Vol 16 No 9, September 2007
1009-1963/2007/16(09)/2616-11
Chinese Physics
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Generation of on–off intermittency based on Rössler chaotic system

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(Received 2 January 2007; revised manuscript received 29 January 2007)

In this paper, one-state on–off intermittency and two-state on–off intermittency are generated in two five-dimensional continuum systems respectively. In each system, a two-dimensional subsystem is driven by the Rössler chaotic system. The parameter conditions under which the on–off intermittency occurs are discussed in detail. The statistical property of the intermittency is investigated. It is shown that the distribution of the laminar phase duration time follows a power law with an exponent of $-3/2$, which is a signature of on–off intermittency. Moreover, the phenomenon of intermingled basins is observed when attractors in the two symmetric invariant subspaces are stable. We provide an effective way to generate on–off intermittency based on a chaotic system, which is important for application and theoretical study.

Keywords: chaotic system, on–off intermittency, transverse Lyapunov exponent, blowout bifurcation, intermingled basins

PACC: 0545

1. Introduction

In recent years, chaos and later reported hyperchaos with their complex dynamics and great potential for applications have become hot topics of research. Much work has focused on their generation,[1−4] control[5,6] and synchronization.[7−10] Intermittency is another common nonlinear phenomenon in dynamic systems such as chaotic systems.[11−15] Different types of intermittencies have been reported, among which is on–off intermittency. It is characterized by the distinct two-state behaviour. The 'off' state, also laminar phase, is nearly constant, and it can remain so for a very long period of time, and the 'on' state, also burst phase, is irregular burst departing from and returning quickly to the 'off' state.[16]

On–off intermittency has a close connection with chaotic synchronization,[17−20] and recently it has been reported in the human balancing task[21] and in the brain activity when the normal states and epilepsy paroxysms alternately occur.[22] Zhou and Lai[23] have studied how to amplify weak signals and stochastic resonance via on–off intermittency. Therefore, the on–off intermittency has attracted extensive attention and shown great potential for applications.

The basic mechanism responsible for the on–off intermittency[13] can be understood as follows: on varying a parameter of a dynamical system which has a chaotic attractor in an invariant subspace, the attractor can lose or gain transverse stability at what is termed a blowout bifurcation point.[24] The bifurcation can be measured by the largest transverse Lyapunov exponent[24−27] (corresponding to perturbations in a direction transverse to the invariant subspace). When the largest transverse Lyapunov exponent is negative, the chaotic attractor in the invariant subspace is also an attractor for the whole system. In this case, if more than one of such invariant subspaces symmetrically exists in the system, the basins of each attractor are intermingled. When the exponent is slightly positive, the attractor in the invariant subspace loses transverse stability and becomes weakly unstable, then the system exhibits on–off intermittency.

The previous studies of on–off intermittency in continuum systems generally concentrate on the synchronization of identical chaotic systems[17−20] or on the nonlinear systems driven by the Duffing oscillator.[24−29] Besides, the on–off intermittency has been also investigated with models of one-dimensional maps that are multiplicatively coupled to either random or chaotic signals.[16,18,30−32]

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In this paper, we consider the Rössler system[30] as a drive system, and construct two five-dimensional continuum models. By calculating the largest transverse Lyapunov exponent, we find the location of blowout bifurcation of each system. Just beyond the bifurcation, one-state on–off intermittency and two-state on–off intermittency are generated in the two systems separately. Moreover, the phenomenon of intermingled basins is observed below the blowout bifurcation. Through the investigation of the statistical property of the intermittency in our systems, we find that for both one-state intermittency and two-state intermittency, the distribution of the laminar phase duration time follows a $-3/2$ power law, and verify that the intermittency is on–off intermittency. Our results show that the Rössler system can be used to drive on–off intermittency, so can the other simple chaotic system.

The remainder of this paper is organized as follows: in Section 2, we introduce the model of one-state on–off intermittency. Through the calculation of the largest Lyapunov exponent, we find the parameter conditions for one-state on–off intermittency. Numerical simulations verify that the distribution of laminar phases lengths approximately follows a $-3/2$ power law. In Section 3, we present the model of two-state on–off intermittency and its parameter conditions. We also observe intermingled basins of the two chaotic attractors in the symmetric subspaces. Finally, concluding remarks are given in Section 4.

2. The one-state on–off intermittency and its characteristics

2.1. Model of one-state on–off intermittency

On–off intermittency usually occurs in drive-response systems. Moreover, the drive system determines the dynamics in the invariant subspaces, while the response system determines the positions and the number of invariant subspaces, that is, the number of laminar phases. We consider the following two-dimensional system:

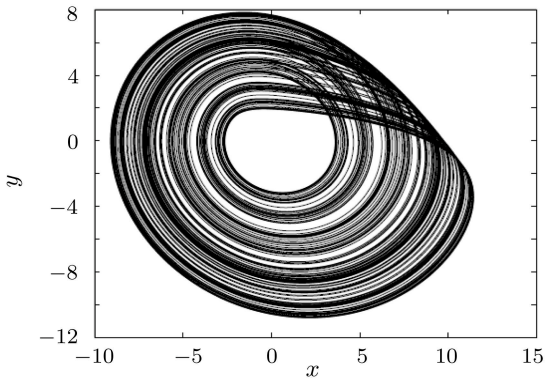
$$\begin{aligned} \frac{\mathrm{d}u}{\mathrm{d}t} &= v, \\ \frac{\mathrm{d}v}{\mathrm{d}t} &= -\gamma u^3 + p_1 S u + p_2 v, \end{aligned} \tag{1}$$

where S is a signal from a driving system, p_1 and p_2 are control parameters, and $\gamma > 0$. Let system (1) be driven by the well-known Rössler system[30]

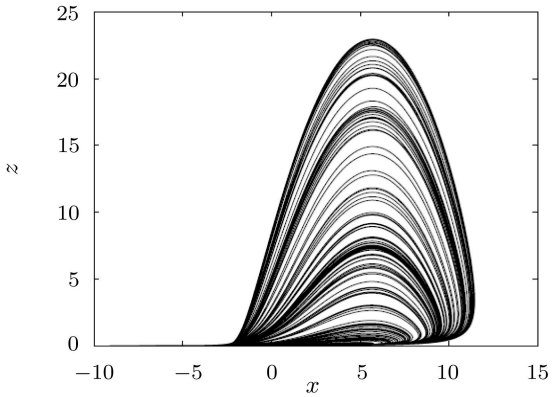
$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= -y - z, \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= x + ay, \\ \frac{\mathrm{d}z}{\mathrm{d}t} &= b + z(x - c). \end{aligned} \tag{2}$$

Here we choose y to be a driving signal of on–off intermittency and combine system (1) with system (2) to obtain the model of one-state on–off intermittency as follows:

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= -y - z, \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= x + ay, \\ \frac{\mathrm{d}z}{\mathrm{d}t} &= b + z(x - c), \\ \frac{\mathrm{d}u}{\mathrm{d}t} &= v, \\ \frac{\mathrm{d}v}{\mathrm{d}t} &= -\gamma u^3 + p_1 y u + p_2 v. \end{aligned} \tag{3}$$



(a) y versus x



(b) z versus x

Fig.1. Phase portraits of system (2) with $a = b = 0.2$, and $c = 5.7$.

