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Exponential–fraction trial function method to the 5th-order mKdV equation*

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This paper obtains some solutions of the 5th-order mKdV equation by using the exponential–fraction trial function method, such as solitary wave solutions, shock wave solutions and the hopping wave solutions. It successfully shows that this method may be valid for solving other nonlinear partial differential equations.

Keywords: 5th-order mKdV equation, nonlinear partial differential equations, exponential–fraction trial function

PACC: 0290, 0340K, 0340G

1. Introduction

It is very important to search for the analytical solutions in nonlinear problems. Generally speaking, it is very difficult to search for the accurate solutions of nonlinear equations. However, we can find out parts of analytical solutions, such as solitary wave solutions, periodic solutions, shock wave solutions and so on. The construction of an exact solution for the associated nonlinear equation plays an important role in understanding the nonlinear problems.[1−3] Investigators have presented many methods for solving nonlinear evolvement equations, such as the inverse scattering transformation,[4] Backlund transforming method,[5] Hirota method,[6] sine–cosine method,[7] Ma method,[8] Sinh–Gordon equation expansion method,[9] constant coefficient Ricatti equation expansion method[10−12] and others;[13−15] these are all very helpful for finding solutions of the nonlinear partial differential equations.

We apply a new method[16] which is the exponential-fraction trial function to solve the 5th-order mKdV equation. And some groups of analytical solutions have been obtained. The method is based on the assumption that travelling wave solutions can be expressed in the symmetrical exponential function form. It is a very simple and straightforward method for solving nonlinear partial differential equations. The solution procedure, by help of Mathematica, is of utter simplicity, and can be easily extended to all kinds of nonlinear equations.

2. The exponential–fraction trial solutions for 5th-order mKdV equation

The 5th-order mKdV equation is an important equation in fluid mechanics and plasma physics. It occurs in the theory of surface waves in a convecting fluid.[17] The form of the nonlinear 5th-order mKdV equation can be written as follows:

$$\frac{\partial U}{\partial t}+\frac{\partial}{\partial x}\left[\left(6U^5+10\alpha\left(U^2\frac{\partial^2 U}{\partial t^2}+U\left(\frac{\partial U}{\partial x}\right)^2\right)+\frac{\partial^4 U}{\partial t^4}\right)\right]=0, \tag{1}$$

where α is a non-zero constant. We use a kind of transformation $\eta = x-ct$ which is a scale transformation called the Gardner–Morikawa transformation, x is the space coordinate, t is the time coordinate, and c is a non-zero

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constant. Substituting $\eta = x - ct$ into Eq.(1) and rearranging the result, we obtain the following equation:

$$\frac{\mathrm{d}^5U}{\mathrm{d}\eta^5} + 10\alpha U^2\frac{\mathrm{d}^3U}{\mathrm{d}\eta^3} + 40\alpha U\frac{\mathrm{d}U}{\mathrm{d}\eta}\frac{\mathrm{d}^2U}{\mathrm{d}\eta^2} + 10\alpha\left(\frac{\mathrm{d}U}{\mathrm{d}\eta}\right)^3 + 30\frac{\mathrm{d}U}{\mathrm{d}\eta}U^4 - c\frac{\mathrm{d}U}{\mathrm{d}\eta} = 0, \tag{2}$$

where U is the function of η ($U = U(\eta)$). According to the exponential–fraction trial function method, we assume that the travelling solutions of Eq.(l) have the symmetry form[18] as:

$$U = U_0 + \frac{A_0 + A_1\exp(k\eta) + A_2\exp(2k\eta) + A_3\exp(3k\eta)}{B_0 + B_1\exp(k\eta) + B_2\exp(2k\eta) + B_3\exp(3k\eta)}, \tag{3}$$

where A_i and B_i ($i = 0, 1, 2, 3$) refer to the constants which will be determined later, and U_0, k are non-zero constants.

