Vol 16 No 9, September 2007
1009-1963/2007/16(09)/2569-09

Chinese Physics

© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

The transfer of the quantum correlation from two-mode nonclassical state field to the supercurrents in two distant SQUID rings*

Peng Zhao-Hui(彭朝辉)[a)b)], Zou Jian(邹 健)[a)†], and Shao Bin(邵 彬)[a)]

[a)] *Department of Physics, Beijing Institute of Technology, Beijing* 100081, *China*

[b)] *Department of Physics, Hunan University of Science and Technology, Xiangtan* 411201, *China*

(Received 7 May 2006; revised manuscript received 14 March 2007)

We have considered two distant mesoscopic superconducting quantum interference device (SQUID) rings A and B in the presence of two-mode nonclassical state fields and investigated the correlation of the supercurrents in the two rings using the normalized correlation function $C_{\rm AB}$. We show that when the parameter α is very small for the separable state with the density matrix $\hat{\rho} = (|\alpha, -\alpha\rangle \langle\alpha, -\alpha| + |-\alpha, \alpha\rangle \langle -\alpha, \alpha|)/2$ and entangled coherent state (ECS) $|u\rangle = N_1(|\alpha, -\alpha\rangle + |-\alpha, \alpha\rangle)$ fields, the dynamic behaviours of the normalized correlation function $C_{\rm AB}$ are similar, but it is quite different for the entangled coherent state $|u'\rangle = N_2(|\alpha, -\alpha\rangle - |-\alpha, \alpha\rangle)$ field. When the parameter α is very large, the dynamic behaviours of $C_{\rm AB}$ are almost the same for the separable state, entangled coherent state $|u\rangle$ and $|u'\rangle$ fields. For the two-mode squeezed vacuum state field the maximum of $C_{\rm AB}$ increases monotonically with the squeezing parameter r, and as $r \to \infty$, $C_{\rm AB} \to 1$. This means that the supercurrents in the two rings A and B are quantum mechanically correlated perfectly. It is concluded that not all the quantum correlations in the two-mode nonclassical state field can be transferred to the supercurrents; and the transfer depends on the state of the two-mode nonclassical state field prepared.

Keywords: SQUID, normalized correlation function, two-mode nonclassical state field
PACC: 0365,7450

1. Introduction

The interaction of a mesoscopic Josephson junction (MJJ) with classical microwaves has been widely studied.[1] In the past few years the interaction between the Josephson junction system and the quantized field was discussed, and some interesting results have been obtained.[2] The superconducting quantum interference devices (SQUIDs) that include one or more MJJs exhibit quantum coherence at the macroscopic level,[3] and have been widely studied. In the past twenty years two-mode nonclassical state fields have been studied and produced experimentally.[4,5] An important feature of two-mode nonclassical state fields is the correlations between the two modes, and a lot of work has classified them into classical correlation (separable) and quantum correlation (entangled).[6] In the past few years the transfer of entanglement between different quantum systems was widely studied.[7−9] Kim *et al* have investigated the entanglement transfer from two-mode continuous-variable state field to a pair of qubits.[7] Recently the entanglement transfer from electrons to photons has been discussed.[8] Zhang *et al* have investigated the conditional transfer of the quantum correlation in the continuous-variable regime.[9] In Ref.[10], Tsomokos *et al* have considered two distant SQUID rings that are irradiated with two-mode nonclassical microwaves produced by a single source. The states of the microwaves they have considered are the number states and coherent states. It has been shown that the supercurrents in the two SQUID rings are correlated after interacting with the two-mode nonclassical microwaves, which irradiate these two rings with correlated photons.[10] It has also been shown that classically correlated (separable) and quantum mechanically correlated (entangled) photons induce different Josephson currents and different tunnelling statistics in the two devices. In Ref.[11], the transfer of the quantum correlation from a two-mode squeezed vacuum state field to the supercurrents in two SQUID rings has also been obtained.

In this paper we further investigate the effects of

*Project supported by the National Natural Science Foundation of China (Grant No 10374007).
†E-mail: zoujian@bit.edu.cn

different two-mode nonclassical fields on the correlation of the supercurrents in two SQUID rings A and B, and aim to investigate the transfer of the quantum correlation from different two-mode nonclassical state fields to the supercurrents. Following Ref.[10] we let the two-mode nonclassical state fields interact with two distant mesoscopic SQUID rings, and investigate the correlation of the supercurrents in the two SQUID rings with the normalized correlation function $C_{\rm AB}$.[12] We find that when α is very small, the dynamic behaviours of the normalized correlation function $C_{\rm AB}$ for the separable state and entangled coherent state (ECS)$|u\rangle$ fields are similar, but the dynamic behaviour for ECS$|u'\rangle$ field is quite different. We also find that when the parameter α is very large, the dynamic behaviours of $C_{\rm AB}$ for the separable state, ECS$|u\rangle$ and ECS$|u'\rangle$ fields are almost the same. For the two-mode squeezed vacuum state field the maximum of $C_{\rm AB}$ increases monotonically with the squeezing parameter r; and when $r \to \infty$, the maximum of $C_{\rm AB}$ is close to unity, which means that the two supercurrents are perfectly correlated. In other words, for the ECS$|u\rangle$ and ECS$|u'\rangle$ fields the quantum correlation cannot be transferred to the supercurrents when the parameter α is very large, but for the two-mode squeezed vacuum state field the quantum correlation can be sufficiently transferred to the supercurrents.

