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Controlled teleportation of high-dimension quantum-states with generalized Bell-state measurement*

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In this paper a scheme for controlled teleportation of arbitrary high-dimensional unknown quantum states is proposed by using the generalized Bell-basis measurement and the generalized Hadamard transformation. As two special cases, two schemes of controlled teleportation of an unknown single-qutrit state and an unknown two-qutrit state are investigated in detail. In the first scheme, a maximally entangled three-qutrit state is used as the quantum channel, while in the second scheme, an entangled two-qutrit state and an entangled three-qutrit state are employed as the quantum channels. In these schemes, an unknown qutrit state can be teleported to either one of two receivers, but only one of them can reconstruct the qutrit state with the help of the other. Based on the case of qutrits, a scheme of controlled teleportation of an unknown qudit state is presented.

Keywords: controlled teleportation, high-dimensional quantum state, generalized Bell-basis measurement, generalized Hadamard transformation

PACC: 0365, 0367

Quantum teleportation, proposed by Bennett *et al*[1] is one of the most important aspects of quantum information. In the teleportation process, unknown quantum state is transferred without material transportation from a sender to a receiver via a quantum channel with the help of some classical information. At present, teleportation has been generalized to many cases,[2−23] and is demonstrated with photon polarized states,[24] optical coherent states,[25] and nuclear magnetic resonance[26] in the experiments. In the original work of Bennett *et al*,[1] quantum teleportation of the information contained in a state of an unknown d-dimensional system was introduced. Recently, the high-dimensional aspects of various topics in quantum information have been studied. For example, quantum cloning,[27] superdense coding,[28] quantum key distribution,[29] and teleportation[30−36] have all been discussed in this context.

Recently, a scheme for controlled quantum teleportation has been presented.[37] In this scheme, an arbitrary single qubit state can be teleported to either one of two receivers by using a GHZ state, but only one of them can reconstruct the qubit state with the help of the other. More recently, a lot of schemes for controlled teleportation has been proposed.[38−41] However, in above schemes of the controlled teleportation the two-dimensional quantum system was only considered. In this paper, we first present a scheme for controlled teleportation of a d-dimensional quantum state. The proposed scheme consumes d-dimensional Bell-basis measurement and the generalized Hadamard gate, this makes our scheme differing from other known schemes.

To present our scheme more clearly, let us first begin with controlled teleportation in three dimensions. Suppose Alice has an unknown qutrit

$$|\psi\rangle_1 = a|0\rangle_1 + b|1\rangle_1 + c|2\rangle_1, \tag{1}$$

with $|a|^2 + |b|^2 + |c|^2 = 1$. Alice want to transmit this state to receiver Bob. Assume that Alice, Bob and Charlie share a maximally entangled state

$$|\psi\rangle_{234} = \frac{1}{\sqrt{3}}(|000\rangle + |111\rangle + |222\rangle)_{234}, \tag{2}$$

and Alice, Bob and Charlie hold qutrits 2, 3 and 4 respectively. The state of the whole system at this moment is

$$\begin{aligned} |\Psi\rangle_{1234} &= |\psi\rangle_1 \otimes |\psi\rangle_{234} \\ &= \frac{1}{3}[|\Psi_{00}\rangle_{12}(a|00\rangle + b|11\rangle + c|22\rangle)_{34} \end{aligned}$$

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$$
\begin{aligned}
&+|\Psi_{10}\rangle_{12}(a|00\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}b|11\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}c|22\rangle)_{34}\\
&+|\Psi_{20}\rangle_{12}(a|00\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}b|11\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}c|22\rangle)_{34}\\
&+|\Psi_{01}\rangle_{12}(a|11\rangle+b|22\rangle+c|00\rangle)_{34}\\
&+|\Psi_{11}\rangle_{12}(a|11\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}b|22\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}c|00\rangle)_{34}\\
&+|\Psi_{21}\rangle_{12}(a|11\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}b|22\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}c|00\rangle)_{34}\\
&+|\Psi_{02}\rangle_{12}(a|22\rangle+b|00\rangle+c|11\rangle)_{34}\\
&+|\Psi_{12}\rangle_{12}(a|22\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}b|00\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}c|11\rangle)_{34}\\
&+|\Psi_{22}\rangle_{12}(a|22\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}b|00\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}c|11\rangle)_{34}], \quad (3)
\end{aligned}
$$

where $|\Psi_{nm}\rangle$ are generalized Bell basis of the Hilbert space of two qutrits[28]

$$|\Psi_{nm}\rangle=\sum_j \mathrm{e}^{2\pi\mathrm{i}jn/3}|j\rangle\otimes|j+m \bmod 3\rangle/\sqrt{3}, \quad (4)$$