Substituting Eq.(3) into Eq.(2) and rearranging the results, we have a polynomial of $\exp(nk\eta)$. Collecting the terms which contain $\exp(nk\eta)$ and setting their coefficients to zero, we obtain seventeen algebraic equations. After some calculations and simplifications, five solutions of these algebraic equations can be obtained. In the case of $U_0 = 0$, the solutions are:

Case 1

$$\begin{aligned} c &= \frac{243k^4}{8}, \quad \alpha = -1, \\ A_0 &= \frac{3kB_0}{2}, \quad A_3 = -\frac{3kB_3}{2}, \\ A_1 &= A_2 = B_1 = B_2 = 0 \end{aligned} \tag{4}$$

and Eq.(3) can be written as

$$U = \frac{3k}{2}\frac{B_0 - B_3\exp(3k\eta)}{B_0 + B_3\exp(3k\eta)}; \tag{5}$$

Case 2

$$\begin{aligned} c &= \frac{243k^4}{8}, \quad \alpha = -1, \\ A_0 &= -\frac{3kB_0}{2}, \quad A_3 = \frac{3kB_3}{2}, \\ A_1 &= A_2 = B_1 = B_2 = 0 \end{aligned} \tag{6}$$

and Eq.(3) can be written as

$$U = -\frac{3k}{2}\frac{B_0 - B_3\exp(3k\eta)}{B_0 + B_3\exp(3k\eta)}; \tag{7}$$

Case 3

$$\begin{aligned} c &= \frac{243k^4}{8}, \quad \alpha = 1, \\ A_3 &= \frac{9}{4}k^2\frac{B_0B_3}{A_0}, \\ A_1 &= A_2 = B_1 = B_2 = 0 \end{aligned} \tag{8}$$

and Eq.(3) can be written as

$$U = \frac{4A_0^2 + 9k^2B_0B_3\exp(3k\eta)}{4A_0\left[B_0 + B_3\exp(3k\eta)\right]}; \tag{9}$$

Case 4

$$\begin{aligned} c &= k^4, \quad \alpha = -1, \\ B_2 &= -\frac{A_1^2}{4k^2B_0}, \\ A_0 &= A_2 = A_3 = B_1 = B_3 = 0 \end{aligned} \tag{10}$$

and Eq.(3) can be written as

$$U = \frac{4k^2A_1B_0\exp(k\eta)}{4k^2B_0^2 - A_1^2\exp(2k\eta)}; \tag{11}$$

Case 5

$$\begin{aligned} c &= k^4, \quad \alpha = 1, \\ B_2 &= \frac{A_1^2}{4k^2B_0}, \\ A_0 &= A_2 = A_3 = B_1 = B_3 = 0 \end{aligned} \tag{12}$$

and Eq.(3) can be written as

$$U = \frac{4k^2A_1B_0\exp(k\eta)}{4k^2B_0^2 + A_1^2\exp(2k\eta)}. \tag{13}$$

We now reduce Eq.(3) to the following form:

$$U = U_0 + \frac{A_0 + A_1\exp(k\eta) + A_2\exp(2k\eta)}{B_0 + B_1\exp(k\eta) + B_2\exp(2k\eta)}. \tag{14}$$

By solving the set of algebraic equations which is obtained by similar steps, we can obtain a family of the constants. The solutions of the constants are:

Case 6

$$\begin{aligned} c &= k^4, \quad A_1 = -2k\sqrt{B_0B_2}, \\ A_0 &= A_2 = B_1 = 0, \quad U_0 = 0 \end{aligned} \tag{15}$$

and Eq.(14) becomes

$$U = \frac{-2k\sqrt{B_0B_2}\exp(k\eta)}{B_0 + B_2\exp(2k\eta)}; \tag{16}$$

Case 7

$$\begin{aligned} c &= k^4, \quad A_1 = 2k\sqrt{B_0B_2}, \\ A_0 &= A_2 = B_1 = 0, \quad U_0 = 0 \end{aligned} \tag{17}$$

and Eq.(14) becomes

$$U = \frac{2k\sqrt{B_0 B_2}\exp(k\eta)}{B_0 + B_2\exp(2k\eta)}. \tag{18}$$

If we simplify Eq.(3) into another symmetrical expansion

$$U = U_0 + \frac{A_0 + A_1\exp(k\eta)}{B_0 + B_1\exp(k\eta)}. \tag{19}$$

as we have done before, the solutions of the constants are as follows:

Case 8

$$A_0 = \sqrt[4]{\frac{c}{30}}B_0 - B_0U_0,$$
$$A_1 = \sqrt[4]{\frac{c}{30}}B_1 - B_1U_0, \tag{20}$$

and Eq.(19) becomes

$$U = U_0 + \frac{\sqrt[4]{\frac{c}{30}}B_0 - B_0U_0 + \sqrt[4]{\frac{c}{30}}B_1 - B_1U_0\exp(k\eta)}{B_0 + B_1\exp(k\eta)}; \tag{21}$$

