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Internal Josephson-like tunnelling in two-component Bose–Einstein condensates affected by sign of the atomic interaction and external trapping potential*

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This paper studies the Josephson-like tunnelling in two-component Bose–Einstein condensates coupled with microwave field, which is in respond to various attractive and repulsive atomic interaction under the various aspect ratio of trapping potential. It is very interesting to find that the dynamic of Josephson-like tunnelling can be controlled from fast damped oscillations to nondamped oscillation, and relative number of atoms changes from asymmetric occupation to symmetric occupation correspondingly.

Keywords: Bose–Einstein, Josephson tunnelling, nonlinear
PACC: 0365, 7450, 0530J, 4660D

The existence of a Josephson current is a direct manifestation of macroscopic quantum phase coherence[1] and has numerous important applications in condensed matter physics, quantum optics and cold atom physics: for example, precision measurement and quantum computation. The physical origin of the Josephson current is the temporal interference of the two systems which must both have a well defined quantum phase and a different average energy per particle respectively. Recently, the experimental realization of multi-component Bose–Einstein condensates (BECs) of weakly interacting alkali atoms has provided a route to study the Josephson effect[2−15] in a controlled and tunable way by means of Feshbach resonance[16−27] so far unattainable in charged systems. The physical origin of the low-energy Feshbach resonances is the low-energy binary collisions described by the difference of the initial and intermediate state energies which can be effectively altered through variations of the strength of an external magnetic field.

In this paper, we study the dynamics of Josephson-like tunnelling in two-component BECs coupled with microwave field under tuning the atomic interaction from attractive case to repulsive case in the various aspect ratio of trapping potential. Through changing the relative value between attractive intra-atomic and inter-atomic interaction, we find that, occupation on relative number of atoms represents two very different behaviour, for which one is fast damped oscillation, but the other is nondamped and irregular oscillation. In particular, when the atomic interaction is changed from attractive to repulsive case, we find that relative number of atoms also represents nondamped and irregular oscillation.

We consider a system of longitudinal elongated two-component BECs coupled by the microwave field with the effective Rabi frequency Ω and finite detune δ, where the Ω and δ are independent upon time and space coordinate as the experimental case.[2,3] At zero temperature, this system can be described by macroscopic wavefunction ψ_j (normalized to unity, i.e. $\int(|\psi_1|^2+|\psi_2|^2)\mathrm{d}z=1$) which obey the dimensionless one-dimensional Gross–Pitaevskii equations as[8,9]

$$\mathrm{i}\frac{\partial}{\partial t}\psi_j(z,t)=\Big[-\frac{\partial^2}{\partial z^2}+V(z)\Big]\psi_j(z,t)+[G_{jj}(t)|\psi_j|^2+G_{jk}(t)|\psi_k|^2]\psi_j(z,t)$$

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$$+(-1)^{j+1}\frac{\delta}{\omega_\perp}\psi_j(z,t)+\frac{\Omega}{\omega_\perp}\psi_k(z,t), \quad (1)$$

where $j,k=1,2$, $j\neq k$, and $\omega_\perp$ is radial harmonic trapping frequency, time and coordinate unit are $2/\omega_\perp$ and $a_\perp=\sqrt{\hbar/m\omega_\perp}$, respectively, and $V(z)$ is the longitudinal confining potential, $G_{jj}=(4N\hbar a_{jj})/(ma_\perp^3\omega_\perp)$ and $G_{jk}=(4N\hbar a_{jk})/(ma_\perp^3\omega_\perp)$ are dimensionless intra- and inter-species effective atomic interaction potentials, respectively, where a_{jj} and a_{jk} are corresponding s-wave scattering lengths, respectively, N is the total atomic number, and m is the atomic mass assumed the same for both two species.

In the following discussion, we consider two-hyperfine spin states of ^{7}Li Bose atom for which the magnitude and sign of the atomic interaction can be tuned to any value via a magnetic-field dependent Feshbach resonance. And we choose dimensionless $V(z)=\gamma^2z^2+\lambda z$, where $\gamma=\omega_z/\omega_\perp$ is aspect ratio of trapping potential and λ determines the strength of the gravitational field, ω_z is longitudinal harmonic trapping frequency and $\omega_\perp=2\pi\times625$ Hz (so, the time and coordinate units are 0.51 ms and $a_\perp=1.51$ μm respectively). Moreover, considering that the condensates will be unstable and collapse when the number of particles is large enough,[24] so we choose $N=6000$, which provides a safe range of parameters for avoiding instability and occurring collapse.

