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Realistic teleportation of coherent states using squeezed vacuum states*

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This paper presents a realistic scheme for the teleportation of coherent states in which a two-mode squeezed vacuum state serves as the quantum channel and the position-sum and momentum-difference of two local modes serve as the measuring observables. The average fidelity of the teleportation of coherent states is derived for finite squeezing parameters and it turns out that fidelity greater than 1/2 cannot be achieved by using a classical channel alone and the probability distribution of the measurement result is a Gaussian distribution around the unknown parameter of the input coherent state with a width given by the squeezing parameter.

Keywords: quantum teleportation, coherent states, squeezed vacuum states
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Quantum teleportation (QT) plays a central role in quantum communication and information processing. Since Bennett *el al*[1] showed that an unknown quantum state of a two-state particle can be teleported from one place to another in 1993, much attentions have been paid to QT. The QT of finite-level systems has been demonstrated[2] and many other aspects such as general schemes[3−6] and some applications[7,8] have also been investigated. On the other hand, the idea of QT for the discrete quantum states has been generalized to continuous variables corresponding to states of infinite-systems.[9,10] The discrete variable and continuous variable teleportation was unified in the framework of canonical quantum teleportation.[11] The QT of optical coherent states was demonstrated experimentally by using squeezed-state entanglement.[12]

In this paper, we present a more realistic scheme for teleportation of coherent states by using a two-mode squeezed vacuum state. The average fidelity and the probability distribution of measurement are investigated. Our scheme also provides a generalized nonideal measurement of the unknown coherent state. A kind of trade-off between the measurement-disturbance and the teleportation fidelity is demonstrated.

Generally speaking, the QT consists of three basic steps: (i) to prepare two systems in Einstein–Podolsky–Rosen (EPR) entangled state or a Bell state and send them to two different places to establish a quantum channel; (ii) at one place, to perform the so-called joint Bell operator measurements with respect to one system involved in the EPR entanglement and a third system in an unknown state to be transferred; (iii) at other place, to perform necessary unitary operations to the other system involved in the EPR entanglement according to the outcomes of Bell operator measurements. By this means the unknown state is transferred from one place to another.

In our proposal, the entangled state produced by a beam splitter[13] is a two-mode squeezed vacuum (TMSV) state

$$\begin{aligned} |\psi\rangle_{23} &= |\mathrm{TMSV}\rangle \\ &= \sqrt{1-|q|^2}\exp\left[-q\hat{a}_2^{\dagger}a_3^{\dagger}\right]|0,0\rangle_{23}\,, \end{aligned} \tag{1}$$

where q is a parameter which stands for the degree of entanglement.

We make use of $|\psi\rangle_{23}$ as a quantum channel so that Alice and Bob will share the entangled state of mode 2 and mode 3. An unknown coherent state to be teleported from Alice to Bob is

$$\begin{aligned} |\alpha\rangle_1 &= \exp\left[\alpha\hat{a}_1^{\dagger}-\alpha^*\hat{a}_1\right]|0\rangle_1 \\ &= \exp\left[-\frac{|\alpha|^2}{2}\right]\exp\left[\alpha\hat{a}_1^{\dagger}\right]|0\rangle_1\,. \end{aligned} \tag{2}$$

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Thus the total initial state of the whole system can be expressed as

$$|\psi\rangle_{123} = |\alpha\rangle_1 |\psi\rangle_{23}. \tag{3}$$

To teleport this unknown state $|\alpha\rangle_1$ to Bob, Alice only makes a local measurement of the position-sum $Q_1 + Q_2$ and the momentum-difference $P_1 - P_2$ on his two modes, where

$$\begin{aligned} Q_1 &= \frac{\hat{a}_1 + \hat{a}_1^\dagger}{\sqrt{2}}, \quad Q_2 = \frac{\hat{a}_2 + \hat{a}_2^\dagger}{\sqrt{2}}; \\ p_1 &= \frac{\hat{a}_1 - \hat{a}_1^\dagger}{\sqrt{2}}, \quad P_2 = \frac{\hat{a}_2 - \hat{a}_2^\dagger}{\sqrt{2}}. \end{aligned} \tag{4}$$