2. The interaction of a single SQUID ring with a single-mode quantized field

First we consider a single SQUID ring interacting with quantized fields, which are carefully prepared in a particular quantum state and are described by a density matrix $\hat{\rho}$. For the quantized electromagnetic field the vector potential $A_{\rm i}$ and the electric field $E_{\rm i}$ are dual quantum variables (operators). Then we can integrate these quantities over the SQUID ring and obtain the magnetic flux and the electromotive force:

$$\phi = \oint A_{\rm i}{\rm d}x_{\rm i}, \qquad V_{\rm EMF} = \oint E_{\rm i}{\rm d}x_{\rm i}. \tag{1}$$

We work in the external field approximation and neglect the back reaction from the SQUID ring on the external fields. In the Heisenberg picture the magnetic flux operator that a single-mode quantized field brings to the SQUID ring can evolve as[10]

$$\hat{\phi}(t) = \frac{\xi}{\sqrt{2}}[\hat{a}^{\dagger}\exp({\rm i}\omega_1 t) + \hat{a}\exp(-{\rm i}\omega_1 t)], \tag{2}$$

where ω_1 is the angular frequency of the electromagnetic field, $\hat{a}^{\dagger}(\hat{a})$ is the creation (annihilation) operator of the field and ξ is a parameter proportional to the area of the SQUID ring. Apart from the nonclassical electromagnetic field, we also impose a classical voltage V_0 which provides a classical magnetic flux $\phi_0 = V_0 t$. Consequently, the phase difference $\hat{\theta}(t)$ across the SQUID ring is an operator:

$$\begin{aligned}\hat{\theta}(t) &= 2e[\phi_0 + \hat{\phi}(t)] \\ &= \omega_0 t + q[\hat{a}^{\dagger}\exp({\rm i}\omega_1 t) + \hat{a}\exp(-{\rm i}\omega_1 t)],\end{aligned} \tag{3}$$

where $\omega_0 = 2eV_0$, $q = \sqrt{2}e\xi$. Therefore

$$\exp[{\rm i}\hat{\theta}(t)] = \exp({\rm i}\omega_0 t)D[{\rm i}q\exp({\rm i}\omega_1 t)], \tag{4}$$

where $D(\alpha) = \exp(\alpha\hat{a}^{\dagger} - \alpha^{*}\hat{a})$ is the displacement operator, and the supercurrent in the SQUID ring is also an operator[2,10]

$$\begin{aligned}\hat{I} &= I_0 \sin\hat{\theta}(t) \\ &= I_0 {\rm Im}\{\exp({\rm i}\omega_0 t)D[{\rm i}q\exp({\rm i}\omega_1 t)]\}.\end{aligned} \tag{5}$$

We can obtain the expectation value of the supercurrent $\langle\hat{I}\rangle$ by taking the trace of the density matrix $\hat{\rho}$, which describes the nonclassical electromagnetic field

$$\begin{aligned}\langle\hat{I}\rangle &= {\rm Tr}(\hat{\rho}\hat{I}) \\ &= I_0 {\rm Im}\{\exp({\rm i}\omega_0 t){\rm Tr}[\hat{\rho}D({\rm i}q\exp({\rm i}\omega_1 t))]\}.\end{aligned} \tag{6}$$

We can also obtain the second moments of supercurrent:

$$\begin{aligned}\langle\hat{I}^2\rangle &= {\rm Tr}(\hat{\rho}\hat{I}^2) \\ &= I_0^2 {\rm Tr}[\hat{\rho}\sin^2(\hat{\theta}(t))] \\ &= \frac{I_0^2}{2} - \frac{I_0^2}{2}{\rm Re}\{\exp({\rm i}2\omega_0 t){\rm Tr}[\hat{\rho}D({\rm i}2q\exp({\rm i}\omega_1 t))]\}.\end{aligned} \tag{7}$$

3. The interaction of two distant SQUID rings with two-mode quantized fields

Then following Ref.[10] we consider two distant mesoscopic SQUID rings A and B, which are irradiated by two-mode quantized fields. Let $\hat{\rho}$ be the density matrix of the two-mode quantized field, and $\hat{\rho}_{\rm A} = {\rm Tr}_{\rm B}\hat{\rho}$, $\hat{\rho}_{\rm B} = {\rm Tr}_{\rm A}\hat{\rho}$ are the reduced density matrices of the fields. Let modes A and B interact with

these two SQUID rings respectively. When $\hat{\rho}$ is factorizable as $\hat{\rho}_{\rm fact} = \hat{\rho}_{\rm A} \otimes \hat{\rho}_{\rm B}$, the two modes are not correlated. If it can be written as $\hat{\rho}_{\rm sep} = \sum_i p_i \hat{\rho}_{{\rm A}i} \otimes \hat{\rho}_{{\rm B}i}$, where p_i are probabilities, it is called separable and the two modes are classically correlated. Density matrices, which cannot be written in one of these two forms, are entangled (quantum mechanically correlated).

In this paper we also impose two classical bias voltages V_{10} and V_{20} on the two SQUID rings A and B respectively. Similar to the derivation of $\langle \hat{I} \rangle$ in the case of a single SQUID ring, we can gain the expectation values of the supercurrents in the two SQUID rings A and B,

$$\begin{aligned}\langle \hat{I}_A \rangle &= I_{10} {\rm Tr}(\hat{\rho}_{\rm A} \sin \hat{\theta}_{\rm A}) \\ &= I_{10} {\rm Im}\{\exp({\rm i}\omega_{10} t) {\rm Tr}[\hat{\rho}_{\rm A} D_1({\rm i}q \exp({\rm i}\omega_1 t))]\}\end{aligned} \tag{8}$$

and

$$\begin{aligned}\langle \hat{I}_B \rangle &= I_{20} {\rm Tr}(\hat{\rho}_{\rm B} \sin \hat{\theta}_{\rm B}) \\ &= I_{20} {\rm Im}\{\exp({\rm i}\omega_{20} t) {\rm Tr}[\hat{\rho}_{\rm B} D_2({\rm i}q \exp({\rm i}\omega_2 t))]\},\end{aligned} \tag{9}$$

where $\omega_{10} = 2eV_{10}$, $\omega_{20} = 2eV_{20}$, $I_{10}(I_{20})$ is the critical current in the SQUID ring A(B), $D_1(\alpha)(D_2(\alpha))$ is the displacement operator for mode A(B), and $\omega_1(\omega_2)$ is the angular frequency of mode A(B). The second moments of the supercurrents in the two SQUID rings are

$$\langle \hat{I}_{\rm A}^2 \rangle = {\rm Tr}(\hat{\rho}_{\rm A} \hat{I}_{\rm A}^2) = \frac{I_{10}^2}{2} - \frac{I_{10}^2}{2} {\rm Re}\{\exp({\rm i}2\omega_{10} t) {\rm Tr}[\hat{\rho}_{\rm A} D_1({\rm i}2q \exp({\rm i}\omega_1 t))]\} \tag{10}$$

and

$$\langle \hat{I}_{\rm B}^2 \rangle = {\rm Tr}(\hat{\rho}_{\rm B} \hat{I}_B^2) = \frac{I_{20}^2}{2} - \frac{I_{20}^2}{2} {\rm Re}\{\exp({\rm i}2\omega_{20} t) {\rm Tr}[\hat{\rho}_{\rm B} D_2({\rm i}2q \exp({\rm i}\omega_2 t))]\}. \tag{11}$$

