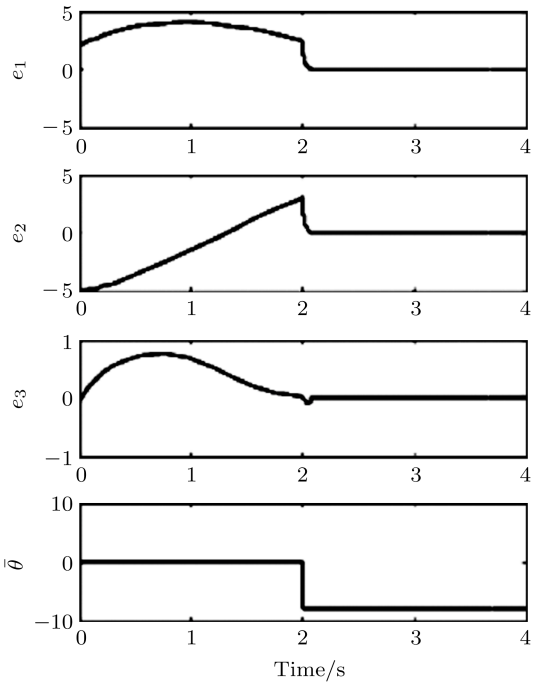


Fig.1. The time response of the error states between the Rossler system and the desired trajectory $x_{\rm d}(t) = [0\ \cos(t)\ \sin(t)]^{\rm T}$, when the controller is deactivated ($t <$ 2s).

Assume the desired orbit be $x_{\rm d}(t) = [0\ \cos(t)\ \sin(t)]^{\rm T}$ and the initial state of the system be $x(0) = [2\quad -4\quad -0.3]^{\rm T}$. According to Eqs.(6)–(7) and (16)–(18), we obtain $V = e^{\rm T}e + \left(e^{\rm T}e\right)^2/2$, $\bar{d} = 2.7 + 6.7\,\|x_{\rm d}\| + 0.5\,\|x_{\rm d}\|^2$. From Theorem 1, the following controller can be obtained:

$$u = \dot{x}_{\rm d} - x_{\rm d} + 2\bar{\theta}(\|e\|^2+1)e - \bar{d}e\,\|e\|^{-1}\,, \tag{19}$$

with the adaptive law

$\dot{\bar{\theta}} = -(\|e\|^2+1)^2\,\|e\|^2$ and $\bar{\theta}(0) = 0.2$.

From Theorem 1, it can be concluded that the state of the Rossler system asymptotically tracks the desired trajectory $x_{\rm d}(t) = [0\cos(t)\sin(t)]^{\rm T}$, as verified by the simulation results shown in Fig.1.

5. Conclusion

In this paper, by employing Lyapunov stability theory and adaptive technique, a novel adaptive robust controller for a class of chaotic system with time-varying uncertainty is proposed, which can asymptotically track the arbitrarily desired bounded trajectory. Simulation on the Rossler chaotic system shows the effectiveness of the proposed controller.

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