Case 9

$$A_0 = -\sqrt[4]{\frac{c}{30}}B_0 - B_0U_0,$$
$$A_1 = -\sqrt[4]{\frac{c}{30}}B_1 - B_1U_0, \tag{22}$$

and Eq.(19) becomes

$$U = U_0 - \frac{\sqrt[4]{\frac{c}{30}}B_0 + B_0U_0 + \sqrt[4]{\frac{c}{30}}B_1 + B_1U_0\exp(k\eta)}{B_0 + B_1\exp(k\eta)}. \tag{23}$$

3. Numerical simulation analysis

We use a new method of the exponential–fraction trial function for solving the 5th-order mKdV equation and have obtained 9 families of solutions. Meanwhile, the figures of these different kinds of solutions will be shown by the numerical simulation analysis. According to analysis, we find the shock waves solutions from Fig.1; Figure 2 shows the solitary wave solutions, in which there are two convex solid waves and one concave solid wave; Figure 3 shows a hopping form wave, a mutative phenomenon arises in the middle of the wave; Figure 4 shows two different trembling wave solutions in which there may be some chaos characteristics.

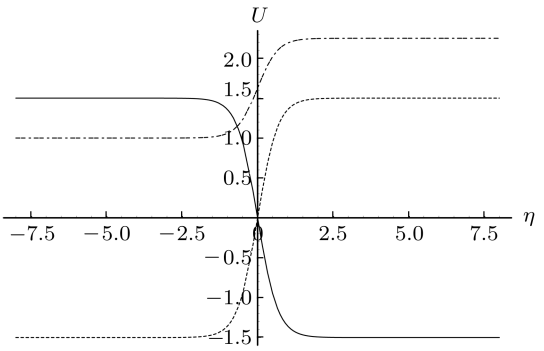


Fig.1. The shock wave solutions of case 1, case 2 and case 3. ($k = 1, A_0 = 1, B_0 = 1, B_3 = 0$).

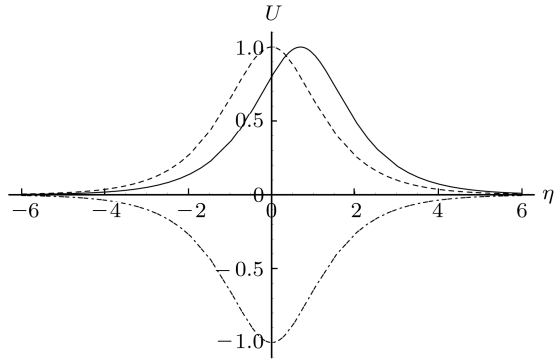


Fig.2. The solid wave solutions of case 5, case 6 and case 7. ($k = 1, A_0 = 1, B_0 = 1, B_2 = 0$).

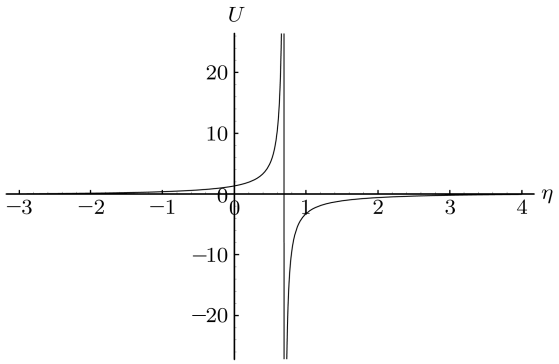


Fig.3. The hopping wave solutions of case 4. ($k = 1, A_1 = 1, B_0 = 1$).

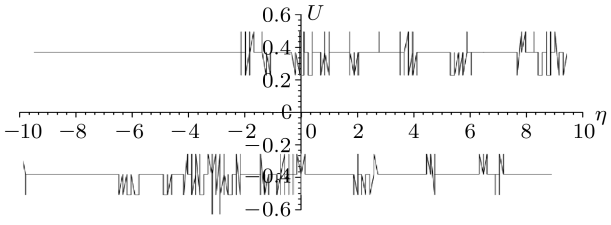


Fig.4. The trembling wave solutions of case 8 and case 9. ($k = 1, U_0 = 1, c = 1, B_0 = 1, B_1 = 1$).

4. Conclusion

In this paper, we have obtained some solutions of the 5th-order mKdV equation with the exponential-fraction trial function method. The method is valid in finding solutions of this nonlinear partial different equation. It also may be useful to solve other nonlinear equations, such as the mKdV equations in plasma physics, the partial different equation of chaos system, equations about the nonlinear dynamical equations, etc.

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