where $n,m,j=0,1,2$. More explicitly,

$$
\begin{aligned}
|\Psi_{00}\rangle&=\frac{1}{\sqrt{3}}(|00\rangle+|11\rangle+|22\rangle),\\
|\Psi_{10}\rangle&=\frac{1}{\sqrt{3}}(|00\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}|11\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}|22\rangle),\\
|\Psi_{20}\rangle&=\frac{1}{\sqrt{3}}(|00\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}|11\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}|22\rangle),\\
|\Psi_{01}\rangle&=\frac{1}{\sqrt{3}}(|01\rangle+|12\rangle+|20\rangle),\\
|\Psi_{11}\rangle&=\frac{1}{\sqrt{3}}(|01\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}|12\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}|20\rangle),\\
|\Psi_{21}\rangle&=\frac{1}{\sqrt{3}}(|01\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}|12\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}|20\rangle),\\
|\Psi_{02}\rangle&=\frac{1}{\sqrt{3}}(|02\rangle+|10\rangle+|21\rangle),\\
|\Psi_{12}\rangle&=\frac{1}{\sqrt{3}}(|02\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}|10\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}|21\rangle),\\
|\Psi_{22}\rangle&=\frac{1}{\sqrt{3}}(|02\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}|10\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}|21\rangle). \quad (5)
\end{aligned}
$$

Through simple calculation, it can be shown that the single-body operators[28]

$$
\begin{aligned}
U_{00}&=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},\\
U_{10}&=\begin{pmatrix}1&0&0\\0&\mathrm{e}^{2\pi\mathrm{i}/3}&0\\0&0&\mathrm{e}^{4\pi\mathrm{i}/3}\end{pmatrix},\\
U_{20}&=\begin{pmatrix}1&0&0\\0&\mathrm{e}^{4\pi\mathrm{i}/3}&0\\0&0&\mathrm{e}^{2\pi\mathrm{i}/3}\end{pmatrix},\\
U_{01}&=\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix},\\
U_{11}&=\begin{pmatrix}0&0&\mathrm{e}^{4\pi\mathrm{i}/3}\\1&0&0\\0&\mathrm{e}^{2\pi\mathrm{i}/3}&0\end{pmatrix},\\
U_{21}&=\begin{pmatrix}0&0&\mathrm{e}^{2\pi\mathrm{i}/3}\\1&0&0\\0&\mathrm{e}^{4\pi\mathrm{i}/3}&0\end{pmatrix},\\
U_{02}&=\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix},\\
U_{12}&=\begin{pmatrix}0&\mathrm{e}^{2\pi\mathrm{i}/3}&0\\0&0&\mathrm{e}^{4\pi\mathrm{i}/3}\\1&0&0\end{pmatrix},\\
U_{22}&=\begin{pmatrix}0&\mathrm{e}^{4\pi\mathrm{i}/3}&0\\0&0&\mathrm{e}^{2\pi\mathrm{i}/3}\\1&0&0\end{pmatrix} \quad (6)
\end{aligned}
$$

will transform $|\Psi_{00}\rangle$ into the corresponding states in Eq.(4) respectively,

$$U_{nm}|\Psi_{00}\rangle=|\Psi_{nm}\rangle. \quad (7)$$

Now let Alice performs a generalized Bell-state measurement on her qutrits 1 and 2. Then she will inform Bob and Charlie of the result of the measurement by the classical channel. Without loss of generality, we assume that the outcome of Alice's measurement is $|\Psi_{21}\rangle_{12}$, the state of qutrits 3 and 4 will collapse into

$$|\psi\rangle_{34}=\frac{1}{3}(a|11\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}b|22\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}c|00\rangle)_{34}. \quad (8)$$

If Charlie would like to help Bob with the teleportation, he should make a measurement on his qutrit 4 under the basis $\{|\Psi_0\rangle, |\Psi_1\rangle, |\Psi_2\rangle\}$, which is given by

$$
\begin{aligned}
|\Psi_0\rangle&=H_{(3)}|0\rangle=\frac{1}{\sqrt{3}}(|0\rangle+|1\rangle+|2\rangle),\\
|\Psi_1\rangle&=H_{(3)}|1\rangle=\frac{1}{\sqrt{3}}(|0\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}|1\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}|2\rangle),\\
|\Psi_2\rangle&=H_{(3)}|2\rangle=\frac{1}{\sqrt{3}}(|0\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}|1\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}|2\rangle), \quad (9)
\end{aligned}
$$