The expectation value of the product of the two supercurrent operators is given by

$$\langle \hat{I}_{\rm A} \hat{I}_{\rm B} \rangle = I_{10} I_{20} {\rm Tr}(\hat{\rho} \sin \hat{\theta}_{\rm A} \sin \hat{\theta}_{\rm B}). \tag{12}$$

In this paper we use the normalized correlation function $C_{\rm AB}$[12] to investigate the correlation of supercurrents in the two SQUID rings for the separable state, two ECS($|u\rangle$ and $|u'\rangle$) state and two-mode squeezed vacuum state fields. From Eqs.(8)–(12) we can obtain the normalized correlation function of the supercurrents in the two SQUID rings:

$$C_{\rm AB} = \frac{\langle \hat{I}_{\rm A} \hat{I}_{\rm B} \rangle - \langle \hat{I}_{\rm A} \rangle \langle \hat{I}_{\rm B} \rangle}{\sqrt{\langle \hat{I}_{\rm A}^2 \rangle - \langle \hat{I}_{\rm A} \rangle^2} \sqrt{\langle \hat{I}_{\rm B}^2 \rangle - \langle \hat{I}_{\rm B} \rangle^2}}, \tag{13}$$

which varies between -1 (perfect anti-correlation) and 1 (perfect correlation). For factorizable density matrices, $\rho_{\rm fact} = \rho_{\rm A} \otimes \rho_{\rm B}$, we have $C_{\rm AB} = 0$.

4. Correlation of the supercurrents in two distant SQUID rings in the presence of a separable state field

Now we consider the separable state field with the density matrix

$$\hat{\rho}_{\rm sep} = \frac{1}{2}(|\alpha, -\alpha\rangle\langle\alpha, -\alpha| + |-\alpha, \alpha\rangle\langle -\alpha, \alpha|), \tag{14}$$

where $|\alpha\rangle$ and $|-\alpha\rangle$ are coherent states. For microwaves in the separable state of Eq.(14) the reduced density matrices are

$$\hat{\rho}_{\rm sep,A} = \hat{\rho}_{\rm sep,B} = \frac{1}{2}(|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle -\alpha|). \tag{15}$$

From Eqs.(8), (9) and (15), the expectation values of the supercurrents in the rings A and B are

$$\left\langle \hat{I}_{\rm A} \right\rangle_{\rm sep} = \frac{I_{10}}{2} \exp\left(-\frac{q^2}{2}\right) \{\sin[\omega_{10} t + 2q|\alpha| \cos(\omega_1 t - \theta_1)] + \sin[\omega_{10} t - 2q|\alpha| \cos(\omega_1 t - \theta_1)]\} \tag{16}$$

and

$$\langle \hat{I}_{\rm B}\rangle_{\rm sep} = \frac{I_{20}}{2}\exp\left(-\frac{q^2}{2}\right)\{\sin[\omega_{20}t + 2q\,|\alpha|\cos(\omega_2 t-\theta_1)] + \sin[\omega_{20}t - 2q\,|\alpha|\cos(\omega_2 t-\theta_1)]\}, \quad (17)$$

where $\theta_1 = \arg(\alpha)$. From Eqs.(10), (11) and (15), we can gain

$$\langle \hat{I}^2_{\rm A}\rangle_{\rm sep} = \frac{I^2_{10}}{2} - \frac{I^2_{10}}{4}\exp\left(-2q^2\right)\{\cos[2\omega_{10}t + 4q\,|\alpha|\cos(\omega_1 t-\theta_1)] + \cos[2\omega_{10}t - 4q\,|\alpha|\cos(\omega_1 t-\theta_1)]\} \quad (18)$$

and

$$\langle \hat{I}^2_{\rm B}\rangle_{\rm sep} = \frac{I^2_{20}}{2} - \frac{I^2_{20}}{4}\exp\left(-2q^2\right)\{\cos[2\omega_{20}t + 4q\,|\alpha|\cos(\omega_2 t-\theta_1)] + \cos[2\omega_{20}t - 4q\,|\alpha|\cos(\omega_2 t-\theta_1)]\}. \quad (19)$$

From Eqs.(12) and (14)，we can also have

$$\begin{aligned}\langle \hat{I}_{\rm A}\hat{I}_{\rm B}\rangle_{\rm sep} = {} & \frac{I_{10}I_{20}}{2}\exp(-q^2)\{\sin[\omega_{10}t + 2q\,|\alpha|\cos(\omega_1 t-\theta_1)]\sin[\omega_{20}t - 2q\,|\alpha|\cos(\omega_2 t-\theta_1)] \\ & + \sin[\omega_{10}t - 2q\,|\alpha|\cos(\omega_1 t-\theta_1)]\sin[\omega_{20}t + 2q\,|\alpha|\cos(\omega_2 t-\theta_1)]\}. \end{aligned} \quad (20)$$

In Fig.1 we plot the normalized correlation function $C_{\rm AB}$ against the parameter α and the scaled time ωt for the separable state field. For simplicity, we let $k_{\rm B} = \hbar = c = q = 1$, $\omega_{10} = \omega_{20} = \omega_1 = \omega_2 \equiv \omega$ and $\theta_1 = 0$.