In this section we take the parameters $a = b = 0.2$, $c = 5.7$, and $\gamma = 1$. We first consider p_1 as a control parameter and p_2 as a constant, here we

choose $p_2 = -2.8$. Note that $u = v = 0$ is an invariant three-dimensional manifold in the five-dimensional phase space, since if $u = v = 0$ at $t = 0$, this remains so for all times of $t > 0$. In this invariant subspace the equations of motion reduce to the Rössler system, which has a chaotic attractor as shown in Fig.1.

2.2. Parameter conditions for one-state on–off intermittency

Take $(\delta u, \delta v)$ as infinitesimal perturbations transverse to the invariant subspace. The largest Lyapunov exponent $h_\perp$ which transverse to the invariant subspace is computed by taking the derivative of both sides of Eq.(1) and setting $u = v = 0$. The motion equation of the perturbation $(\delta u, \delta v)$ near the subspace is

$$\begin{aligned} \frac{\mathrm{d}\delta u}{\mathrm{d}t} &= \delta v, \\ \frac{\mathrm{d}\delta v}{\mathrm{d}t} &= p_1 y \delta u + p_2 \delta v, \end{aligned} \tag{4}$$

where y is a chaotic trajectory produced by system (3) in the invariant subspaces. The exponent $h_\perp$ is calculated from

$$h_\perp = \lim_{t\to\infty} \frac{1}{t} \ln[\delta(t)/\delta(0)], \tag{5}$$

where $\delta(t) = \{[\delta u(t)]^2 + [\delta v(t)]^2\}^{1/2}$. A plot of $h_\perp$ versus p_1 for $p_1 \in [1.6, 2.6]$, $p_2 = -2.8$ is shown in Fig.2. We see that $h_\perp$ passes through zero at $p_{c1} \approx 1.995$, $p_{c2} \approx 2.242$, and $p_{c3} \approx 2.343$. Thus the system has three blowout bifurcation points in the parameter range of our interest.

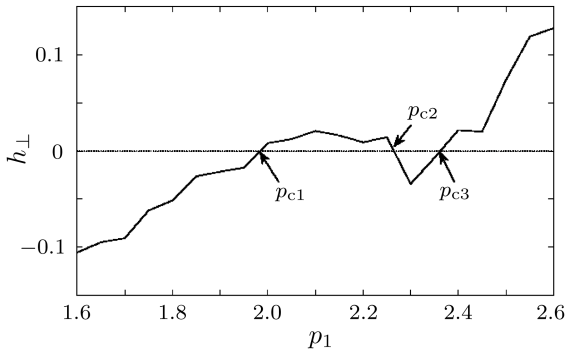


Fig.2. $h_\perp$ versus p_1 for $p_1 \in [1.6, 2.6]$ and $p_2 = -2.8$.

Since the nonlinear term $-\gamma u^3$ in the last equation of system (1) prevents u and v from going too far from the invariant manifold, there is no attractor other than Rössler attractor in $u = v = 0$. According to Ref.[24], the bifurcation is of hysteretic type and the attraction basin of the attractor will not be riddled.

Near each bifurcation point in Fig.2, while p_1 takes values corresponding to slightly positive $h_\perp$, system (3) exhibits one-state on–off intermittency. With $p_1 = 2.2$ and $p_2 = -2.8$, time series of $u(t)$ and $v(t)$ from an arbitrary initial condition are plotted in Fig.3. which show typical on–off intermittent behaviour.

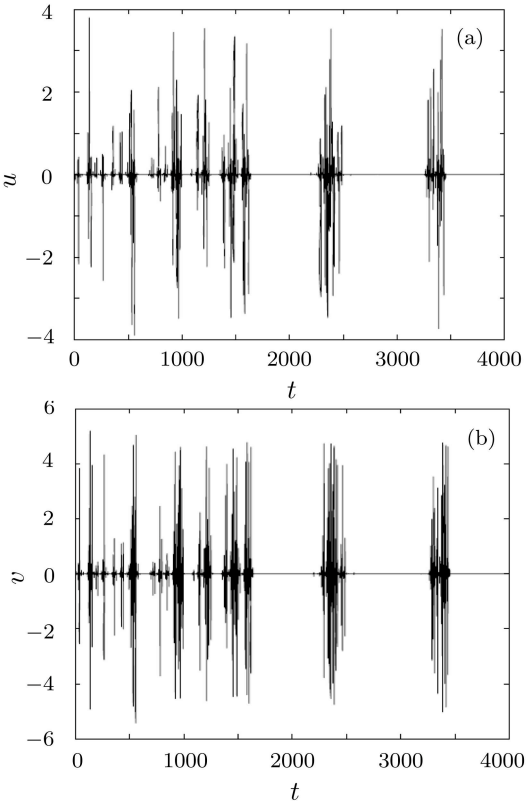


Fig.3. Time series of $u(t)$ (a) and time series of $v(t)$ (b) for $p_1 = 2.2$ and $p_2 = -2.8$.

In fact, if we choose p_2 as a control parameter and p_1 as a constant, say, $p_1 = 3$, the model also can exhibit on–off intermittency while p_2 corresponds to slightly positive $h_\perp$. A plot of $h_\perp$ versus p_2 for $p_2 \in [-3.7, -3.2]$ and $p_1 = 3$ is shown in Fig.4(a), from which we can see that in this parameter range $h_\perp$ passes through a zero point from negative to positive at one blowout bifurcation point $p_c \approx -3.4482$. Near this bifurcation point, when p_2 takes a value of -3.43, the largest transverse Lyapunov exponent $h_\perp \approx 0.0108$, and the time series of $u(t)$ presents on–off intermittency as shown in Fig.4(b).

Note that we have not given all blowout bifurcation points of system (3). Any combination of p_1 with p_2 that satisfy $h_\perp \cong 0$ corresponds to a bifurcation point. Therefore, it is likely there exist other parameter ranges for on–off intermittency. The typical phase portraits of on–off intermittency in system (3) with parameter settings of $p_1 = 2.2$ and $p_2 = -2.8$ are shown in Figs.5(a)–5(p).