where $H_{(3)}$ is the Hadamard operator in three dimensional Hilbert space. After above measurement, Charlie sends the measurement result to Bob via a classical channel. Since the state $|\psi\rangle_{34}$ can be written as

$$
\begin{aligned}
|\psi\rangle_{34}=\frac{1}{3\sqrt{3}}[&|\Psi_0\rangle_4(a|1\rangle_3+\mathrm{e}^{2\pi\mathrm{i}/3}b|2\rangle_3\\
&+\mathrm{e}^{4\pi\mathrm{i}/3}c|0\rangle_3)+\mathrm{e}^{4\pi\mathrm{i}/3}|\Psi_1\rangle_4(a|1\rangle_3
\end{aligned}
$$

$$+ b|2\rangle_3 + c|0\rangle_3) + \mathrm{e}^{2\pi \mathrm{i}/3}|\Psi_2\rangle_4(a|1\rangle_3 + \mathrm{e}^{4\pi \mathrm{i}/3}b|2\rangle_3 + \mathrm{e}^{2\pi \mathrm{i}/3}c|0\rangle_3), \quad (10)$$

it is clear that, for instance, if the measurement outcome of Charlie is $|\Psi_0\rangle_4$, the state of qutrit 3 will become

$$a|1\rangle_3 + \mathrm{e}^{2\pi \mathrm{i}/3}b|2\rangle_3 + \mathrm{e}^{4\pi \mathrm{i}/3}c|0\rangle_3. \quad (11)$$

According to Charlie's measurement result, Bob should operate an unitary transformation U_{22} on Eq.(11) to obtain the original state

$$|\psi\rangle_3 = a|0\rangle_3 + b|1\rangle_3 + c|2\rangle_3. \quad (12)$$

The relation of generalized Bell-State measurement outcome $|\Psi_{nm}\rangle_{12}$ performed by Alice on the qutrits 1 and 2, measurement results of Charlie $|\Psi_j\rangle_4$ $(j = 0, 1, 2)$ and the unitary transformation $(U_{nm})_3$ corresponding to the states of qutrit 3 performed by Bob are given in Table 1. There are a total of 27 kinds.

Table 1. Relation of measurement results of qutrits 1 and 2 $|\Psi_{nm}\rangle_{12}$, measurement results of qutrits 4 $|\Psi_j\rangle_4$ $(j = 0, 1, 2)$ and the unitary operation $(U_{nm})_3$.

	$\|\Psi_0\rangle_4$	$\|\Psi_1\rangle_4$	$\|\Psi_2\rangle_4$
$\|\Psi_{00}\rangle_{12}$	$(U_{00})_3$	$(U_{20})_3$	$(U_{10})_3$
$\|\Psi_{10}\rangle_{12}$	$(U_{10})_3$	$(U_{00})_3$	$(U_{20})_3$
$\|\Psi_{20}\rangle_{12}$	$(U_{20})_3$	$(U_{10})_3$	$(U_{00})_3$
$\|\Psi_{01}\rangle_{12}$	$(U_{02})_3$	$(U_{12})_3$	$(U_{22})_3$
$\|\Psi_{11}\rangle_{12}$	$(U_{22})_3$	$(U_{02})_3$	$(U_{12})_3$
$\|\Psi_{21}\rangle_{12}$	$(U_{12})_3$	$(U_{22})_3$	$(U_{02})_3$
$\|\Psi_{02}\rangle_{12}$	$(U_{01})_3$	$(U_{21})_3$	$(U_{11})_3$
$\|\Psi_{12}\rangle_{12}$	$(U_{11})_3$	$(U_{01})_3$	$(U_{21})_3$
$\|\Psi_{22}\rangle_{12}$	$(U_{21})_3$	$(U_{11})_3$	$(U_{01})_3$

Now we will generalize the previous scheme for controlled teleportation of an unknown bi-partite entangled state in three dimensions. Suppose that the sender Alice wants to transmit an unknown bi-partite entangled state with three dimensions to the receiver Bob. The unknown entangled state that will be teleported can be expressed as

$$|\phi\rangle_{12} = a|00\rangle_{12} + b|11\rangle_{12} + c|22\rangle_{12}, \quad (13)$$

where $|a|^2 + |b|^2 + |c|^2 = 1$. Assume Alice, Bob and Charlie share two maximally entangled states in three dimensions

$$|\varphi\rangle_{34} = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)_{34},$$

$$|\varphi\rangle_{567} = \frac{1}{\sqrt{3}}(|000\rangle + |111\rangle + |222\rangle)_{567}, \quad (14)$$