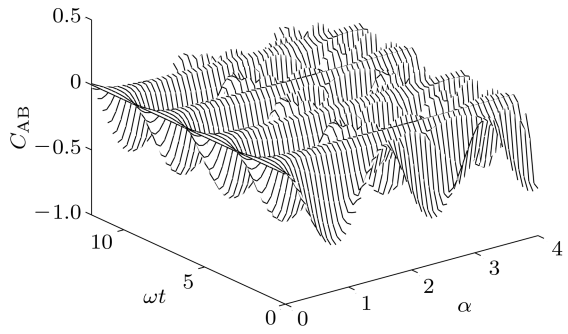


Fig.1. A plot of the normalized correlation function $C_{\rm AB}$ for the separable state field against the parameter α and the scaled time ωt for $\theta_1 = 0$, $\omega_{10} = \omega_{20} = \omega_1 = \omega_2 \equiv \omega$, and $k_{\rm B} = \hbar = c = q = 1$.

5. Correlation of the supercurrents in two distant SQUID rings in the presence of an ECS field

We consider the ECS$|u\rangle = N_1(|\alpha,\ -\alpha\rangle + |-\alpha,\ \alpha\rangle)$,[5] with the density matrix

$$\begin{aligned}\hat{\rho}_u = {} & 2N_1^2\hat{\rho}_{\rm sep} + N_1^2(|\alpha,-\alpha\rangle\langle -\alpha,\alpha| \\ & + |-\alpha,\alpha\rangle\langle\alpha,-\alpha|), \end{aligned} \quad (21)$$

where the normalization constant N_1 is $[2+2E_1]^{-\frac{1}{2}}$ with $E_1 = \exp(-4\,|\alpha|^2)$. For microwaves in the state of Eq.(21) the reduced density matrices are

$$\begin{aligned}\hat{\rho}_{\rm A} = {} & \hat{\rho}_{\rm B} \\ = {} & N_1^2(|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle -\alpha| \\ & + \tau\,|\alpha\rangle\langle -\alpha| + \tau^*\,|-\alpha\rangle\langle\alpha|) \\ = {} & 2N_1^2\hat{\rho}_{{\rm sep},A} + N_1^2\tau\,|\alpha\rangle\langle -\alpha| \\ & + N_1^2\tau^*\,|-\alpha\rangle\langle\alpha|, \end{aligned} \quad (22)$$

where $\tau = \langle\alpha\mid -\alpha\rangle = \exp(-2\,|\alpha|^2) = \tau^*$. The degree of entanglement of the ECS$|u\rangle$ can be expressed as[5]

$$E(\alpha) = -\lambda_1\log\lambda_1 - \lambda_2\log\lambda_2, \quad (23)$$

with $\lambda_1 = \dfrac{(1+\tau)^2}{2(1+\tau^2)}$ and $\lambda_2 = \dfrac{(1-\tau)^2}{2(1+\tau^2)}$. We can also express the entanglement in terms of concurrence[5]

$$C = \frac{1-\tau^2}{1+\tau^2}. \quad (24)$$

We can see that the degree of entanglement for ECS$|u\rangle$ increases with the increase of the parameter α, and as $\alpha\to\infty$, we have $C\to 1$.

From Eqs.(8) and (22), the expectation value of the supercurrent in the SQUID ring A is

$$\langle \hat{I}_{\rm A}\rangle = 2N_1^2\left\langle \hat{I}_{\rm A}\right\rangle_{\rm sep} + N_1^2\exp\left(-\frac{q^2}{2}\right)E_1F_1I_{10}, \quad (25)$$

where $F_1 = [\exp(2q\,|\alpha|\,S_{{\rm A},1}) + \exp(-2q\,|\alpha|\,S_{{\rm A},1})]\sin(\omega_{10}t)$ with $S_{{\rm A},1} = \sin(\omega_1 t-\theta_1)$. Similarly, we can obtain the expectation value of supercurrent in the SQUID ring B,

$$\langle \hat{I}_{\rm B}\rangle = 2N_1^2\left\langle \hat{I}_{\rm B}\right\rangle_{\rm sep} + N_1^2\exp\left(-\frac{q^2}{2}\right)E_1F_2I_{20}, \quad (26)$$

where $F_2 = [\exp(2q|\alpha|S_{\mathrm{A},2}) + \exp(-2q|\alpha|S_{\mathrm{A},2})]\sin(\omega_{20}t)$ with $S_{\mathrm{A},2} = \sin(\omega_2 t - \theta_1)$. From Eqs.(10), (11) and (22), we have

$$\langle \hat{I}_{\mathrm{A}}^2 \rangle = 2N_1^2 \left\langle \hat{I}_{\mathrm{A}}^2 \right\rangle_{\mathrm{sep}} + N_1^2 E_1 F_3 I_{10}^2, \tag{27}$$

where $F_3 = 1 - \frac{\exp(-2q^2)}{2}\cos(2\omega_{10}t)[\exp(4q|\alpha|S_{\mathrm{A},1}) + \exp(-4q|\alpha|S_{\mathrm{A},1})]$, and

$$\langle \hat{I}_{\mathrm{B}}^2 \rangle = 2N_1^2 \left\langle \hat{I}_{\mathrm{B}}^2 \right\rangle_{\mathrm{sep}} + N_1^2 E_1 F_4 I_{20}^2, \tag{28}$$

with $F_4 = 1 - \frac{\exp(-2q^2)}{2}\cos(2\omega_{20}t)[\exp(4q|\alpha|S_{\mathrm{A},2}) + \exp(-4q|\alpha|S_{\mathrm{A},2})]$. From Eqs.(12) and (21), we can also obtain

$$\begin{aligned}\langle \hat{I}_{\mathrm{A}}\hat{I}_{\mathrm{B}} \rangle = {} & 2N_1^2 \left\langle \hat{I}_{\mathrm{A}}\hat{I}_{\mathrm{B}} \right\rangle_{\mathrm{sep}} \\ & + N_1^2 \mathrm{Tr}(|\alpha,-\alpha\rangle\langle -\alpha,\alpha| \hat{I}_{\mathrm{A}}\hat{I}_{\mathrm{B}}) \\ & + N_1^2 \mathrm{Tr}(|-\alpha,\alpha\rangle\langle \alpha,-\alpha| \hat{I}_A\hat{I}_{\mathrm{B}}).\end{aligned} \tag{29}$$

