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An integrable Hamiltonian hierarchy and associated integrable couplings system*

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This paper establishes a new isospectral problem. By making use of the Tu scheme, a new integrable system is obtained. It gives integrable couplings of the system obtained. Finally, the Hamiltonian form of a binary symmetric constrained flow of the system obtained is presented.

Keywords: integrable system, Hamiltonian structure, loop algebra
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1. Introduction

It is an important and interesting aspect in soliton theory to search for new integrable systems in recent years. Various efficient approaches have been used to obtain many integrable systems such as AKNS hierarchy, KN hierarchy, Schrödinger system and so on.[1−22] Tu proposed a simple and efficient method for generating integrable Hamiltonian hierarchies of soliton equations in the frame of the zero curvature equation, which is called the Tu scheme.[5] By using this technique, some interesting results with physical significance have appeared such as TA, TB, TC, BTP and coupled KdV hierarchy, etc.[5−15] Moreover, bi-Hamiltonian structure and tri-Hamiltonian structure[11,12] have been applied to some soliton hierarchies by using the recursion operators. Cao[16] presented a powerful approach (the technique of nonlinearization of Lax pair or the restricted flow) and provided us with a bridge from the infinite dimensional Hamiltonian systems to the finite dimensional integrable Hamiltonian systems. Recently, Guo and Zhang further developed Tu scheme to look for some new integrable hierarchies and integrable couplings systems by using some subalgebras of loop algebra $\widetilde{A_1}$ and $\widetilde{A_2}$. Integrable couplings are a quite new interesting aspect in the field of soliton theory.[13] Using the method proposed in Refs.[21,22] some authors have obtained integrable couplings of many systems. These integrable couplings obtained are based on loop algebra $\widetilde{A_1}$. Regarding the systems with basis on loop $\widetilde{A_2}$, one can refer to Ref.[22], where a proper algebraic transformation is taken to turn the loop algebra $\widetilde{A_2}$ into its equivalent loop algebra like the form of $\widetilde{A_1}$.

In this paper, we would like to introduce a subalgebra of loop algebra $\widetilde{A_2}$. Therefore, a new isospectral problem is designed by using a new subalgebra of loop algebra $\widetilde{A_2}$. By making use of Tu scheme, a new integrable system is obtained. Furthermore, integrable couplings of the system are given. Finally, by establishing a binary symmetric constraint, the Hamiltonian form of the constrained flows are given.

2. A new loop algebra and an integrable system

Taking the loop algebra $\widetilde{A_2}$ with the basis

$$e_1(n) = \begin{pmatrix} \lambda^n & 0 & \lambda^n \\ 0 & -2\lambda^n & 0 \\ \lambda^n & 0 & \lambda^n \end{pmatrix},$$

$$e_2(n) = \begin{pmatrix} 0 & \lambda^n & 0 \\ \lambda^n & 0 & \lambda^n \\ 0 & \lambda^n & 0 \end{pmatrix},$$

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$$e_3(n)=\begin{pmatrix}0&\lambda^n&0\\-\lambda^n&0&-\lambda^n\\0&\lambda^n&0\end{pmatrix},$$

we have

$$[e_1(m),e_2(n)]=4e_3(m+n),$$
$$[e_1(m),e_3(n)]=4e_2(m+n),$$
$$[e_2(m),e_3(n)]=-2e_1(m+n),$$
$$\deg e_i(n)=n\quad(i=1,2,3).$$

Consider the spectral problem

$$\varphi_x=U\varphi,\quad \lambda_t=0,$$
$$U=U(u,\lambda)=\begin{pmatrix}\lambda&u+v&\lambda\\u-v&-2\lambda&u-v\\\lambda&u+v&\lambda\end{pmatrix}$$
$$=e_1(1)+ue_2(0)+ve_3(0),$$
$$\varphi=(\varphi_1,\varphi_2,\varphi_3)^{\mathrm{T}},\quad u=(u,v)^{\mathrm{T}}. \tag{1}$$

Let

$$V=ae_1(0)+be_2(0)+ce_3(0)$$
$$=\begin{pmatrix}a&b+c&a\\b-c&-2a&b-c\\a&b+c&a\end{pmatrix},$$

where

$$a=\sum_{m=0}^{\infty}a_m\lambda^{-m},\quad b=\sum_{m=0}^{\infty}b_m\lambda^{-m},$$
$$c=\sum_{m=0}^{\infty}c_m\lambda^{-m}.$$