and Alice holds qutrits 3 and 5, Bob holds qutrits 4 and 6 and Charlie holds qutrit 7, respectively. The total initial state now is $|\Phi\rangle = |\phi\rangle_{12} \otimes |\varphi\rangle_{34} \otimes |\varphi\rangle_{567}$. In order to realize teleportation, Alice performs generalized Bell-state measurement on her qutrits (1,3) and (2,5), respectively. Without loss of generality, if the measurement results of Alice are $|\Psi_{12}\rangle_{13}$ and $|\Psi_{01}\rangle_{25}$, the state of qutrits 4, 6 and 7 collapses into the following state:

$$|\phi\rangle_{467} = \frac{1}{9}(a|211\rangle + \mathrm{e}^{4\pi \mathrm{i}/3}b|022\rangle + \mathrm{e}^{2\pi \mathrm{i}/3}c|100\rangle)_{467}. \quad (15)$$

Next, Alice informs Bob and Charlie of her measured results via a classical channel. After Charlie receives measured results of Alice, if Charlie would like to help Bob with the teleportation, he should make a measurement on his qutrit 7 under the basis $\{|\Psi_0\rangle, |\Psi_1\rangle, |\Psi_2\rangle\}$ and transfer the result of his measurement to Bob via a classical channel.

Because the state can be written as

$$\begin{aligned}|\phi\rangle_{467} = \frac{1}{9\sqrt{3}}[&|\Psi_0\rangle_7(a|21\rangle + \mathrm{e}^{4\pi \mathrm{i}/3}b|02\rangle + \mathrm{e}^{2\pi \mathrm{i}/3}c|10\rangle)_{46} \\ &+ |\Psi_1\rangle_7 \mathrm{e}^{2\pi \mathrm{i}/3}(a|21\rangle + b|02\rangle + c|10\rangle)_{46} \\ &+ |\Psi_2\rangle_7 \mathrm{e}^{4\pi \mathrm{i}/3}(a|21\rangle + \mathrm{e}^{2\pi \mathrm{i}/3}b|02\rangle \\ &+ \mathrm{e}^{4\pi \mathrm{i}/3}c|10\rangle)_{46}, \end{aligned} \quad (16)$$

it is clear that if the measured result of Charlie is $|\Psi_0\rangle_7$, the state of qutrits 4 and 6 will become

$$a|21\rangle_{46} + \mathrm{e}^{4\pi \mathrm{i}/3}b|02\rangle_{46} + \mathrm{e}^{2\pi \mathrm{i}/3}c|10\rangle_{46}. \quad (17)$$

According to the measurement outcome of Charlie, Bob can operate an unitary transformation $((U_{11})_4 \otimes (U_{12})_6)$ on Eq.(17) and has

$$|\phi\rangle_{46} = a|00\rangle_{46} + b|11\rangle_{46} + c|22\rangle_{46}, \quad (18)$$

the teleportation is successfully realized.

For the other cases, the relation of generalized Bell-state measurement results $|\Psi_{nm}\rangle_{13}|\Psi_{n'm'}\rangle_{25}$ $(n, m, n', m' = 0, 1, 2)$ performed by Alice on the qutrits (1,3) and (2,5), measurement outcomes of Charlie $|\Psi_j\rangle_7$ $(j = 0, 1, 2)$ and the unitary operations $(U_{nm})_4 \otimes (U_{n'm'})_6$ performed by Bob on his qutrits 4 and 6 are given in Table 2. There are a total of 243 kinds.

Table 2. Relation of measurement results of qutrits (1,3) and (2,5) $|\Psi_{nm}\rangle_{13}|\Psi_{n'm'}\rangle_{25}$ $(n,m,n',m'=0,1,2)$, measurement results of qutrit 7 $|\Psi_j\rangle_7$ $(j=0,1,2)$ and the unitary operation $(U_{nm})_4\otimes(U_{n'm'})_6$.