It can be seen that the last two terms in Eq.(29) are conjugate, so we only need calculate one of them. Using the following formulas

$$\sin\hat{\theta}_{\mathrm{A}} = \frac{1}{2\mathrm{i}}(\mathrm{e}^{\mathrm{i}\hat{\theta}_{\mathrm{A}}} - \mathrm{e}^{-\mathrm{i}\hat{\theta}_{\mathrm{A}}}) \tag{30}$$

and

$$D(\alpha)D(\beta) = \exp\left[\frac{1}{2}(\alpha\beta^* - \alpha^*\beta)\right] D(\alpha+\beta), \tag{31}$$

we can obtain

$$\langle -\alpha|\sin\hat{\theta}_{\mathrm{A}}|\alpha\rangle = \frac{1}{2\mathrm{i}}\exp\left(-\frac{q^2}{2} - 2|\alpha|^2\right)\{\exp(\mathrm{i}\omega_{10}t + 2q|\alpha|S_{\mathrm{A},1}) - \exp(-\mathrm{i}\omega_{10}t - 2q|\alpha|S_{\mathrm{A},1})\}. \tag{32}$$

Then we can write Eq.(29) as follows:

$$\langle \hat{I}_{\mathrm{A}}\hat{I}_{\mathrm{B}} \rangle = 2N_1^2 \left\langle \hat{I}_{\mathrm{A}}\hat{I}_{\mathrm{B}} \right\rangle_{\mathrm{sep}} + N_1^2 \frac{I_{10}I_{20}}{2}\exp(-q^2)E_1F_5, \tag{33}$$

where

$$\begin{aligned}F_5 = {} & \{-\cos(\omega_{10}t + \omega_{20}t)[\exp(2q|\alpha|S_{\mathrm{A},1} - 2q|\alpha|S_{\mathrm{A},2}) + \exp(2q|\alpha|S_{\mathrm{A},2} - 2q|\alpha|S_{\mathrm{A},1})] \\ & + \cos(\omega_{10}t - \omega_{20}t)[\exp(2q|\alpha|S_{\mathrm{A},1} + 2q|\alpha|S_{\mathrm{A},2}) + \exp(-2q|\alpha|S_{\mathrm{A},1} - 2q|\alpha|S_{\mathrm{A},2})]\}.\end{aligned}$$

Now we consider another ECS $|u'\rangle = N_2(|\alpha, -\alpha\rangle - |-\alpha,\alpha\rangle)$,[5] with the density matrix

$$\begin{aligned}\hat{\rho}_{u'} = {} & 2N_2^2\hat{\rho}_{\mathrm{sep}} - N_2^2(|\alpha,-\alpha\rangle\langle -\alpha,\alpha| \\ & + |-\alpha,\alpha\rangle\langle\alpha,-\alpha|),\end{aligned} \tag{34}$$

where $N_2 = [2-2E_1]^{-\frac{1}{2}}$. It has been pointed out that the degree of entanglement of this state is independent of the parameters involved and the concurrence of this state is $C = 1$, which means that the entanglement is exactly one ebit.[5] Similar to the derivation in the case of the ECS$|u\rangle$, we can obtain the normalized correlation function C_{AB} for the ECS$|u'\rangle$ easily.

In Figs.2 and 3 we plot the normalized correlation function C_{AB} against the parameter α and the scaled time ωt for the two ECS fields, and for simplicity we let $\omega_{10} = \omega_{20} = \omega_1 = \omega_2 \equiv \omega$, $\theta_1 = 0$ and $k_{\mathrm{B}} = \hbar = c = q = 1$. From Figs.1–3 we can see that when α is very small, the dynamic behaviours of the normalized correlation function C_{AB} for the separable state and ECS$|u\rangle$ fields are very similar, but for the ECS$|u'\rangle$ its dynamic behaviour is quite different, and when $\alpha \sim 1$, the normalized correlation functions C_{AB} for the separable state, ECS$|u\rangle$ and ECS$|u'\rangle$ fields are quite different. From Figs.1–3 we can also see that the dynamic behaviours of C_{AB} are almost the same for the separable state, ECS$|u\rangle$ and ECS$|u'\rangle$ fields when α is very large.

We can simply analyse the above results as follows. When α is very small, we can expand the coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ in the first order of α

$$|\alpha\rangle \approx |0\rangle + \alpha|1\rangle, |-\alpha\rangle \approx |0\rangle - \alpha|1\rangle. \tag{35}$$

So we can write the ECS$|u\rangle$ approximately as $|u\rangle \approx |0,0\rangle$, which is a product state obviously. We can also obtain the same result for the separable state. So when α is very small, the ECS$|u\rangle$ and the separable state fields are almost factorizable, and the correla-

tions of the supercurrents in the two SQUID rings transferred from these two fields are close to zero. When α is very small, we can also write the ECS$|u'\rangle$ approximately

$$|u'\rangle \approx \frac{1}{\sqrt{2}}(|1,0\rangle - |0,1\rangle) = \left|\psi^-\right\rangle . \tag{36}$$

In this case the ECS$|u'\rangle$ is very close to the Bell state $|\psi^-\rangle$. As we know, the correlation between two modes of the Bell state $|\psi^-\rangle$ is the quantum correlation, so the correlation of the supercurrents transferred from the ECS$|u'\rangle$ field might be the quantum correlation. As we know, the degree of entanglement of the ECS$|u\rangle$ increases with the increase of the parameter α, and in Fig.2 we can see that $C_{\rm AB}$ increases with the increase of the parameter α, which is different from that for the separable state. In this case the correlation of the supercurrents might be the quantum correlation.

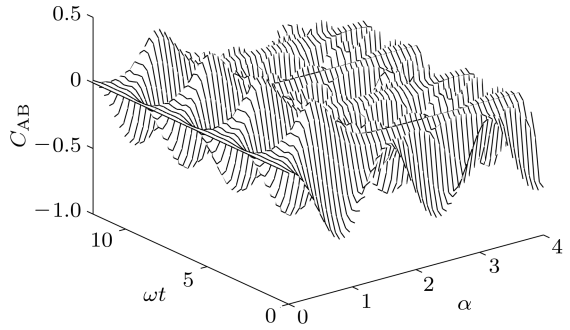


Fig.2. A plot of the normalized correlation function $C_{\rm AB}$ for the ECS$|u\rangle = N_1(|\alpha,-\alpha\rangle + |-\alpha,\alpha\rangle)$ field against the parameter α and the scaled time ωt for $\omega_{10} = \omega_{20} = \omega_1 = \omega_2 \equiv \omega$, $\theta_1 = 0$ and $k_{\rm B} = \hbar = c = q = 1$.