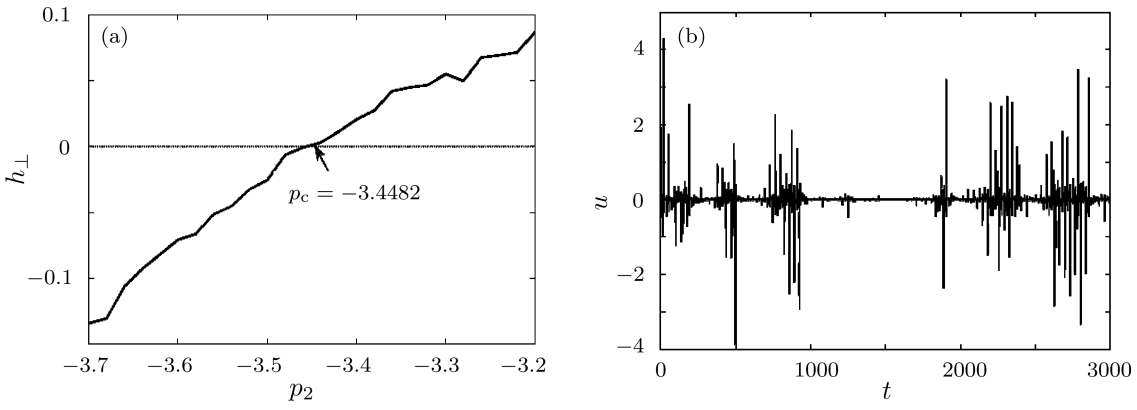
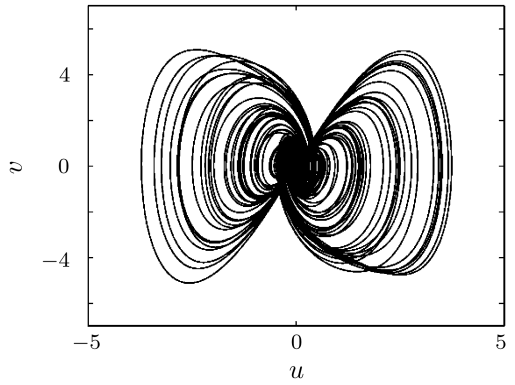
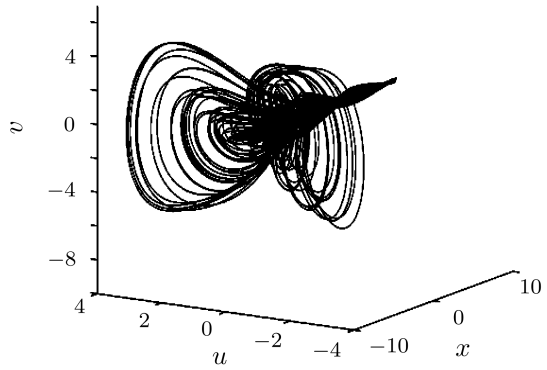


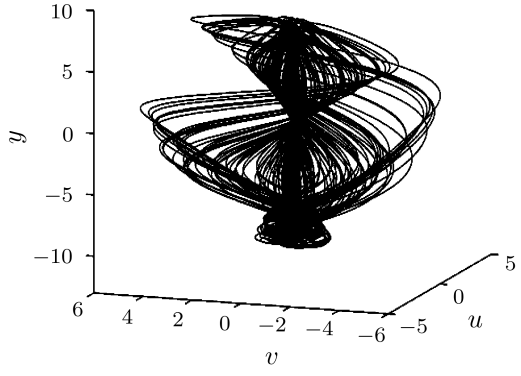
Fig.4. $h_\perp$ versus p_2 for $p_2 \in [-3.7, -3.2]$ and $p_1 = 3$ (a) and time series of $u(t)$ for $p_1 = 3$ and $p_2 = -3.43$ (b).