	$\|\Psi_0\rangle_7$	$\|\Psi_1\rangle_7$	$\|\Psi_2\rangle_7$
$\|\Psi_{00}\rangle_{13}\|\Psi_{00}\rangle_{25}$(or$\|\Psi_{10}\rangle_{13}\|\Psi_{20}\rangle_{25}$, or$\|\Psi_{20}\rangle_{13}\|\Psi_{10}\rangle_{25}$)	$(U_{00})_4\otimes(U_{00})_6$	$(U_{00})_4\otimes(U_{20})_6$	$(U_{00})_4\otimes(U_{10})_6$
$\|\Psi_{00}\rangle_{13}\|\Psi_{10}\rangle_{25}$(or$\|\Psi_{10}\rangle_{13}\|\Psi_{00}\rangle_{25}$, or$\|\Psi_{20}\rangle_{13}\|\Psi_{20}\rangle_{25}$)	$(U_{00})_4\otimes(U_{10})_6$	$(U_{00})_4\otimes(U_{00})_6$	$(U_{00})_4\otimes(U_{20})_6$
$\|\Psi_{00}\rangle_{13}\|\Psi_{20}\rangle_{25}$(or$\|\Psi_{10}\rangle_{13}\|\Psi_{10}\rangle_{25}$, or$\|\Psi_{20}\rangle_{13}\|\Psi_{00}\rangle_{25}$)	$(U_{00})_4\otimes(U_{20})_6$	$(U_{00})_4\otimes(U_{10})_6$	$(U_{00})_4\otimes(U_{00})_6$
$\|\Psi_{00}\rangle_{13}\|\Psi_{01}\rangle_{25}$(or$\|\Psi_{10}\rangle_{13}\|\Psi_{21}\rangle_{25}$, or$\|\Psi_{20}\rangle_{13}\|\Psi_{11}\rangle_{25}$)	$(U_{00})_4\otimes(U_{02})_6$	$(U_{20})_4\otimes(U_{02})_6$	$(U_{10})_4\otimes(U_{02})_6$
$\|\Psi_{00}\rangle_{13}\|\Psi_{11}\rangle_{25}$(or$\|\Psi_{10}\rangle_{13}\|\Psi_{01}\rangle_{25}$, or$\|\Psi_{20}\rangle_{13}\|\Psi_{21}\rangle_{25}$)	$(U_{10})_4\otimes(U_{02})_6$	$(U_{00})_4\otimes(U_{02})_6$	$(U_{20})_4\otimes(U_{02})_6$
$\|\Psi_{00}\rangle_{13}\|\Psi_{21}\rangle_{25}$(or$\|\Psi_{10}\rangle_{13}\|\Psi_{11}\rangle_{25}$, or$\|\Psi_{20}\rangle_{13}\|\Psi_{01}\rangle_{25}$)	$(U_{20})_4\otimes(U_{02})_6$	$(U_{10})_4\otimes(U_{02})_6$	$(U_{00})_4\otimes(U_{02})_6$
$\|\Psi_{00}\rangle_{13}\|\Psi_{02}\rangle_{25}$(or$\|\Psi_{10}\rangle_{13}\|\Psi_{22}\rangle_{25}$, or$\|\Psi_{20}\rangle_{13}\|\Psi_{12}\rangle_{25}$)	$(U_{00})_4\otimes(U_{01})_6$	$(U_{20})_4\otimes(U_{01})_6$	$(U_{10})_4\otimes(U_{01})_6$
$\|\Psi_{00}\rangle_{13}\|\Psi_{12}\rangle_{25}$(or$\|\Psi_{10}\rangle_{13}\|\Psi_{02}\rangle_{25}$, or$\|\Psi_{20}\rangle_{13}\|\Psi_{22}\rangle_{25}$)	$(U_{10})_4\otimes(U_{01})_6$	$(U_{00})_4\otimes(U_{01})_6$	$(U_{20})_4\otimes(U_{01})_6$
$\|\Psi_{00}\rangle_{13}\|\Psi_{22}\rangle_{25}$(or$\|\Psi_{10}\rangle_{13}\|\Psi_{12}\rangle_{25}$, or$\|\Psi_{20}\rangle_{13}\|\Psi_{02}\rangle_{25}$)	$(U_{20})_4\otimes(U_{01})_6$	$(U_{10})_4\otimes(U_{01})_6$	$(U_{00})_4\otimes(U_{01})_6$