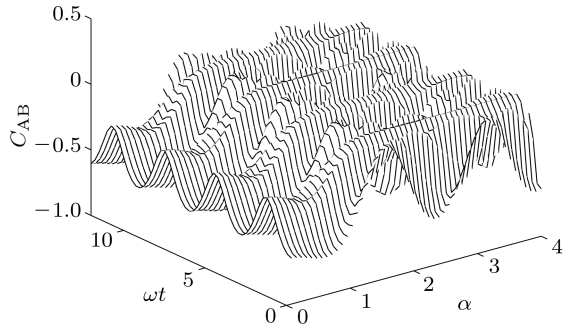


Fig.3. A plot of the normalized correlation function $C_{\rm AB}$ for the ECS$|u'\rangle = N_2(|\alpha,-\alpha\rangle - |-\alpha,\alpha\rangle)$ field against the parameter α and the scaled time ωt for $\omega_{10} = \omega_{20} = \omega_1 = \omega_2 \equiv \omega$, $\theta_1 = 0$ and $k_{\rm B} = \hbar = c = q = 1$.

We can see that Eqs.(25)–(28) and (33) are composed of two terms. One term is the same as that for the separable state, and the other term includes F_1, F_2, F_3, F_4, F_5 and E_1. When the parameter α is very large, F_1, F_2, F_3, F_4, F_5 and E_1 are close to zero, and the difference among the results for the separable state, ECS$|u\rangle$ and ECS$|u'\rangle$ can be neglected. It is noted that when α is very large, for the ECS$|u\rangle$ the concurrence is close to unity, and for the ECS$|u'\rangle$ it is always equal to unity. But the correlations of supercurrents transferred from these two ECS fields are almost the same as that transferred from the separable state field, so they must be the classical correlation. This means that the transfer of the quantum correlation is not efficient when α is very large, and as $\alpha \to \infty$, it tends to zero.

In Fig.4 we plot the normalized correlation functions $C_{\rm AB}$ against the parameter α at $\omega t \approx 1.7$ for the separable state, ECS$|u\rangle$ and ECS$|u'\rangle$ fields respectively. For simplicity, we let $\omega_{10} = \omega_{20} = \omega_1 = \omega_2 \equiv \omega$, $\theta_1 = 0$ and $k_{\rm B} = \hbar = c = q = 1$. From Fig.4(a) we can see that $C_{\rm AB}$ is about zero for the separable state no matter what the value of α is, which means that there is no correlation transferred from the separable state field to the supercurrents at this time. As we know, there are only diagonal terms in Eq.(14) (the separable state), but besides the diagonal terms there are non-diagonal terms in Eqs.(21) (the ECS$|u\rangle$) and (34) (the ECS$|u'\rangle$). From Fig.4(a) we can see that the diagonal terms in Eq.(14) have no contribution to the correlation of the supercurrents at this time. The nor-

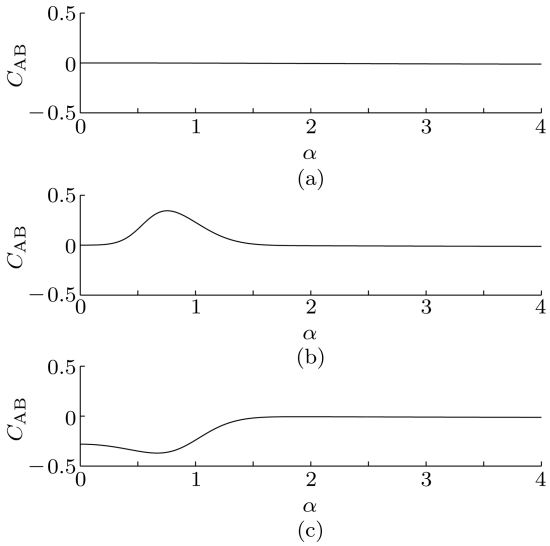


Fig.4. Plots of the normalized correlation functions $C_{\rm AB}$ against the parameter α at $\omega t = 1.7$ for (a) the separable state, (b) ECS$|u\rangle = N_1(|\alpha,-\alpha\rangle + |-\alpha,\alpha\rangle)$ and (c) ECS$|u'\rangle = N_2(|\alpha,-\alpha\rangle - |-\alpha,\alpha\rangle)$ at $\omega_{10} = \omega_{20} = \omega_1 = \omega_2 \equiv \omega$, $\theta_1 = 0$ and $k_{\rm B} = \hbar = c = q = 1$.

malized correlation function $C_{\rm AB} \neq 0$ in the case of $\alpha \sim 1$ for the ECS$|u\rangle$ field and small α for the ECS$|u'\rangle$ field is just the contribution from the non-diagonal terms in Eqs.(21) and (34). So the correlation of the supercurrents for the ECS$|u\rangle$ ($|u'\rangle$) field at this time is the quantum correlation. From Figs.4(b) and 4(c) we can see that as $\alpha \to \infty$, $C_{\rm AB}$ is almost equal to zero at this time. When $\alpha \to \infty$, the contributions of the non-diagonal terms in Eqs.(21) and (34) to the correlation of the supercurrents are close to zero, and then the differences among the separable state, ECS$|u\rangle$ and ECS$|u'\rangle$ can be neglected.