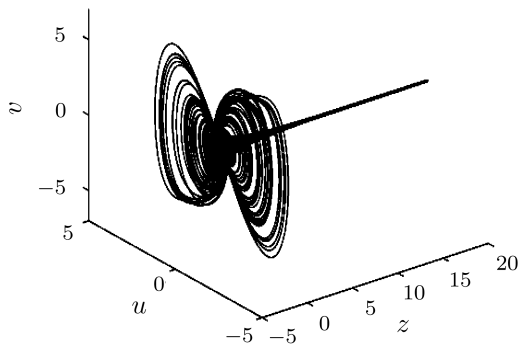
(a) v versus u



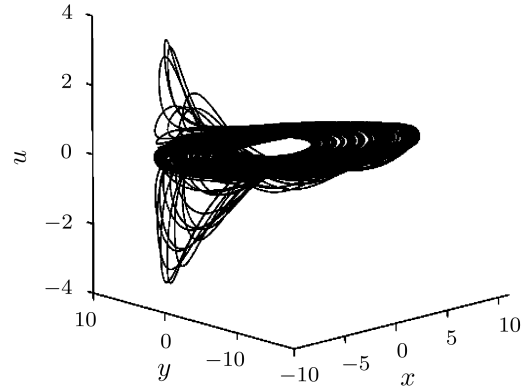
(b) v versus x and u



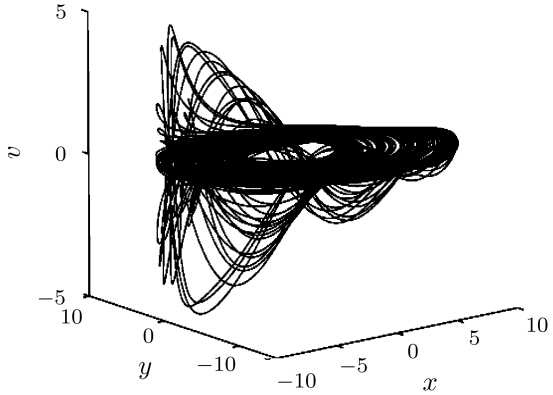
(c) y versus v and u



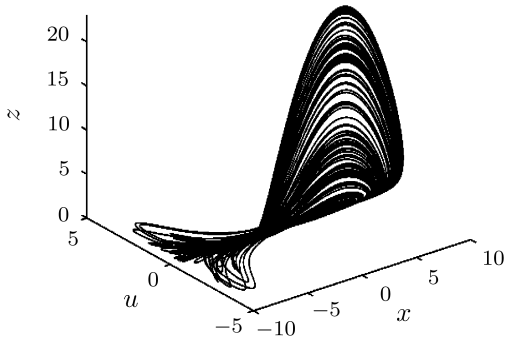
(d) v versus u and z



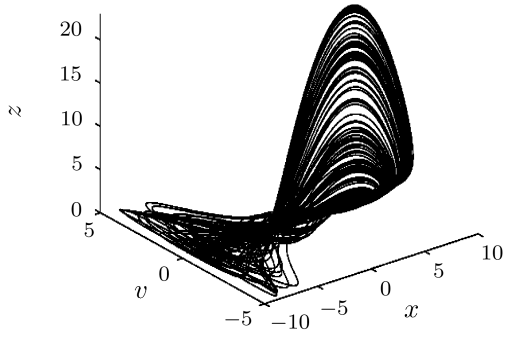
(e) u versus y and x



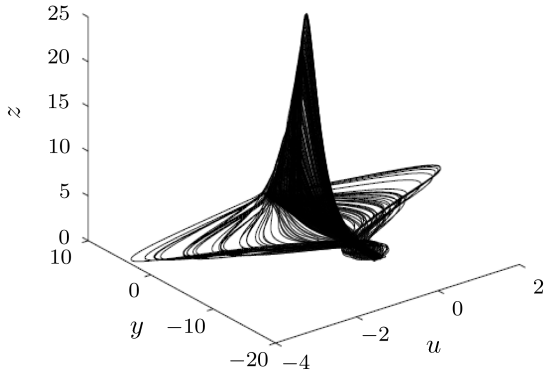
(f) v versus y and x



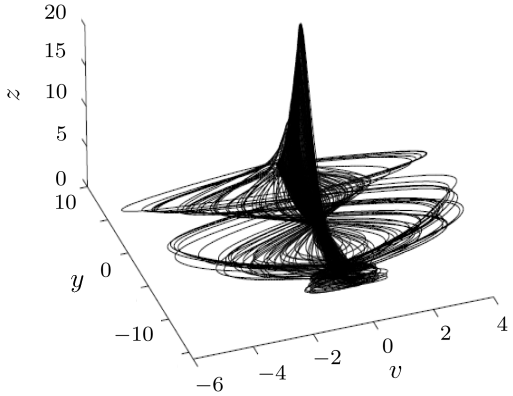
(g) z versus u and x



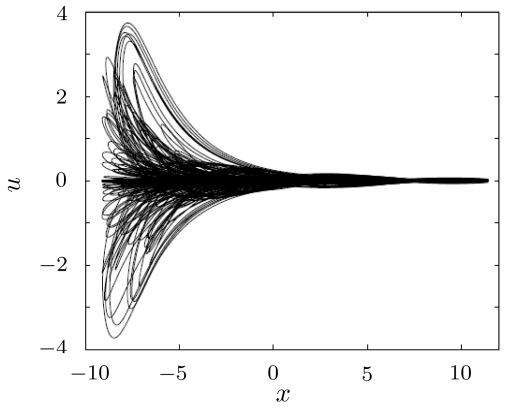
(h) z versus v and x



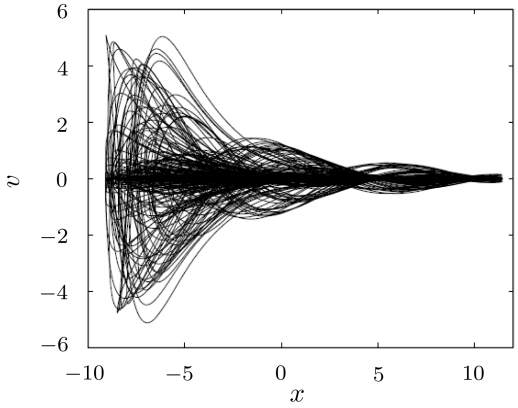
(i) z versus y and u



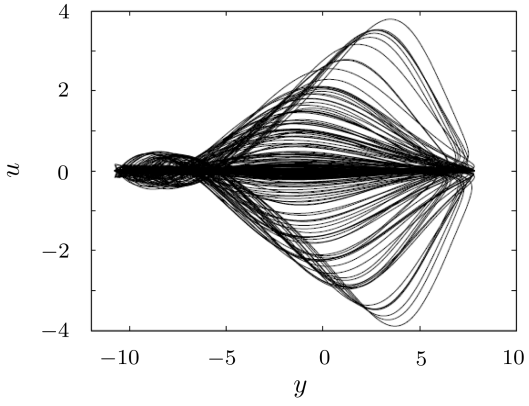
(j) z versus y and v



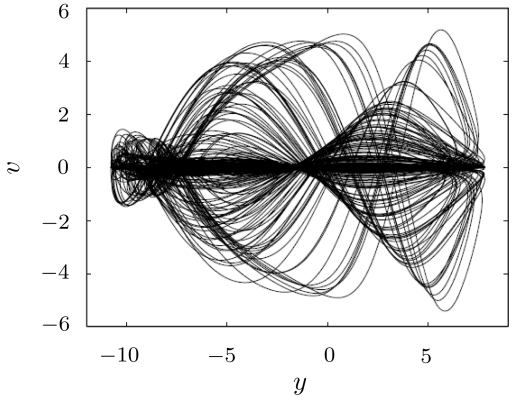
(k) u versus x



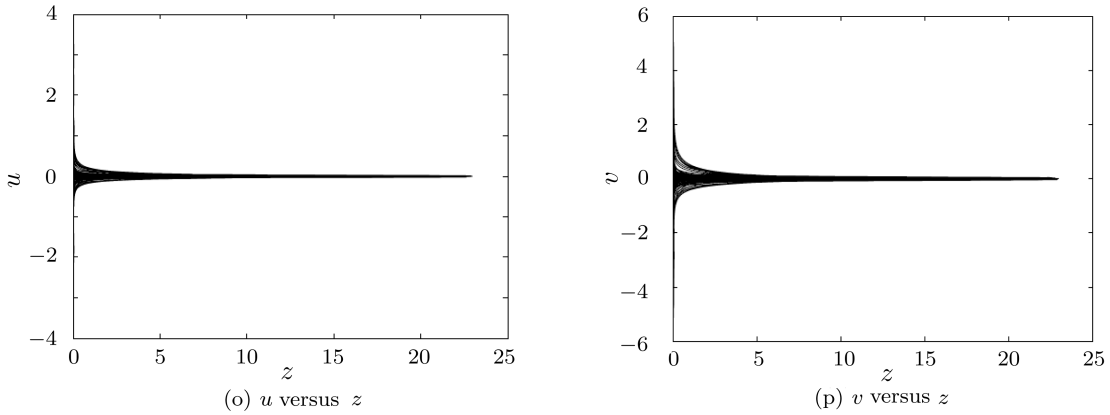
(l) v versus x



(m) u versus y



(n) v versus y



(o) u versus z (p) v versus z

Fig.5. Phase portraits of system (3) with $p_1 = 2.2$ and $p_2 = -2.8$.

In Fig.5(a) it is clearly seen that the projection of chaotic attractor of system (3) in the $u - v$ plane has two rolls which joint at the invariant space $u = v = 0$. In these figures when the orbits stay in or near the Rössler attractor (dense area), the on–off intermittency signal is in a laminar state and when the orbits escape from the Rössler attractor (sparse orbits), the on–off intermittency signal is in a burst state. Anyway, the escaping orbits will return to the Rössler attractor and repeat the above process.