$\|\Psi_{01}\rangle_{13}\|\Psi_{00}\rangle_{25}$(or$\|\Psi_{11}\rangle_{13}\|\Psi_{01}\rangle_{25}$, or$\|\Psi_{21}\rangle_{13}\|\Psi_{10}\rangle_{25}$)	$(U_{02})_4\otimes(U_{00})_6$	$(U_{02})_4\otimes(U_{20})_6$	$(U_{02})_4\otimes(U_{10})_6$
$\|\Psi_{01}\rangle_{13}\|\Psi_{10}\rangle_{25}$(or$\|\Psi_{11}\rangle_{13}\|\Psi_{00}\rangle_{25}$, or$\|\Psi_{21}\rangle_{13}\|\Psi_{20}\rangle_{25}$)	$(U_{02})_4\otimes(U_{10})_6$	$(U_{02})_4\otimes(U_{00})_6$	$(U_{02})_4\otimes(U_{20})_6$
$\|\Psi_{01}\rangle_{13}\|\Psi_{20}\rangle_{25}$(or$\|\Psi_{11}\rangle_{13}\|\Psi_{10}\rangle_{25}$, or$\|\Psi_{21}\rangle_{13}\|\Psi_{00}\rangle_{25}$)	$(U_{02})_4\otimes(U_{20})_6$	$(U_{02})_4\otimes(U_{10})_6$	$(U_{02})_4\otimes(U_{00})_6$
$\|\Psi_{01}\rangle_{13}\|\Psi_{01}\rangle_{25}$(or$\|\Psi_{11}\rangle_{13}\|\Psi_{21}\rangle_{25}$, or$\|\Psi_{21}\rangle_{13}\|\Psi_{11}\rangle_{25}$)	$(U_{02})_4\otimes(U_{02})_6$	$(U_{22})_4\otimes(U_{02})_6$	$(U_{12})_4\otimes(U_{02})_6$
$\|\Psi_{01}\rangle_{13}\|\Psi_{11}\rangle_{25}$(or$\|\Psi_{11}\rangle_{13}\|\Psi_{01}\rangle_{25}$, or$\|\Psi_{21}\rangle_{13}\|\Psi_{21}\rangle_{25}$)	$(U_{12})_4\otimes(U_{02})_6$	$(U_{02})_4\otimes(U_{02})_6$	$(U_{22})_4\otimes(U_{02})_6$
$\|\Psi_{01}\rangle_{13}\|\Psi_{21}\rangle_{25}$(or$\|\Psi_{11}\rangle_{13}\|\Psi_{11}\rangle_{25}$, or$\|\Psi_{21}\rangle_{13}\|\Psi_{01}\rangle_{25}$)	$(U_{22})_4\otimes(U_{02})_6$	$(U_{12})_4\otimes(U_{02})_6$	$(U_{02})_4\otimes(U_{02})_6$
$\|\Psi_{01}\rangle_{13}\|\Psi_{02}\rangle_{25}$(or$\|\Psi_{11}\rangle_{13}\|\Psi_{22}\rangle_{25}$, or$\|\Psi_{21}\rangle_{13}\|\Psi_{12}\rangle_{25}$)	$(U_{02})_4\otimes(U_{01})_6$	$(U_{22})_4\otimes(U_{01})_6$	$(U_{12})_4\otimes(U_{01})_6$
$\|\Psi_{01}\rangle_{13}\|\Psi_{12}\rangle_{25}$(or$\|\Psi_{11}\rangle_{13}\|\Psi_{02}\rangle_{25}$, or$\|\Psi_{21}\rangle_{13}\|\Psi_{22}\rangle_{25}$)	$(U_{12})_4\otimes(U_{01})_6$	$(U_{02})_4\otimes(U_{01})_6$	$(U_{22})_4\otimes(U_{01})_6$
$\|\Psi_{01}\rangle_{13}\|\Psi_{22}\rangle_{25}$(or$\|\Psi_{11}\rangle_{13}\|\Psi_{12}\rangle_{25}$, or$\|\Psi_{21}\rangle_{13}\|\Psi_{02}\rangle_{25}$)	$(U_{22})_4\otimes(U_{01})_6$	$(U_{12})_4\otimes(U_{01})_6$	$(U_{02})_4\otimes(U_{01})_6$