The common eigenstates $\{|\eta\rangle\}$ of these two commuting observables are given by the following entangled states of two-mode optical field[14]

$$|\eta\rangle_{12} = \exp\left[-\frac{|\eta|^2}{2} + \eta\hat{a}_1^\dagger + \eta^*\hat{a}_2^\dagger - \hat{a}_1^\dagger\hat{a}_2^\dagger\right]|0\rangle_1 |0\rangle_2, \tag{5}$$

in which $\eta = \eta_1 + \mathrm{i}\eta_2$ is an arbitrary complex number. And we have $(Q_1 + Q_2)|\eta\rangle = \sqrt{2}\eta_1|\eta\rangle$, and $(P_1 - P_2)|\eta\rangle = \sqrt{2}\eta_2|\eta\rangle$. Because the state $|\eta\rangle$ is complete $\int \frac{\mathrm{d}^2\eta}{\pi}|\eta\rangle\langle\eta| = 1$ and orthonormal $\langle\eta'|\,\eta\rangle = \pi\delta(\eta - \eta')(\eta^* - \eta'^*)$, it is proper to be considered as continuous Bell basis. After the measurement made by Alice with outcome η the projected state for mode 3 shared by Bob is

$$\begin{aligned} |\psi\rangle_3 &= {}_{12}\langle\eta|\,\alpha\rangle_1 |\psi\rangle_{23} \\ &= \sum_m {}_1\langle m|_2 \langle m|(-1)^m \exp\left[a_1\eta^* - a_1^\dagger\eta\right]|\alpha\rangle_1 \sqrt{1-q^2}\sum_n (-q)^n |n\rangle_2 |n\rangle_3 \\ &= \sum_{m,n} {}_1\langle m|(-1)^m D_1^\dagger(\eta)|\alpha\rangle_1 \sqrt{1-q^2}_2\langle m|\,n\rangle_2 (-q)^n |n\rangle_3 \\ &= \sqrt{1-q^2}\sum_n q^n |n\rangle_3 \langle n| D_3^\dagger(\eta)|\alpha\rangle_3 \\ &= \sqrt{1-q^2}q^{\hat{N}_3} D_3^\dagger(\eta)|\alpha\rangle_3, \end{aligned} \tag{6}$$

where $\hat{N}_3 = \hat{a}_3^\dagger\hat{a}_3$ is a particle number operator acting on mode 3. In the limit of infinite squeezing ($q \to 1$) the TMSV state which serves as the quantum channel tends to EPR state $|\psi_{\mathrm{E}}\rangle_{23} \propto \exp\left[-\hat{a}_2^\dagger\hat{a}_3^\dagger\right]|0\rangle_2 |0\rangle_3$ and we obtain

$$|\psi_{\mathrm{E}}\rangle_3 = {}_{12}\langle\eta|\,\alpha\rangle_1 |\psi_{\mathrm{E}}\rangle_{23} \propto D_3^\dagger(\eta)|\alpha\rangle_3. \tag{7}$$

We note that $D_3^\dagger$ is an unitary transformation. If Alice now sends the result of measurement (η) to Bob via a classical channel, Bob is able to perform unitary operation D_3 to mode 3 and mode 3 is then at the unknown state $|\alpha\rangle_3$ though no one knows what the unknown state is.

To consider the effects of finite squeezing, we compute the average fidelity of the teleported state with respect to the initial coherent state. Because the normalized probability of finding the outcome η after the measurement is $p(\eta) = \frac{1}{\pi}{}_3\langle\psi|\,\psi\rangle_3$, one is able to describe the fidelity of teleportation $F(\eta)$ and the average fidelity $\bar{F}$ as follows:

$$F(\eta) = \frac{1}{\pi P(\eta)}\left|{}_3\langle\psi_{\mathrm{E}}|\,\psi\rangle_3\right|^2, \tag{8}$$

$$\bar{F} = \int \mathrm{d}^2\eta P(\eta) F(\eta) = \int \frac{\mathrm{d}^2\eta}{\pi}\left|{}_3\langle\psi_{\mathrm{E}}|\,\psi\rangle_3\right|^2. \tag{9}$$