Solving the stationary zero curvature equation

$$V_x=[U,V], \tag{2}$$

we obtain the recursive relations:

$$a_{mx}=2vb_m-2uc_m,\quad b_{mx}=4c_{m+1}-4va_m,\quad c_{mx}=4b_{m+1}-4ua_m;$$
$$b_0=0,\quad c_0=0,\quad a_0=\alpha,\quad b_1=\alpha u,\quad c_1=\alpha v,\quad a_1=0,\quad b_2=\frac{\alpha}{4}v_x,\quad c_2=\frac{\alpha}{4}u_x,\quad a_2=\frac{\alpha}{4}(v^2-u^2),$$
$$b_3=\frac{\alpha}{16}u_{xx}+\frac{\alpha}{4}(uv^2-u^3),\quad c_3=\frac{\alpha}{16}v_{xx}+\frac{\alpha}{4}(v^3-vu^2),\quad a_3=\frac{\alpha}{8}(vu_x-uv_x),$$
$$b_4=\frac{\alpha}{64}v_{xxx}+\frac{3\alpha}{16}v^2v_x-\frac{3\alpha}{16}u^2v_x,\quad c_4=\frac{\alpha}{64}u_{xxx}-\frac{3\alpha}{16}u^2u_x+\frac{3\alpha}{16}v^2u_x.$$

Noting that

$$V_+^{(n)}=\sum_{m=0}^{n}(a_me_1(n-m)+b_me_2(n-m)+c_me_3(n-m)),\quad V_-^{(n)}=\lambda^nV-V_+^{(n)},$$

we can write Eq.(2) as

$$-V_{+x}^{(n)}+[U,V_+^{(n)}]=V_{-x}^{(n)}-[U,V_-^{(n)}]. \tag{3}$$

It is easy to verify that the terms on the left-hand side of Eq.(3) are of degree ≥ 0, while the terms on the right-hand side are of degree ≤ 1. Therefore, the terms of both sides are of degree 0,1. So

$$-V_{+x}^{(n)}+[U,V_+^{(n)}]=-4b_{m+1}e_3(0)-4c_{m+1}e_2(0).$$

Taking $V^{(n)}=V_+^{(n)}$, we find that the zero curvature equation determines the Lax integrable system as follows:

$$u_{tm}=\begin{pmatrix}u\\v\end{pmatrix}_{tm}=\begin{pmatrix}0&-1\\1&0\end{pmatrix}\begin{pmatrix}4b_{m+1}\\-4c_{m+1}\end{pmatrix}$$
$$=JG_{m+1}.$$

$$G_m=\begin{pmatrix}2u\partial^{-1}v&-\frac{1}{4}\partial+2u\partial^{-1}u\\-\frac{1}{4}\partial-2v\partial^{-1}v&-2v\partial^{-1}u\end{pmatrix}$$
$$=LG_{m-1}. \tag{4}$$

Taking $m=2$ and $m=3$ in Eq.(4) we get two nonlinear equations

$$u_t=\frac{\alpha}{4}v_{xx}+\alpha(v^3-vu^2),$$
$$v_t=\frac{\alpha}{4}u_{xx}+\alpha(uv^2-u^3),$$
$$u_t=\frac{\alpha}{16}u_{xxx}-\frac{3\alpha}{4}u^2u_x+\frac{3\alpha}{4}v^2u_x,$$
$$v_t=\frac{\alpha}{16}v_{xxx}+\frac{3\alpha}{4}v^2v_x-\frac{3\alpha}{4}u^2v_x,$$

where the latter is a coupled nonlinear Schrödinger equation.

A direct calculation gives

$$\left\langle V, \frac{\partial U}{\partial u}\right\rangle = 4b, \quad \left\langle V, \frac{\partial U}{\partial v}\right\rangle = -4c, \quad \left\langle V, \frac{\partial U}{\partial \lambda}\right\rangle = 8a.$$

Substituting the above formulate into trace identity we get

$$\begin{pmatrix} \dfrac{\delta}{\delta u} \\ \dfrac{\delta}{\delta v} \end{pmatrix} (8a) = \lambda^{-\gamma}\frac{\partial}{\partial\lambda}\lambda^{\gamma}\begin{pmatrix} 4b \\ -4c \end{pmatrix}.$$

Comparing the coefficients of λ^{-m-1} we have

$$\frac{\delta}{\delta u}8a_{m+1} = (\gamma - m)G_m.$$

Let $m = 1$, then it leads to $\gamma = 0$. Therefore,

$$u_t = \begin{pmatrix} u \\ v \end{pmatrix}_t = J\frac{\delta H_{m+1}}{\delta u}, \quad H_m = -\frac{8a_{m+1}}{m}. \tag{5}$$

It is easy to verify that

$$JL = L^*J = \begin{pmatrix} \frac{1}{4}\partial + 2v\partial^{-1}v & 2v\partial^{-1}u \\ 2u\partial^{-1}v & -\frac{1}{4}\partial + 2u\partial^{-1}u \end{pmatrix},$$

thus, the system (4) is integrable in the Liouville sense.