$\|\Psi_{02}\rangle_{13}\|\Psi_{00}\rangle_{25}$(or$\|\Psi_{12}\rangle_{13}\|\Psi_{20}\rangle_{25}$, or$\|\Psi_{22}\rangle_{13}\|\Psi_{10}\rangle_{25}$)	$(U_{01})_4\otimes(U_{00})_6$	$(U_{01})_4\otimes(U_{20})_6$	$(U_{01})_4\otimes(U_{10})_6$
$\|\Psi_{02}\rangle_{13}\|\Psi_{10}\rangle_{25}$(or$\|\Psi_{12}\rangle_{13}\|\Psi_{00}\rangle_{25}$, or$\|\Psi_{22}\rangle_{13}\|\Psi_{20}\rangle_{25}$)	$(U_{01})_4\otimes(U_{10})_6$	$(U_{01})_4\otimes(U_{00})_6$	$(U_{01})_4\otimes(U_{20})_6$
$\|\Psi_{02}\rangle_{13}\|\Psi_{20}\rangle_{25}$(or$\|\Psi_{12}\rangle_{13}\|\Psi_{10}\rangle_{25}$, or$\|\Psi_{22}\rangle_{13}\|\Psi_{00}\rangle_{25}$)	$(U_{01})_4\otimes(U_{20})_6$	$(U_{01})_4\otimes(U_{10})_6$	$(U_{01})_4\otimes(U_{00})_6$
$\|\Psi_{02}\rangle_{13}\|\Psi_{01}\rangle_{25}$(or$\|\Psi_{12}\rangle_{13}\|\Psi_{20}\rangle_{25}$, or$\|\Psi_{22}\rangle_{13}\|\Psi_{11}\rangle_{25}$)	$(U_{01})_4\otimes(U_{02})_6$	$(U_{21})_4\otimes(U_{02})_6$	$(U_{11})_4\otimes(U_{02})_6$
$\|\Psi_{02}\rangle_{13}\|\Psi_{11}\rangle_{25}$(or$\|\Psi_{12}\rangle_{13}\|\Psi_{01}\rangle_{25}$, or$\|\Psi_{22}\rangle_{13}\|\Psi_{21}\rangle_{25}$)	$(U_{11})_4\otimes(U_{02})_6$	$(U_{01})_4\otimes(U_{02})_6$	$(U_{21})_4\otimes(U_{02})_6$
$\|\Psi_{02}\rangle_{13}\|\Psi_{21}\rangle_{25}$(or$\|\Psi_{12}\rangle_{13}\|\Psi_{11}\rangle_{25}$, or$\|\Psi_{22}\rangle_{13}\|\Psi_{01}\rangle_{25}$)	$(U_{21})_4\otimes(U_{02})_6$	$(U_{11})_4\otimes(U_{02})_6$	$(U_{01})_4\otimes(U_{02})_6$
$\|\Psi_{02}\rangle_{13}\|\Psi_{02}\rangle_{25}$(or$\|\Psi_{12}\rangle_{13}\|\Psi_{22}\rangle_{25}$, or$\|\Psi_{22}\rangle_{13}\|\Psi_{12}\rangle_{25}$)	$(U_{01})_4\otimes(U_{01})_6$	$(U_{21})_4\otimes(U_{01})_6$	$(U_{11})_4\otimes(U_{01})_6$
$\|\Psi_{02}\rangle_{13}\|\Psi_{12}\rangle_{25}$(or$\|\Psi_{12}\rangle_{13}\|\Psi_{02}\rangle_{25}$, or$\|\Psi_{22}\rangle_{13}\|\Psi_{22}\rangle_{25}$)	$(U_{11})_4\otimes(U_{01})_6$	$(U_{01})_4\otimes(U_{01})_6$	$(U_{21})_4\otimes(U_{01})_6$
$\|\Psi_{02}\rangle_{13}\|\Psi_{22}\rangle_{25}$(or$\|\Psi_{12}\rangle_{13}\|\Psi_{12}\rangle_{25}$, or$\|\Psi_{22}\rangle_{13}\|\Psi_{02}\rangle_{25}$)	$(U_{21})_4\otimes(U_{01})_6$	$(U_{11})_4\otimes(U_{01})_6$	$(U_{01})_4\otimes(U_{01})_6$