Using Eqs.(6) and (7), we derive

$$\begin{aligned} \bar{F} &= \int \frac{\mathrm{d}^2\eta}{\pi}(1-q^2)\left|\langle\alpha|D(\eta)q^{\hat{N}}D^\dagger(\eta)|\alpha\rangle\right|^2 \\ &= \int \frac{\mathrm{d}^2\eta}{\pi}(1-q^2)\left|\exp\left[-(1-q)\left(|\eta|^2 - \alpha^*\eta - \alpha\eta^* + |\alpha|^2\right)\right]\right|^2 \end{aligned}$$

$$= \int \frac{\mathrm{d}^2\eta}{\pi} \left(1-q^2\right) \mathrm{e}^{-2(1-q)|\eta-\alpha|^2}$$
$$= \frac{1+q}{2}. \tag{10}$$

It is easy to see that as $q \to 1$, $\bar{F} \to 1$, i.e., a perfect teleportation is performed. Moreover, $\bar{F} = 0.5$ sets a boundary beyond which entanglement is required in continuous variable teleportation of coherent states; in other words, fidelity greater than 1/2 could not have been achieved through the use of a classical channel alone.

Now, let us analyse the probability of measuring the result η. From Eq.(6) we have

$$\begin{aligned} p(\eta) &= \frac{1}{\pi}\, {}_3\langle\psi|\, \psi\rangle_3 \\ &= \frac{1}{\pi}\left(1-q^2\right) \langle\alpha|\, D(\eta)\, q^{2\hat{N}} D^{\dagger}(\eta)\, |\alpha\rangle \\ &= \frac{1}{\pi}\left(1-q^2\right) \exp\left[-\left(1-q^2\right)\left(|\eta|^2 - \alpha^*\eta - \alpha\eta^* + |\alpha|^2\right)\right] \\ &= \frac{1-q^2}{\pi} \mathrm{e}^{-\left(1-q^2\right)|\eta-\alpha|^2}. \end{aligned} \tag{11}$$

It is clear that the probability density function $p(\eta)$ is a Gaussian distribution which has a maximum at $\eta = \alpha$. This means that Alice's measurement value η has a maximum probability for the coherent states. If parameter q is made larger, then the maximum value of $p(\eta)$ is smaller and values drop off less rapidly with distance of η from α. Hence, larger q means greater spread and smaller q means greater concentration of probability mass about $\eta = \alpha$. This provides a non-ideal measurement of the parameter α. This information gain and the fidelity of the teleportation are complementary in the sense that the more accurate Alice knows about the unknown state via the measurement the less possible Bob can obtain the state and vise versa.

In summary, we have shown how a two-mode squeezed vacuum state can be used for teleportation of an unknown coherent state with measurement of the joint quadrature phase quantities, $Q_1 + Q_2$ and $P_1 - P_2$. If the entanglement source is squeezed infinitely, perfect teleportation can be achieved. We calculate the average fidelity and the probability of measurement, and find that the average fidelity that exceeds 1/2 must have come about through the use of some entanglements, while the measuring probability is a Gaussian distribution which has a maximum at $\eta = \alpha$ and parameter q gives an indication of how the probability is spread around the centre of the distribution.

References

[1] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wooters W K 1993 *Phys. Rev. Lett.* **70** 1895

[2] Bouwmeaster D, Pan J W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 *Nature* **390** 575

[3] Bennett C H, Brassaed G, Popescu S, Schumacher B, Smolin J A and Wooters W K 1996 *Phys. Rev. Lett.* **76** 722

[4] Stenholm S and Bardroff P J 1998 *Phys. Rev.* A **58** 4373

[5] Cai X H 2003 *Chin. Phys.* **12** 1066

[6] Li Y, Zhang T C, Zhang J X and Xie C D 2003 *Chin. Phys.* **12** 0861

[7] Maierle C S, Lidar D A and Harris R A 1998 *Phys. Rev. Lett.* **81** 5928

[8] Zubairy M S 1998 *Phys. Rev.* A **58** 4368

[9] Vaidman L 1994 *Phys. Rev.* A **49** 1473

[10] Braunstein S L and Kimble H J 1998 *Phys. Rev. Lett.* **80** 869

[11] Yu S X and Sun C P 2000 *Phys. Rev.* A **61** 022310

[12] Furusawa A, Sorensen J L, Braunstein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706

[13] Scheel S and Welsch D G 2001 *Phys. Rev.* A **64** 063811

[14] Fan H Y and Klauder J R 1994 *Phys. Rev.* A **49** 704