3. Integrable couplings of hierarchy (5)

Integrable coupling, a notion presented by Fuchsteiner, originates from an investigation on centreless Virasoro symmetry algebras of integrable systems or soliton equations.[20] Some papers of integrable couplings have been published.[21,22] Using the method proposed in Ref.[22], one can obtain integrable couplings of the system (5). For this purpose, we take a Lie algebra A_1:

$$\widetilde{e_1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \widetilde{e_2} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$
$$\widetilde{e_3} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad [\widetilde{e_1}, \widetilde{e_2}] = -2\widetilde{e_3},$$
$$[\widetilde{e_1}, \widetilde{e_3}] = -2\widetilde{e_2}, \qquad [\widetilde{e_2}, \widetilde{e_3}] = -2\widetilde{e_1}.$$

Let us make the scalar transformation: $e_i \to \alpha_i\widetilde{e_i}$, $i = 1, 2, 3$, where α_i are to be determined. Solving the following algebraic equations:

$$\alpha_1\alpha_2 = -2\alpha_3, \quad \alpha_1\alpha_3 = -2\alpha_2, \quad \alpha_2\alpha_3 = \alpha_1,$$

we get a set of solutions: $\alpha_1 = -2$, $\alpha_2 = \sqrt{2}\,\mathrm{i}$, $\alpha_3 = \sqrt{2}\,\mathrm{i}$. Thus, we have a subalgebra of Lie algebra A_1:

$$\overline{e_1} = \begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix}, \quad \overline{e_2} = \begin{pmatrix} 0 & \sqrt{2}\,\mathrm{i} \\ -\sqrt{2}\,\mathrm{i} & 0 \end{pmatrix},$$
$$\overline{e_3} = \begin{pmatrix} \sqrt{2}\,\mathrm{i} & 0 \\ 0 & -\sqrt{2}\,\mathrm{i} \end{pmatrix}, \quad [\overline{e_1}, \overline{e_2}] = 4\overline{e_3},$$
$$[\overline{e_1}, \overline{e_3}] = 4\overline{e_2}, \quad [\overline{e_2}, \overline{e_3}] = -2\overline{e_1}. \tag{6}$$

Hence Eq.(6) has the same communicative relations as the loop algebra $\widetilde{A_2}$. Therefore, by making use of loop algebra Eq.(6), one also deduces the system (4). Now we expand Eq.(6) into the following high dimensional loop algebra $\widetilde{G}$:

$$e_1(n) = \begin{pmatrix} 0 & -2\lambda^n & 0 \\ -2\lambda^n & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$e_2(n) = \begin{pmatrix} 0 & \sqrt{2}\,\mathrm{i}\lambda^n & 0 \\ -\sqrt{2}\,\mathrm{i}\lambda^n & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$e_3(n) = \begin{pmatrix} \sqrt{2}\,\mathrm{i}\lambda^n & 0 & 0 \\ 0 & -\sqrt{2}\,\mathrm{i}\lambda^n & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$e_4(n) = \begin{pmatrix} 0 & 0 & \lambda^n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad e_5(n) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \lambda^n \\ 0 & 0 & 0 \end{pmatrix},$$

with

$$[e_1(m), e_2(n)] = 4e_3(m+n),$$
$$[e_1(m), e_3(n)] = 4e_2(m+n),$$
$$[e_2(m), e_3(n)] = -2e_1(m+n),$$
$$[e_1(m), e_4(n)] = -2e_5(m+n),$$
$$[e_1(m), e_5(n)] = -2e_4(m+n),$$
$$[e_2(m), e_4(n)] = -\sqrt{2}\,\mathrm{i}e_5(m+n),$$
$$[e_5(m), e_2(n)] = -\sqrt{2}\,\mathrm{i}e_4(m+n),$$
$$[e_3(m), e_4(n)] = \sqrt{2}\,\mathrm{i}e_4(m+n),$$
$$[e_3(m), e_5(n)] = -\sqrt{2}\,\mathrm{i}e_5(m+n),$$
$$[e_4(m), e_5(n)] = 0.$$

Let

$$\widetilde{G_1} = \mathrm{span}\{e_1(n), e_2(n), e_3(n)\},$$
$$\widetilde{G_2} = \mathrm{span}\{e_4(n), e_5(n)\}.$$

then $\widetilde{G_1}$ is isomorphic to $\widetilde{A_1}$,

$$[\widetilde{G_1}, \widetilde{G_2}] \subset \widetilde{G_2}. \tag{7}$$

Take the linear forms as follows:

$$\Psi_x = [U, \Psi], \quad \lambda_t = 0,$$
$$U = e_1(1) + u_1e_2(0) + u_2e_3(0) + u_3e_4(0) + u_4e_5(0). \tag{8}$$