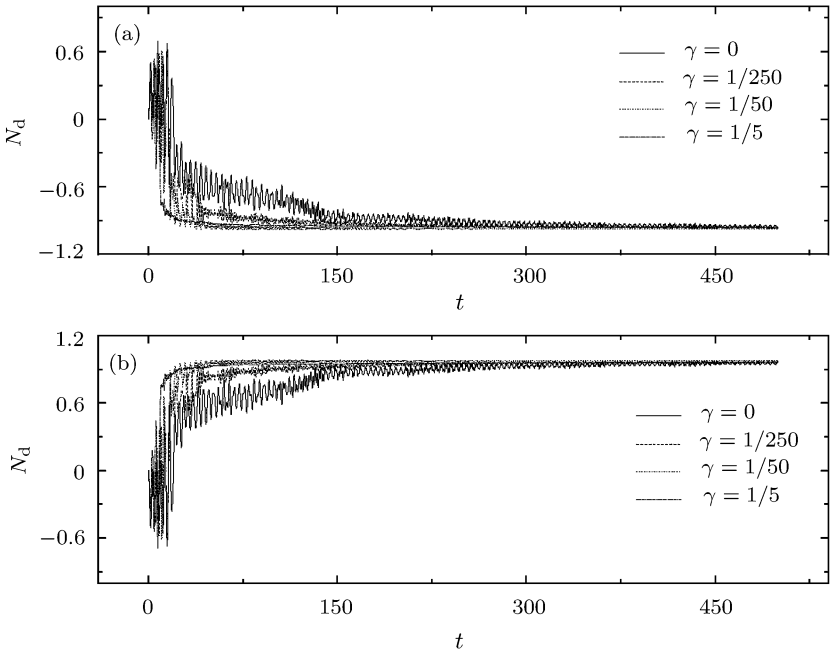


Fig.1. Temporal evolution of relative number of atoms N_d for initial Gaussian wavefunction in response to various aspect ratio of trapping potential γ. (a) Corresponding to attractive atomic interactions for the parameters as $G_{11}=G_{22}=4G_{12}=-10$ and $\Omega=1.5\omega_\perp$, $\delta=0.5\omega_\perp$ (b) Corresponding to attractive atomic interactions for the parameters as $G_{11}=G_{22}=4G_{12}=-10$ and $\Omega=1.5\omega_\perp$, $\delta=-0.5\omega_\perp$.

First of all, we consider the attractive case, i.e., intra- and inter-species effective atomic interaction potentials are time-independent attractive case, which can be easily realized by using Feshbach resonance experimentally. The effective Rabi frequency and finite detune are chosen as

$$\Omega=1.5\omega_\perp \quad \text{and} \quad \delta=0.5\omega_\perp$$

corresponding to Fig.1(a) and $\delta=-0.5\omega_\perp$ corresponding to Fig.1(b). Here, we have chosen $a_{11}=a_{22}$, which just considers the experimental feasibility and assumes that tuning atomic interaction wouldn't destroy the condensates in this system. We denote relative number of atoms between two species as N_d, i.e.

$$N_d=N_1-N_2,$$

where

$$N_1=\int|\psi_1|^2\mathrm{d}z \quad \text{and} \quad N_2=\int|\psi_2|^2\mathrm{d}z.$$

We study Josephson-like tunnelling in this case and the results are shown in Fig.1, which is based on numerical simulation of Eq.(1) for the initial Gaussian

wavefunction under the various aspect ratio of trapping potential. As shown in Fig.1, during the evolution process, exchange of atoms is always damped very fast and almost suppressed completely irrespective of aspect ratio of trapping potential. In particular, it is important to notice that occupation on the each components represents very asymmetric behaviour and the atoms almost transform to second component completely in the case of $\delta = 0.5\omega_\perp$, but almost transform to first component completely in the case of $\delta = -0.5\omega_\perp$. In this case, system has lower energy through exchanging the atoms from first to second components under the condition $\delta = 0.5\omega_\perp$, but from second to first components under the condition $\delta = -0.5\omega_\perp$.