From the phase portraits of on–off intermittency, we see that when there exists on–off intermittency, the chaotic attractor on the invariant manifold becomes a bursting attractor. In fact, as the control parameter approaches bifurcation point, the frequency of bursting away from the attractor becomes lower and lower, approaching zero.

2.3. Scaling law of one-state on–off intermittency

The statistical property of on–off intermittency in system (3) is discussed here. Scaling laws are used widely to describe different intermittent phenomena. It is known that in a range of moderate lengths, the distribution of laminar phase lengths of single-state on–off intermittency approaches a scaling power law with an exponent of $-3/2$.[21] This power law is widely used to characterize on–off intermittency. The intermittency in system (3) is shown to share the same power law.

Let τ denote the length of the laminar phase which is defined as $|u(t)| \leq 0.001$, and $P(\tau)$, the distribution function of τ. We analyse a time-series data of 5×10^7 sample points for $p_1 = 2.4$ and $p_2 = -2.8$, and collect about 10000 laminar phases. Numerical simulations show that the relation between τ and $P(\tau)$ follows a power law

$$P(\tau) \propto \tau^{\alpha}. \tag{6}$$

Figure 6 shows the distribution of the laminar phase lengths for on–off intermittency driven by the Rössler system. In the figure the slope of the dash-dot line is $-2/3$. We see that in a range of moderate laminar phase lengths, the exponent α is very close to $-2/3$. Thus our numerical simulations verify that the intermittency in system (3) is on–off intermittency.

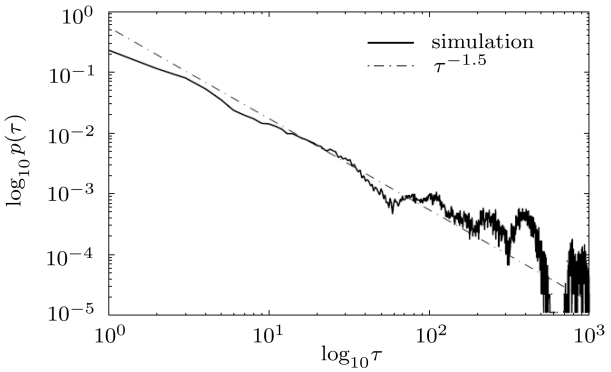


Fig.6. Distribution of laminar phases τ with $p_1 = 2.4$ and $p_2 = -2.8$.

3. The two-state on–off intermittency and its characteristics

3.1. Model of two-state on–off intermittency

If a chaotic system has two low-dimensional symmetric invariant subspaces, it is likely that the system shows two-state on–off intermittency or intermingled basins near the blowout bifurcation.

Modify the response subsystem in Section 2 to make it have two symmetric invariant subspaces, then we will obtain the following response system:

$$\begin{aligned}\frac{\mathrm{d}u}{\mathrm{d}t} &= v,\\ \frac{\mathrm{d}v}{\mathrm{d}t} &= -\gamma u(u^2-1)^3 + p_1 S u(u^2-1) + p_2 v,\end{aligned} \tag{7}$$

where S is the driving signal; and p_1 and p_2 are control parameters; $\gamma > 0$. Let system (7) be driven by the Rössler system and replace S by y, then we will obtain a model of two-state on–off intermittency as follows:

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= -y - z,\\ \frac{\mathrm{d}y}{\mathrm{d}t} &= x + ay,\\ \frac{\mathrm{d}z}{\mathrm{d}t} &= b + z(x-c),\\ \frac{\mathrm{d}u}{\mathrm{d}t} &= v,\\ \frac{\mathrm{d}v}{\mathrm{d}t} &= -\gamma u(u^2-1)^3 + p_1 y u(u^2-1) + p_2 v,\end{aligned} \tag{8}$$

where the parameters $\gamma = 0.3$, $a = b = 0.2$ and $c = 5.7$. Letting $\mathrm{d}u/\mathrm{d}t = 0$ and $\mathrm{d}v/\mathrm{d}t = 0$, we have two three-dimensional symmetric invariant subspaces of system (8): $u = 1$ and $v = 0$; $u = -1$ and $v = 0$. In both invariant subspaces, the dynamics are determined by the Rössler chaotic attractor.

3.2. Parameter conditions for two-state on–off intermittency and intermingled basins

As in the case of one-state on–off intermittency model, the control parameter can be either p_1 or p_2. Here we only consider p_1 as a control parameter.

In order to find the appropriate parameter settings for two-state on–off intermittency, the largest transverse Lyapunov exponent $h_\perp$ needs computing.

Represent $(\delta u, \delta v)$ as infinitesimal perturbations transverse to the invariant subspaces and take the derivative of Eq.(7) at $u = \pm 1$ and $v = 0$, then we will obtain the motion equation of the perturbation $(\delta u, \delta v)$ near both invariant subspaces as follows:

$$\begin{aligned}\frac{\mathrm{d}\delta u}{\mathrm{d}t} &= \delta v,\\ \frac{\mathrm{d}\delta v}{\mathrm{d}t} &= 2p_1 y \delta u + p_2 \delta v.\end{aligned} \tag{9}$$

Thus the two subspaces have the same transverse stability and exponent $h_\perp$ which is computed from Eq.(5). Figure 7 shows a plot of $h_\perp$ versus the parameter p_1 for $p_1 \in [0.9, 1.24]$ and $p_2 = -2.8$, in which the exponent $h_\perp$ passes through zero points at $p_{c1} \approx 1.005$, $p_{c2} \approx 1.118$ and $p_{c3} \approx 1.183$. Thus in this parameter range, system (8) has three blowout bifurcations at p_{c1}, p_{c2}, and p_{c3}.

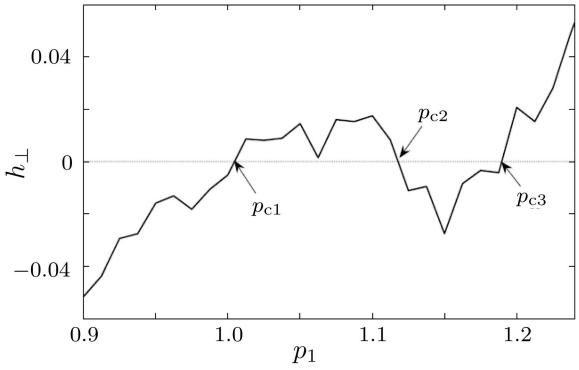


Fig.7. $h_\perp$ versus p_1 for $p_1 \in [0.9, 1.24]$ and $p_2 = -2.8$.