Based on the above scheme for controlled teleportation using qutrits, we will generalize the scheme to arbitrarily high dimensions. We denote the dimension as d. The general Bell-basis states are

$$|\Psi_{nm}^{(d)}\rangle=\sum_j \mathrm{e}^{2\pi \mathrm{i}jn/d}|j\rangle\otimes|j+m \bmod d\rangle/\sqrt{d}\,, \quad (19)$$

where $n,m,j=0,1,2,\ldots,d-1$. The matrix elements of the unitary transformation $U_{nm}^{(d)}$ may be explicitly written as

$$(U_{nm}^{(d)})_{j'j}=\mathrm{e}^{2\pi \mathrm{i}jn/d}\delta_{j',j+m \bmod d}, \quad (20)$$

where $n,m,j,j'=0,1,2,\ldots,d-1$. Obviously, there exist single-body operators $U_{nm}^{(d)}$ on states (19) that satisfy $U_{nm}^{(d)}|\Psi_{00}^{(d)}\rangle=|\Psi_{nm}^{(d)}\rangle$.

Suppose that Alice want to transmit an unknown qudit to Bob

$$|\phi\rangle_1=\sum_j a_j|j\rangle_1, \quad (21)$$

where $j=0,1,2,\ldots,d-1$ and $\sum_j|a_j|^2=1$. Assume that Alice, Bob and Charlie share a maximally entangled state

$$|\phi\rangle_{234}=\sum_j|j\rangle_2\otimes|j\rangle_3\otimes|j\rangle_4/\sqrt{d}, \quad (22)$$

where $j=0,1,2,\ldots,d-1$, and Alice, Bob and Charlie hold qudits 2, 3 and 4, respectively. The state for the whole system can be expressed as

$$\begin{aligned}|\Phi\rangle_{1234}&=|\phi\rangle_1\otimes|\phi\rangle_{234}\\&=\frac{1}{d}\sum_{n,m=0}^{d-1}(|\Psi_{nm}^{(d)}\rangle_{12}\otimes|\Psi_{mm}^{\prime(d)}\rangle_{34}),\end{aligned} \quad (23)$$

where

$$\begin{aligned}|\Psi_{mm}^{\prime(d)}\rangle_{34}=&\sum_j a_j\mathrm{e}^{-2\pi \mathrm{i}jn/d}|j+m \bmod d\rangle\\&\otimes|j+m \bmod d\rangle,\end{aligned} \quad (24)$$

where $j, n, m = 0, 1, 2, \ldots, d-1$. Now Alice performs a generalized Bell-state measurement on qudits 1 and 2 based on Eq.(19), and informs Bob and Charlie of the measure outcome. Assume that Alice's measurement result is $|\Psi^{(d)}_{n'm'}\rangle_{12}$, the state of qudits 3 and 4 will become

$$|\Psi'^{(d)}_{m'm'}\rangle_{34} = \sum_{j=0}^{d-1} a_j \mathrm{e}^{-2\pi \mathrm{i} jn'/d} |j+m' \bmod d\rangle_3 \otimes |j+m' \bmod d\rangle_4, \quad (25)$$

where $n', m' = 0, 1, 2, \ldots, d-1$. In order to make the teleportation successful, Charlie should make a measurement on qudit 4 using the basis $\{|\Psi^{(d)}_0\rangle, |\Psi^{(d)}_1\rangle, \ldots, |\Psi^{(d)}_l\rangle, \ldots |\Psi^{(d)}_{d-1}\rangle, \}$, which is given by

$$|\Psi^{(d)}_l\rangle = H_{(d)}|l\rangle = \sum_{j=0}^{d-1} \mathrm{e}^{2\pi \mathrm{i} jl/d}|j\rangle\sqrt{d}\,, \quad (26)$$

where $l = 0, 1, 2, \ldots, d-1$, and $H_{(d)}$ is the generalized Hadamard operator in d-dimension system[29,42]

$$H_{(d)} := \frac{1}{\sqrt{d}} \sum_{l,j=0}^{d-1} \mathrm{e}^{2\pi \mathrm{i} jl/d}|j\rangle\langle l|. \quad (27)$$

Based on Eq.(26), the state $|\Psi'^{(d)}_{m'm'}\rangle_{34}$ can be written as

$$|\Psi'^{(d)}_{m'm'}\rangle_{34} = \frac{1}{\sqrt{d}}|\Psi^{(d)}_l\rangle_4 \sum_{j=0}^{d-1} a_j \mathrm{e}^{-2\pi \mathrm{i} j[n'j+(j+m')l]/d} \times |j+m' \bmod d\,\rangle_3, \quad (28)$$

where $l = 0, 1, 2, \ldots, d-1$. Without loss of generality, if Charlie obtain $|\Psi^{(d)}_{l'}\rangle_4$, the state of qudit 3 in Bob's side is

$$\sum_{j=0}^{d-1} a_j \mathrm{e}^{-2\pi \mathrm{i} j[n'j+(j+m')l']/d}|j+m' \bmod d\rangle_3, \quad (29)$$

where $l' = 0, 1, 2, \ldots, d-1$. Then Bob should operate an unitary transformation U_{fk} (it is easy to verify that $f = (n'+l') \bmod d$, and $k = (d+m') \bmod d$, respectively) on Eq.(29) to give the original state

$$|\phi\rangle_3 = \sum_{j=0}^{d-1} a_j|j\rangle_3. \quad (30)$$

This means that the controlled teleportation in d-dimensional system is successfully realized.

In conclusion, we have proposed a scheme for controlled teleporting an unknown d-dimensional quantum state. Firstly, we have investigated a scheme of controlled teleportation of an unknown three-dimensional quantum state in detail. Then we have studied a scheme for controlled teleportation of unknown bi-partite entangled state in three dimensions by using two maximally three-dimensional entangled states as quantum channels. Based on the controlled teleportation of qutrits, we have presented a scheme of controlled teleportation for an unknown d-dimensional quantum state. In these schemes, generalized Bell-state measurement and generalized Hadamard transformation are needed. Finally, we point out that, with a similar method, one can generalize the scheme to controlled teleportation of an unknown bi-partite (or N-partite) d-dimensional entangled state.

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