6. Correlation of the supercurrents in two distant SQUID rings in the presence of a two-mode squeezed vacuum state field

A two-mode squeezed vacuum state that can be generated using a non-degenerate optical parametric amplifier is an entangled continuous-variable state.[13] The two-mode squeezed vacuum state can be defined as follows:

$$\hat{S}_{12}(r)\,|0,0\rangle = \frac{1}{\cosh r}\sum_{n=0}^{\infty}(\tanh r)^n\,|n,n\rangle\,, \quad (37)$$

where $\hat{S}_{12}(r) = \exp(r\hat{a}_1^{\dagger}\hat{a}_2^{\dagger} - r^*\hat{a}_1\hat{a}_2)$ with $r = |r|\exp(\mathrm{i}\vartheta)$. Here $\hat{a}_1$ and $\hat{a}_2$ ($\hat{a}_1^{\dagger}$ and $\hat{a}_2^{\dagger}$) are bosonic annihilation (creation) operators and the parameter r is a measure of the amount of squeezing. For $r \to \infty$ the two-mode squeezed vacuum state evolves in the same way as the initial EPR state and it is maximally squeezed and fully entangled.[13] The degree of entanglement can be expressed as[14]

$$\begin{aligned} E(r) \;=\; & \cosh^2 r\log(\cosh^2 r) \\ & -\sinh^2 r\log(\sinh^2 r). \end{aligned} \quad (38)$$

In the previous work[11] we have used the ratio $R = \langle\hat{I}_{\rm A}\hat{I}_{\rm B}\rangle/\langle\hat{I}_{\rm A}\rangle\langle\hat{I}_{\rm B}\rangle$ to investigate the correlation of the supercurrents in two SQUID rings A and B. We have found that the correlation of the supercurrents increases with the degree of entanglement of the two-mode squeezed vacuum state field, but the ratio R is divergent at some time and as $r \to \infty$, $R \to \infty$. In order to exhibit the correlation of supercurrents in the two SQUID rings A and B more explicitly, in this paper we reinvestigate the correlation of the supercurrents against the normalized correlation function $C_{\rm AB}$. In Ref.[11] we have obtained the expectation value of the supercurrents in the SQUID rings A and B:

$$\langle\hat{I}_{\rm A}\rangle = I_{10}\exp\left[-\frac{q^2}{2}(\cosh^2|r| + \sinh^2|r|)\right]\sin(\omega_{10}t) \quad (39)$$

and

$$\langle\hat{I}_{\rm B}\rangle = I_{20}\exp\left[-\frac{q^2}{2}(\cosh^2|r| + \sinh^2|r|)\right]\sin(\omega_{20}t). \quad (40)$$

Also in Ref.[11] we have obtained the expectation value of the product of the two supercurrents operators:

$$\begin{aligned} \langle\hat{I}_{\rm A}\hat{I}_{\rm B}\rangle \;=\; & \frac{I_{10}I_{20}}{2}\exp[-q^2(\cosh^2|r| + \sinh^2|r|)] \\ & \times\{\cos(\omega_{10}t - \omega_{20}t)B_1 \\ & - \cos(\omega_{10}t + \omega_{20}t)/B_1\}, \end{aligned} \quad (41)$$

with $B_1 = \exp\{2q^2\sinh|r|\cosh|r|\cos[(\omega_1+\omega_2)t - \vartheta]\}$. From Eqs.(10), (11) and (37), we can obtain the second moments of the supercurrents in the SQUID rings A and B:

$$\begin{aligned} \langle\hat{I}_{\rm A}^2\rangle \;=\; & \frac{I_{10}^2}{2} - \frac{I_{10}^2}{2}\exp[-2q^2(\sinh^2|r| + \cosh^2|r|)] \\ & \times\cos(2\omega_{10}t) \end{aligned} \quad (42)$$

and

$$\begin{aligned} \langle\hat{I}_{\rm B}^2\rangle \;=\; & \frac{I_{20}^2}{2} - \frac{I_{20}^2}{2}\exp[-2q^2(\sinh^2|r| + \cosh^2|r|)] \\ & \times\cos(2\omega_{20}t). \end{aligned} \quad (43)$$

In Fig.5 we plot the normalized correlation function $C_{\rm AB}$ against the squeezing parameter r and the scaled time ωt for the two-mode squeezed vacuum state field. For simplicity, we let $\omega_{10} = \omega_{20} = \omega_1 = \omega_2 \equiv \omega$, $\vartheta = 0$ and $k_{\rm B} = \hbar = c = q = 1$. From Fig.5 we can see that the maximum of $C_{\rm AB}$ increases monotonically with the squeezing parameter r, and as $r \to \infty$, $C_{\rm AB} \to 1$. This means that in the case of large r the supercurrents in the two SQUID rings are almost perfectly correlated. From Eqs.(39)–(43) we can see that as $r \to \infty$, $\langle\hat{I}_{\rm A}\rangle \to 0$, $\langle\hat{I}_{\rm B}\rangle \to 0$, $\langle\hat{I}_{\rm A}^2\rangle/I_{10}^2 \to 0.5$, $\langle\hat{I}_{\rm B}^2\rangle/I_{20}^2 \to 0.5$, and the maximum of $\langle\hat{I}_{\rm A}\hat{I}_{\rm B}\rangle/I_{10}I_{20}$ is 0.5. Therefore, the maximum of $C_{\rm AB}$ approaches unity when r is very large. It is pointed out that the degree of entanglement of the two-mode squeezed vacuum state increases with the squeezing parameter r too. This means that the correlation of the supercurrents increases with the degree of the entanglement of the field, and the supercurrents in the

two SQUID rings are perfectly quantum mechanically correlated as $r \to \infty$. The entanglement transfer from a continuous-variable state field to two qubits has been investigated and the two-mode squeezed vacuum state field is generally used as the source of entanglement.[7] In Ref.[7] it has been found that the degree of entanglement between two qubits is not a simple increasing function of the parameter r and it decreases as r gets larger after its peak, and the entanglement transferred from the continuous-variable state fields to the two qubits approaches zero as $r \to \infty$. It is noticed that in this paper what we want to investigate is the transfer of the quantum correlation from the continuous-variable state field to the supercurrents, which is also a continuous-variable. So the transfer of entanglement from a continuous-variable system to another continuous-variable system is different from that from a continuous-variable system to a discrete variable system.