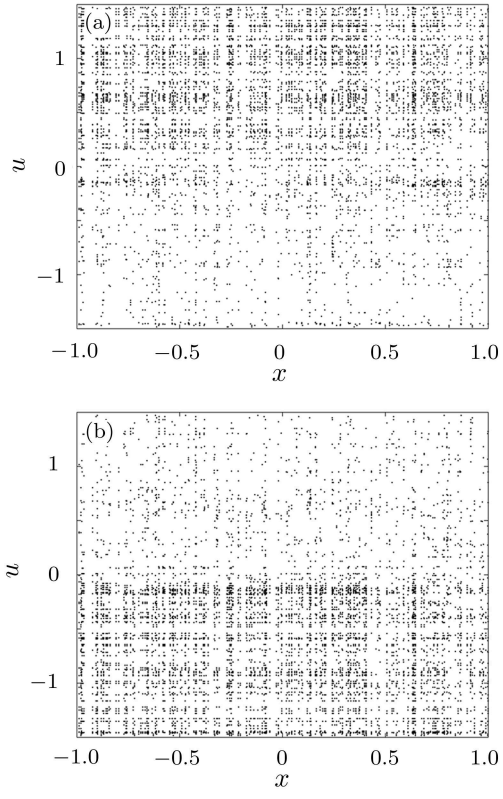


Fig.8. Basins of the attractor at $u = 1$ and $v = 0$ (a), and at $u = -1$ and $v = 0$ (b), with initial conditions taken from the two-dimensional region $-1 \le x \le 1$ and $-1.5 \le u \le 1.5$. The two attraction basins are intermingled.

When p_1 takes values corresponding to a negative $h_\perp$, the Rössler attractors in the subspaces $u = 1$, $v = 0$ and $u = -1$, $v = 0$ are also global attractors. We observe the phenomenon of intermingled basins.

According to Ref.[24], the blowout bifurcation is of hysteretic type. Figure 8 shows the attraction basins of attractors at $u = 1$, $v = 0$ and at $u = -1$, $v = 0$ for initial conditions taken from the two-dimensional region $-1 \leq x \leq 1$ and $-1.5 \leq u \leq 1.5$ with $y = z = v = 0$. In the figure 49000 randomly chosen initial conditions in the region are integrated to determine their destinations. We see that the two basins of attraction are intermingled. Near any initial point in the basin of one attractor, there exist points which belong to the basin of the other attractor. This can be seen clearly in Fig.9, where a blowout of Fig.8 for the region $-0.5 \leq x \leq 0.5$ and $0.5 \leq u \leq 1$ is shown.

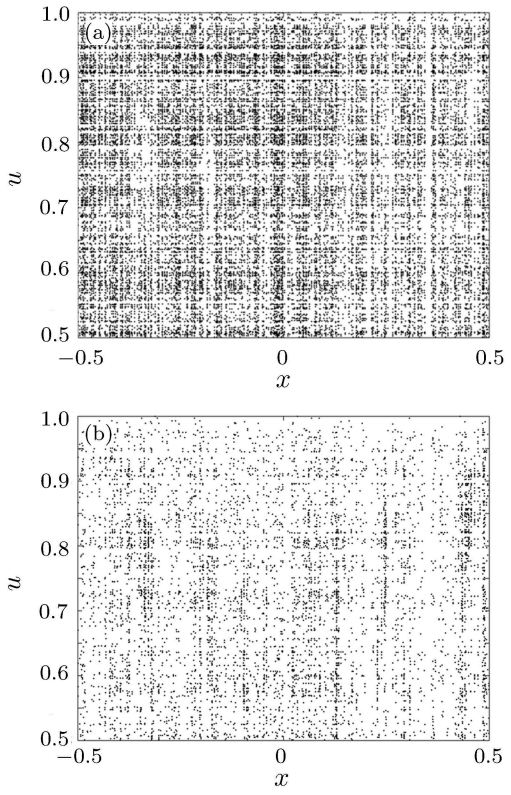


Fig.9. A blowup of Fig.8 in a region $-0.5 \leq x \leq 0.5$ and $0.5 \leq u \leq 1$. (a) Basin of the attractor at $u = 1$ and $v = 0$; (b) basin of the attractor at $u = -1$, $v = 0$.

Here the effect of γ on the emergence of intermingled basins is discussed. When γ takes a relatively large value (e.g. $\gamma > 0.5$), there will be no intermingled basins even though $h_\perp$ is negative. This is because the nonlinear term $-\gamma u(u^2 - 1)^3 (\gamma > 0)$ in Eq.(7) serves to guarantee that $u(t)$ and $v(t)$ are bounded and to place an upper bound on how far away u and v can go from their nearby invariant manifold. And the larger γ is, the greater this effect is. Thus if γ is large enough, it will force $u(t)$ and $v(t)$ to shift directly towards the nearest invariant manifold from the initial point. Therefore, the intermingled basins cannot be observed in this case.

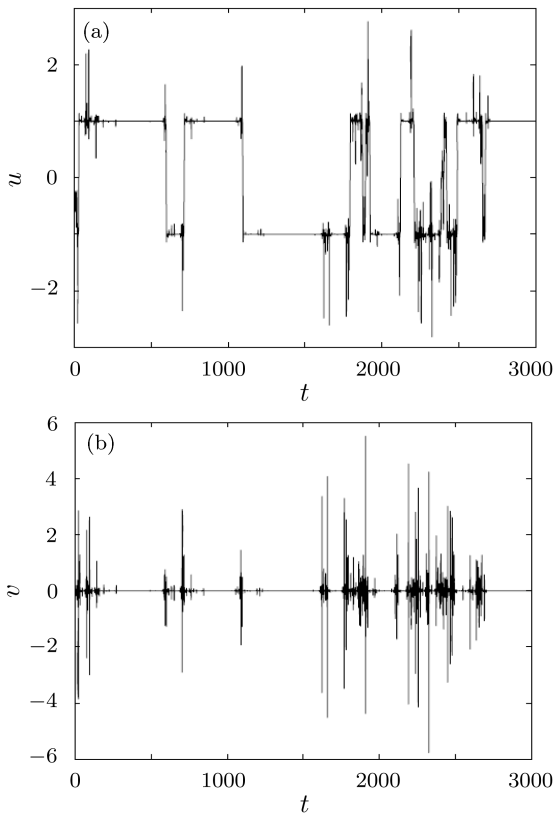
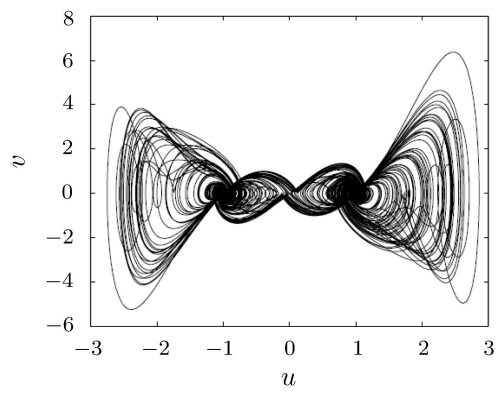