From the experimental points of view, it is very important to study the dynamics of Josephson-like tunnelling under the tuning atomic interaction which provides a route to study the Josephson effect in a controlled and tunable way by means of Feshbach resonance so far unattainable in charged systems. So, we consider that the attractive intra- and inter-species effective atomic interaction potentials become $4G_{11} = 4G_{22} = G_{12} = -10$. The results are shown in Fig.2, which is based on numerical simulation of Eq.(1) for the initial Gaussian wavefunction under the various aspect ratio of trapping potential. We can see that amplitude of oscillation increases with the increase of the aspect ratio of trapping potential γ. Especially, contrary to first case, occupation on the each components almost represents very symmetric behaviour and the atoms are not likely to stay on only one of the two components because the energy of the system in asymmetric occupation is larger than symmetric occupation on each of the components.

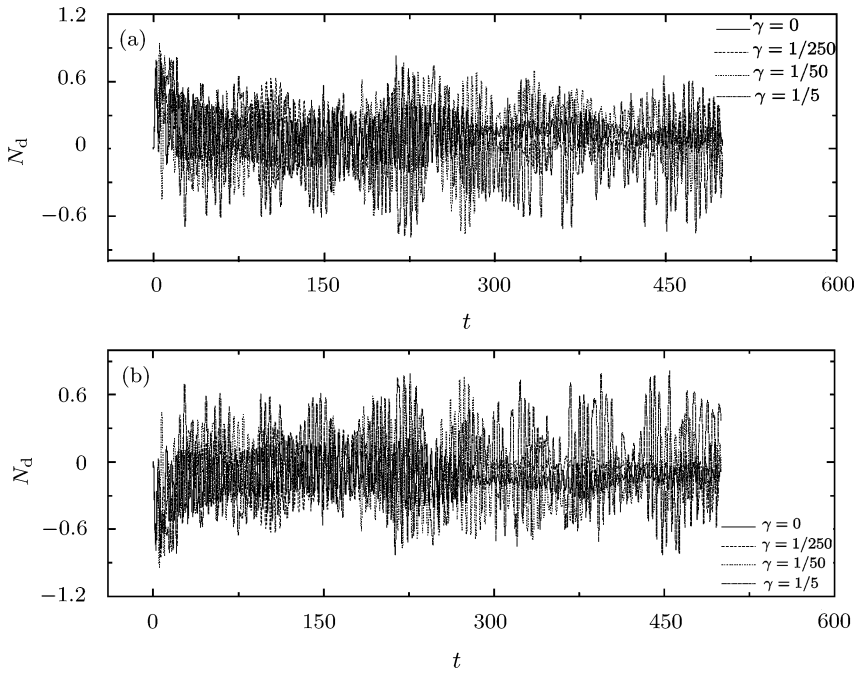


Fig.2. Temporal evolution of relative number of atoms N_d for initial Gaussian wavefunction in response to various aspect ratio of trapping potential γ. (a) Corresponding to attractive atomic interactions for the parameters as $4G_{11} = 4G_{22} = G_{12} = -10$ and $\Omega = 1.5\omega_\perp$, $\delta = 0.5\omega_\perp$. (b) Corresponding to attractive atomic interactions for the parameters as $4G_{11} = 4G_{22} = G_{12} = -10$ and $\Omega = 1.5\omega_\perp$, $\delta = -0.5\omega_\perp$.

Next, we study the dynamics of Josephson-like tunnelling under the repulsive atomic interaction in response to various aspect ratio of trapping potential. As shown in Fig.3(a), corresponding to the case $G_{11} = G_{22} = 4G_{12} = 10$ and $\Omega = 1.5\omega_\perp$, $\delta = -0.5\omega_\perp$, amplitude of oscillation increases with the increase of the aspect ratio of trapping potential γ, and occupation on the each components also represents symmetric behaviour approximately, which is like the case shown in Fig.2. However, oscillation becomes slower with increasing the aspect ratio of trapping potential γ, which is very different from the case shown in Fig.2. It is very interesting to notice that Fig.3(b), corresponding to the case $4G_{11} = 4G_{22} = G_{12} = 10$, represents very different phenomena from Fig.3(a).