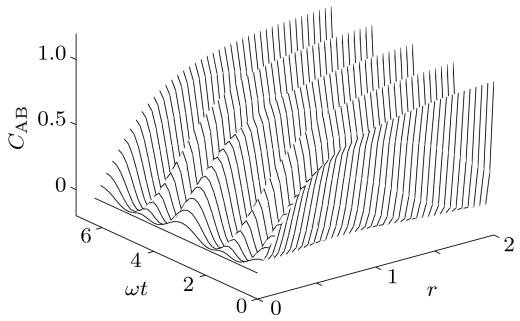


Fig.5. A plot of the normalized correlation function $C_{\rm AB}$ for the two-mode squeezed vacuum state field against the squeezing parameter r and the scaled time ωt at $\omega_{10} = \omega_{20} = \omega_1 = \omega_2 \equiv \omega$, $\vartheta = 0$ and $k_{\rm B} = \hbar = c = q = 1$.

7. Conclusions

In this paper we have considered two distant mesoscopic SQUID rings A and B in the presence of the two-mode nonclassical state fields, and found that the quantum correlation and classical correlation can be transferred from the two-mode nonclassical state fields to the supercurrents in two SQUID rings. We have shown that when the parameter α is very small for the separable state and ECS$|u\rangle$ fields, the dynamic behaviours of the normalized correlation function $C_{\rm AB}$ are very similar, but quite different from that for the ECS$|u'\rangle$ field. When the parameter α is very large, the dynamic behaviours of $C_{\rm AB}$ are almost the same for the separable state, ECS$|u\rangle$ and ECS$|u'\rangle$ fields. For the two-mode squeezed vacuum state field the maximum of $C_{\rm AB}$ increases monotonically with the squeezing parameter r, and as $r \to \infty$, $C_{\rm AB} \to 1$. As $\alpha \to 0$, the ECS$|u'\rangle$ is close to the Bell state $|\psi^-\rangle$ which is a maximally entangled discrete variable state, and the correlation between its two modes is completely the quantum correlation. So the correlation of the supercurrents transferred from the field is the quantum correlation in this case. As α is very large, the ECS$|u'\rangle$ is a continuous-variable state and the degree of entanglement is still one ebit. But the dynamic behaviours of the normalized correlation functions $C_{\rm AB}$ are almost the same for the separable state and ECS$|u'\rangle$ state, so the quantum correlation has not been transferred from the ECS$|u'\rangle$ field to the supercurrents. The two-mode squeezed vacuum state is also an entangled continuous-variable state, but the degree of entanglement increases with r. The maximum of the normalized correlation function $C_{\rm AB}$ also increases with r, and the supercurrents in the two SQUID rings are perfectly quantum mechanically correlated as $r \to \infty$. In summary, not all the quantum correlation in the two-mode nonclassical state field can be transferred to the supercurrents. This depends on the state of the two-mode nonclassical state field. For the maximally entangled discrete variable state field (the Bell state field), the quantum correlation can be transferred from the field to the supercurrents. For the entangled continuous-variable state field, the ECS$|u'\rangle$ and ECS$|u\rangle$ fields in the case of large α, the quantum correlation cannot be transferred to the supercurrents. But for the entangled continuous-variable state field (the two-mode squeezed vacuum state field) the quantum correlation can be transferred to the supercurrents sufficiently. It has been shown that the correlation of supercurrents depends on the state of the two-mode nonclassical state field, so the correlation of supercurrents can reflect the correlation between the two modes in the two-mode nonclassical state field.

References

[1] Schön G and Zaikin A D 1990 *Phys. Rep.* **198** 237
Kastner M A 1992 *Rev. Mod. Phys.* **64** 849
Koch H and Lubbing H 1992 *Single Electron Tunneling and Mesoscopic Devices* (Berlin: Springer)

[2] Vourdas A 1994 *Phys. Rev.* B **49** 12040
Kuang L M, Wang Y and Ge M L 1996 *Phys. Rev.* B **53** 11764
Zou J, Shao B and Xing X S 1997 *Phys. Rev.* B **56** 14116
Zhan Y B 2005 *Chin. Phys.* **14** 1044
Zhan Y B 2004 *Chin. Phys.* **13** 234
Shao B and Zou J 1999 *Chin. Phys.* **8** 368

[3] Byers N and Yang C N 1961 *Phys. Rev. Lett.* **7** 46
Bloch F 1970 *Phys. Rev.* B **2** 109

[4] Reid M D and Drummond P D 1988 *Phys. Rev. Lett.* **60** 2731
Reid M D 1989 *Phys. Rev.* A **40** 913
Ou Z Y, Pereira S F, Kimble H J and Peng K C 1992 *Phys. Rev. Lett.* **68** 3663
Scheel S and Welsch D G 2001 *Phys. Rev.* A **64** 063811

[5] Sanders B C 1992 *Phys. Rev.* A **45** 6811
Van Enk S J and Hirota O 2001 *Phys. Rev.* A **64** 022313
Wang X G 2002 *J. Phys.* A **35** 165

[6] Werner R F 1989 *Phys. Rev.* A **40** 4277
Vedral V, Plenio M B, Rippin M A and Knight P L 1997 *Phys. Rev. Lett.* **78** 2275
Vedral V 2002 *Rev. Mod. Phys.* **74** 197

[7] Paternostro M, Son W and Kim M S 2004 *Phys. Rev. Lett.* **92** 197901
Paternostro M, Son W, Kim M S, Falci G and Palma G M 2004 *Phys. Rev.* A **70** 022320
Son M, Kim M S, Lee J and Ahn D 2002 *J. Mod. Opt.* **49** 1739

[8] Titov M, Trauzettel B, Michaelis B and Beenakker C W J 2005 cond-mat/0503676
Cerletti V, Gywat O and Loss D 2004 cond-mat/0411235

[9] Zhang Y, Hayasaka K and Kasai K 2005 quant-ph/0504032

[10] Tsomokos D I, Chong C C and Vourdas A 2004 *J. Phys.: Condens. Matter* **16** 9169

[11] Peng Z H, Zou J, Shao B and Cai J F 2005 *Int. J. Mod. Phys.* B **19** 1965

[12] Treps N and Fabre C 2004 quant-ph/0407214

[13] Braunstein S L and Loock P V 2005 *Rev. Mod. Phys.* **77** 513
Walls D F and Milburn G J 1994 *Quantum Optics* (Berlin: Springer)

[14] Van Enk S J 1999 *Phys. Rev.* A **60** 5095