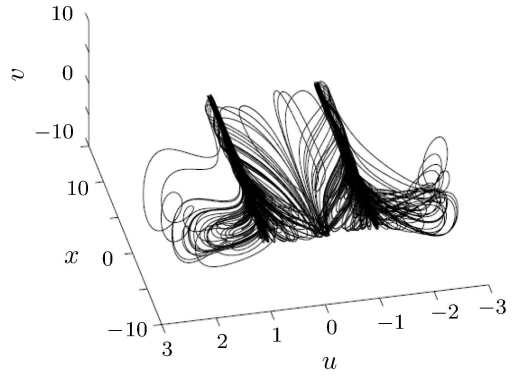
Fig.10. Two-state on–off intermittent time series of $u(t)$ (a) and one-state on–off intermittent time series of $v(t)$ (b) with $p_1 = 1.12$ and $p_2 = -2.8$.

When $p_1 > p_{c1}$, $p_1 < p_{c2}$ or $p_1 > p_{c3}$ and p_1 is very close to one of these bifurcation points, $h_\perp$ is slightly positive and the two attractors in the subspaces lose transverse stabilities simultaneously, then two-state on–off intermittency occurs. Figure 10 shows the time series of $u(t)$ and $v(t)$ with $p_1 = 1.12$ and $p_2 = -2.8$. We see that the on–off intermittent behaviour of $u(t)$ has two laminar states at $u = 1$ and $u = -1$ while $v(t)$ has one laminar state at $v = 0$. We present the phase portraits of two-state model with $p_1 = 1.2$ and $p_2 = -2.8$ as shown in Figs.11(a)–11(p).

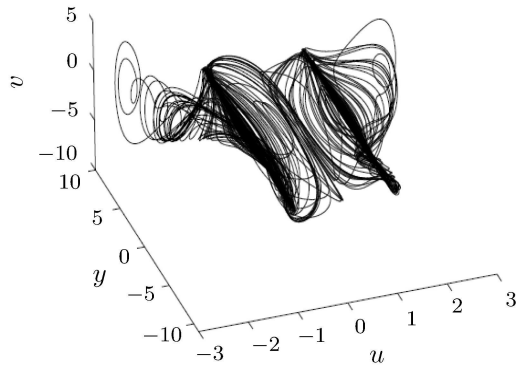
Figure 11(a) shows that the projection of chaotic attractor of system (7) in the $v - u$ plane has four rolls which joint at the invariant spaces $u = 1$, $v = 0$ and $u = -1$, $v = 0$. From the phase portraits of system (7), we can see that the orbits of system (7) spend most time in the two invariant subspaces at $u = \pm 1$, $v = 0$ (dense area, corresponds to a laminar state) and irregularly burst away from the invariant subspaces (corresponds to a burst state). Thus the symmetric existence of two invariant subspaces is a precondition for the onset of two-state on–off intermittency.