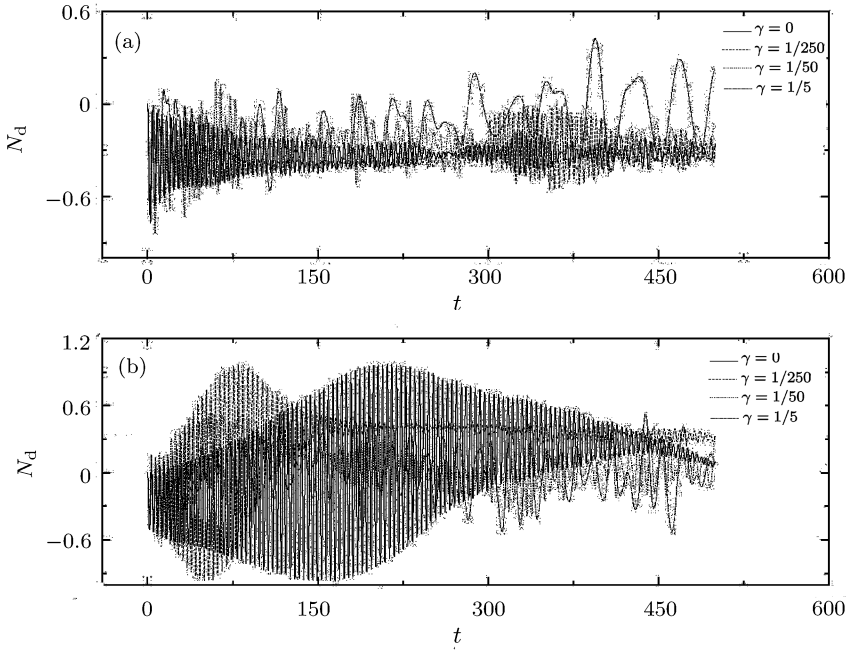


Fig.3. Temporal evolution of relative number of atoms N_d for initial Gaussian wavefunction in response to various aspect ratio of trapping potential γ. (a) Corresponding to repulsive atomic interactions for the parameters as $G_{11} = G_{22} = 4G_{12} = 10$ and $\Omega = 1.5\omega_\perp$, $\delta = -0.5\omega_\perp$. (b) Corresponding to repulsive atomic interactions for the parameters as $4G_{11} = 4G_{22} = G_{12} = 10$ and $\Omega = 1.5\omega_\perp$, $\delta = -0.5\omega_\perp$.

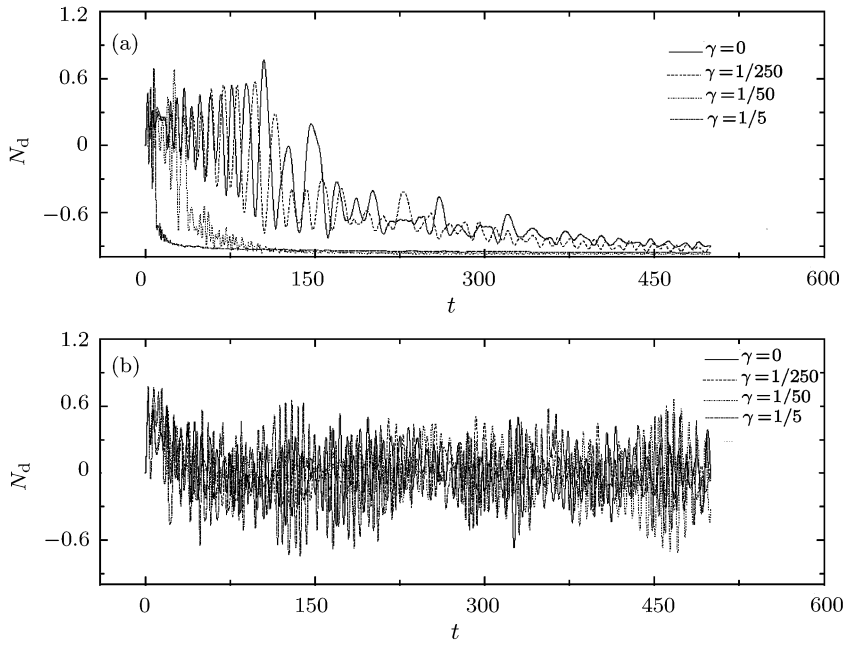


Fig.4. Under the gravitational field, temporal evolution of relative number of atoms N_d for initial Gaussian wavefunction in response to various aspect ratio of trapping potential γ. (a) Corresponding to attractive atomic interactions for the parameters as $G_{11} = G_{22} = 4G_{12} = -10$ and $\Omega = 1.5\omega_\perp$, $\delta = 0.5\omega_\perp$. (b) Corresponding to attractive atomic interactions for the parameters as $4G_{11} = 4G_{22} = G_{12} = -10$ and $\Omega = 1.5\omega_\perp$, $\delta = 0.5\omega_\perp$.