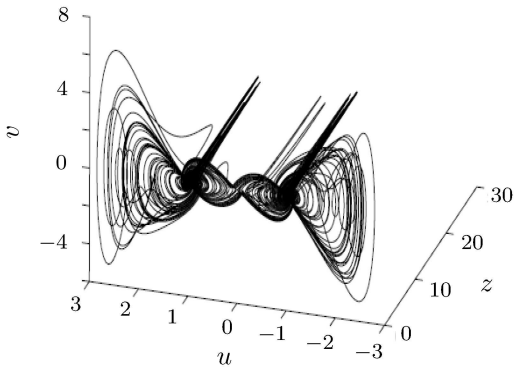
(a) v versus u



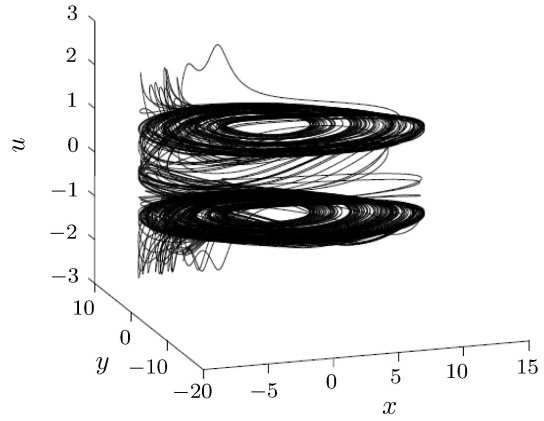
(b) v versus x and u



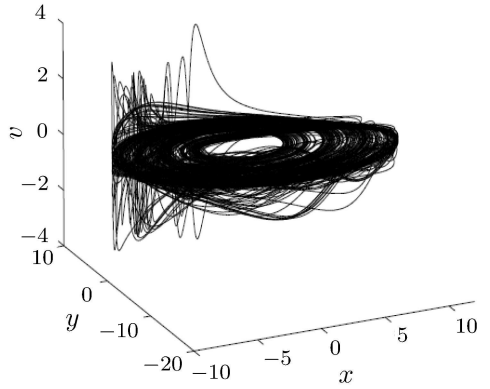
(c) v versus y and u



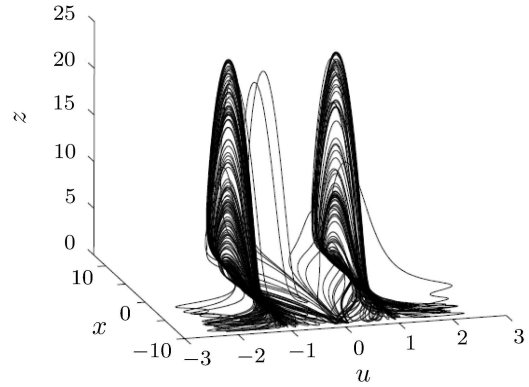
(d) v versus u and z



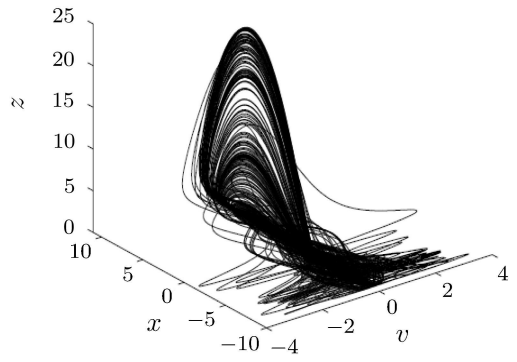
(e) u versus y and x



(f) v versus y and x



(g) z versus x and u



(h) z versus x and v

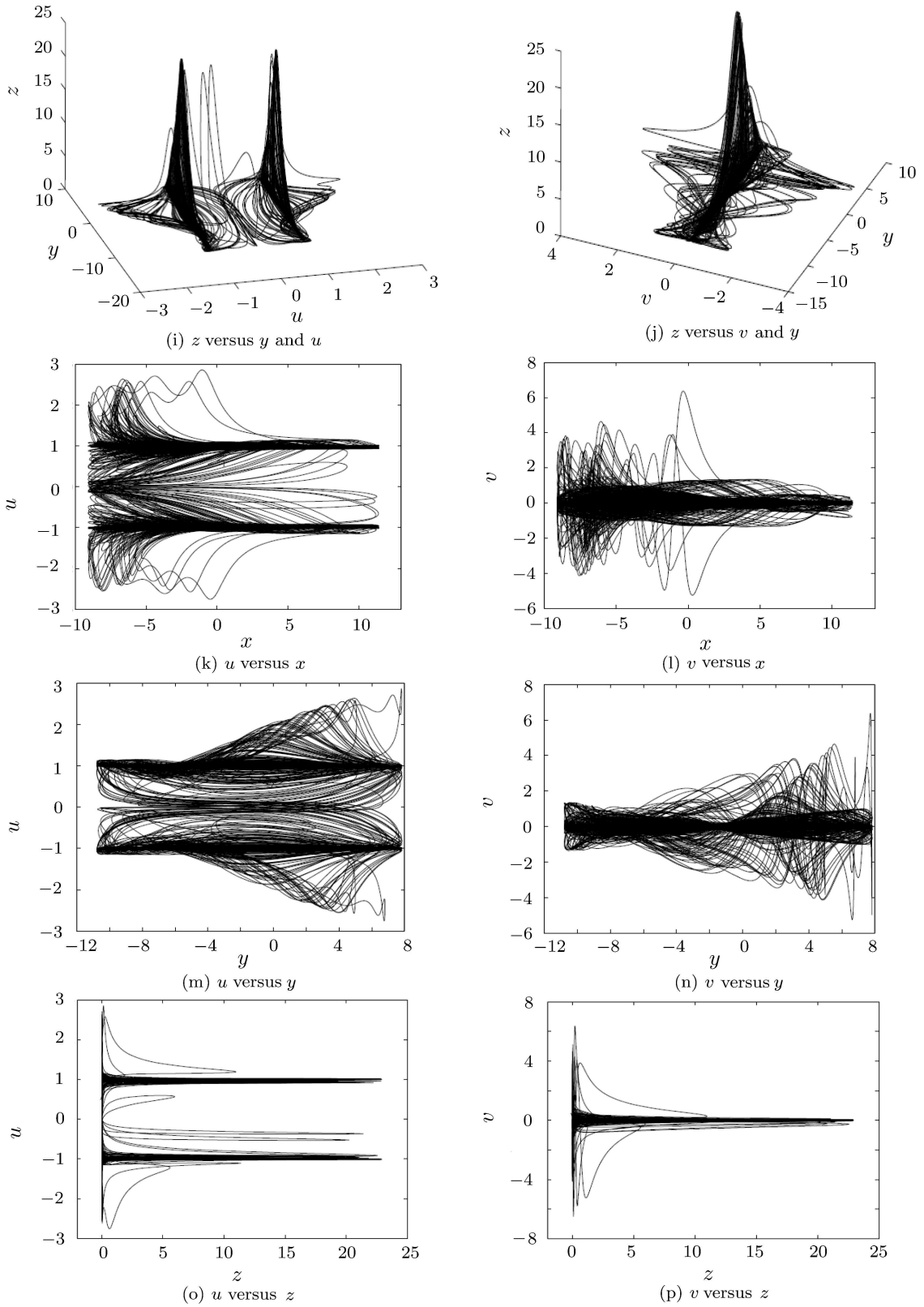


Fig.11. Phase portraits of system (8), with $p_1 = 1.2$ and $p_2 = -2.8$.

3.3. Scaling law of two-state on–off intermittency

The statistical property of two-state on–off intermittency observed in system (8) is analysed here. We investigate the distribution of the laminar phase duration time of the intermittency. Define the laminar phases as $|u(t)| \leq 0.001$, and represent the duration of the laminar phase by $\tau_n (n = 1, -1)$, where τ_1 is

associated with the laminar phase near the attractor at $u = 1$, $v = 0$, and τ_{-1} associated with the laminar phase near the attractor at $u = -1$, $v = 0$. In the case of $p_1 = 1.2$ and $p_2 = -2.8$, for each laminar phase level, we take a time series of $u(t)$ and collect about 23600 laminar phases. Our numerical analysis shows that the distributions of τ_1 and τ_{-1} share the same power law

$$P(\tau_n) \propto \tau_n^{\alpha} \tag{10}$$

as shown in Fig.12, which is because the largest transverse Lyapunov exponents for both invariant subspaces are calculated from Eq.(9).

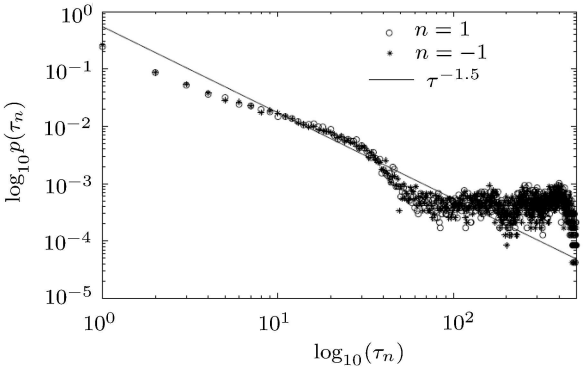


Fig.12. Distribution of laminar phases τ_n $(n = 1, -1)$. τ_1 is associated with the attractor at $u = 1$, $v = 0$; τ_{-1} is associated with the attractor at $u = -1$, $v = 0$.

In the figure, the slope of the solid power law line is $-3/2$. We see that the exponent α is of very close to $-2/3$ in a range of moderate laminar phase lengths. Our numerical analysis verifies that the intermittency in system (8) is on–off intermittency and demonstrates that the universal $-3/2$ power law distribution of laminar phases still holds true for the two-state on–off intermittency in system (8).

4. Conclusion

In this paper, we consider the Rössler system as a drive system, and construct two five-dimensional continuum models. By adjusting two control parameters, one-state on–off intermittency and two-state on–off intermittency are generated in the two systems separately. We discuss the parameter conditions for the onset of on–off intermittency and verify that the distribution of the laminar phase lengths of on–off intermittency approaches a scaling power law with an exponent of $-3/2$. Our results show that the simple Rössler system can be used to generate on–off intermittent signal. It is an effective way to generate on–off intermittency based on a chaotic system, which is important for application and theoretical study.

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