Another interesting case is to study the dynamics of Josephson-like tunnelling under the gravitational field, so $V(z)$ in Eq.(1) becomes $\gamma^2 z^2 + \lambda z$, where λ determines the strength of the gravitational field

and equals 0.839 in unit of $(1/2)\hbar\omega_\perp$. As shown in Fig.4(a), corresponding to attractive atomic interaction and $G_{11} = G_{22} = 4G_{12} = -10$, $\Omega = 1.5\omega_\perp$, $\delta = 0.5\omega_\perp$, we can see that the relative number of atoms is damped slower than the case without the gravitational field when γ is very small, but is damped with the same speed as the case without the gravitational field when γ is large as shown in Fig.1. In Fig.4(b), corresponding to attractive atomic interaction and $4G_{11} = 4G_{22} = G_{12} = -10$, $\Omega = 1.5\omega_\perp$, $\delta = 0.5\omega_\perp$, we can see the same phenomena as the case without the gravitational field as shown in Fig.2, except that the amplitude and frequency of oscillation of relative number of atoms have some differences.

When the atomic interaction becomes repulsive under the gravitational field, the dynamics of Josephson-like tunnelling is as shown in Fig.5. We can see that under the atomic interaction satisfying $G_{11} = G_{22} = 4G_{12} = -10$ corresponding to Fig.5(a), the dynamics of Josephson-like tunnelling under the gravitational field represents the same phenomena as the case without the gravitational field as shown in Fig.3(a), except that the amplitude and frequency of oscillation of relative number of atoms have some differences. But under the atomic interaction satisfying $4G_{11} = 4G_{22} = G_{12} = 10$ corresponding to Fig.5(b) and γ very small, the dynamics of Josephson-like tunnelling under the gravitational field is very different from the case without the gravitational field as shown in Fig.3(b).

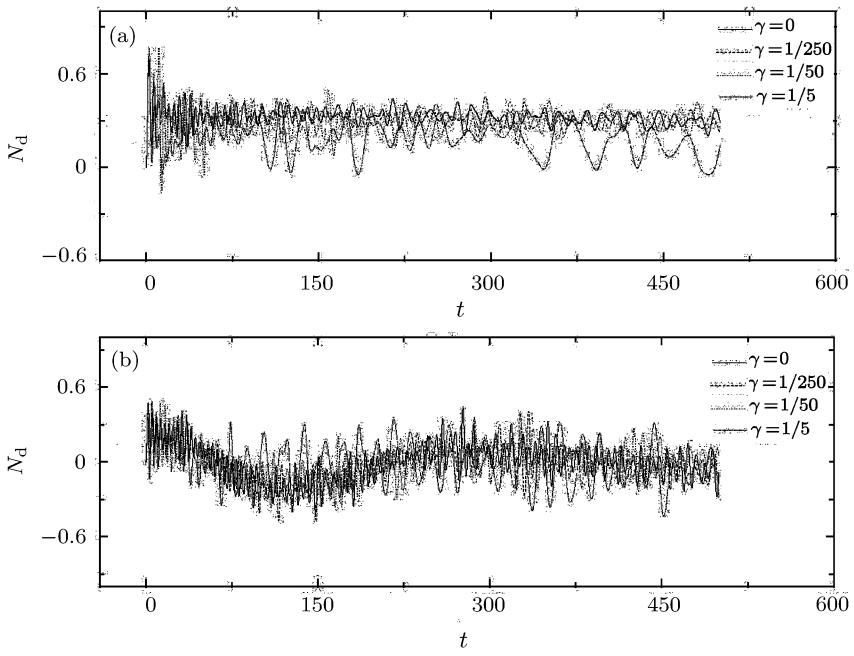


Fig.5. Under the gravitational field, temporal evolution of relative number of atoms N_d for initial Gaussian wavefunction in response to various aspect ratio of trapping potential γ. (a) Corresponding to attractive atomic interactions for the parameters as $G_{11} = G_{22} = 4G_{12} = 10$ and $\Omega = 1.5\omega_\perp$, $\delta = 0.5\omega_\perp$. (b) Corresponding to attractive atomic interactions for the parameters as $4G_{11} = 4G_{22} = G_{12} = 10$ and $\Omega = 1.5\omega_\perp$, $\delta = 0.5\omega_\perp$.

In conclusion, we find a robust way to control the dynamics of Josephson-like tunnelling from fast damped oscillations and asymmetric occupation to nondamped oscillation and symmetric occupation in two-component BECs coupled with microwave field. Recent developments of controlling the scattering length and realization of multi-component BECs in the experiments allow for the experimental investigation of our prediction in the future.

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