Solving the auxiliary equation (2), we obtain

$$a_{mx} = 2u_2b_m - 2u_1c_m, \quad b_{mx} = 4c_{m+1} - 4u_2a_m, \quad c_{mx} = 4b_{m+1} - 4u_1a_m,$$
$$d_{mx} = \sqrt{2}\,\mathrm{i}u_2d_m - 2f_{m+1} + \sqrt{2}\,\mathrm{i}u_1f_m - \sqrt{2}\,\mathrm{i}u_3c_m + 2u_4a_m + \sqrt{2}\,\mathrm{i}u_4b_m,$$
$$f_{mx} = -\sqrt{2}\,\mathrm{i}u_1d_m - 2d_{m+1} - \sqrt{2}\,\mathrm{i}u_2f_m + \sqrt{2}\,\mathrm{i}u_3b_m + 2u_3a_m + \sqrt{2}\,\mathrm{i}u_4c_m;$$
$$-V_{+x}^{(n)} + [U, V_+^{(n)}] = -4b_{m+1}e_3(0) - 4c_{m+1}e_2(0) + 2f_{m+1}e_4(0) + 2d_{m+1}e_5(0).$$

Set $V^{(n)} = V_+^{(n)}$, then the zero curvature equation determines the Lax integrable system as follows:

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}_t = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 \end{pmatrix} \begin{pmatrix} 4b_{m+1} \\ -4c_{m+1} \\ d_{m+1} \\ -f_{m+1} \end{pmatrix} = J \begin{pmatrix} 4b_{m+1} \\ -4c_{m+1} \\ d_{m+1} \\ -f_{m+1} \end{pmatrix} = JL \begin{pmatrix} 4b_m \\ -4c_m \\ d_m \\ -f_m \end{pmatrix},$$

$$L = \begin{pmatrix} 2u_1\partial^{-1}u_2 & -\frac{1}{4}\partial + 2u_1\partial^{-1}u_1 & 0 & 0 \\ -\frac{1}{4}\partial - 2u_2\partial^{-1}u_2 & -2u_2\partial^{-1}u_1 & 0 & 0 \\ \frac{\sqrt{2}\,\mathrm{i}}{8}u_3 + \frac{1}{2}u_3\partial^{-1}u_2 & \frac{1}{2}u_3\partial^{-1}u_1 - \frac{\sqrt{2}\,\mathrm{i}}{8}u_4 & -\frac{\sqrt{2}\,\mathrm{i}}{2}u_1 & -\frac{1}{2}\partial - \frac{\sqrt{2}\,\mathrm{i}}{2}u_2 \\ -\frac{\sqrt{2}\,\mathrm{i}}{8}u_4 - \frac{1}{2}u_4\partial^{-1}u_2 & -\frac{\sqrt{2}\,\mathrm{i}}{8}u_3 + \frac{1}{2}u_4\partial^{-1}u_1 & \frac{1}{2}\partial - \frac{\sqrt{2}\,\mathrm{i}}{2}u_2 & \frac{\sqrt{2}\,\mathrm{i}}{2}u_1 \end{pmatrix},$$

$$u_t = JL^n(4\alpha u_1, -4\alpha u_2, \alpha u_3, -\alpha u_4)^{\mathrm{T}}. \tag{9}$$

Comparing the structures of J and L in Eq.(9) with those in Eq.(4) and according to the definition of integrable couplings, we can conclude that Eq.(9) are integrable couplings of the system (5).

4. Binary symmetric constrained flows

Take the following isospectral problem:

$$\Psi_x = [U^*, \Psi], \quad \lambda_t = 0, \quad \psi = (\psi_1, \psi_2, \psi_3)^{\mathrm{T}},$$
$$U^* = \begin{pmatrix} -\lambda & -(q-r) & -\lambda \\ -(q+r) & 2\lambda & -(q+r) \\ -\lambda & -(q-r) & -\lambda \end{pmatrix}. \tag{10}$$

Let φ_i, ψ_i $(i = 1, 2, 3)$ and their derivatives in Eqs.(1) and (10) tend to zero when $\|x\| \to \infty$. A direct calculation shows that

$$\frac{\delta\lambda}{\delta u} = \frac{1}{E} \begin{pmatrix} \varphi_1\psi_2 + \varphi_2\psi_1 + \varphi_2\psi_3 + \varphi_3\psi_2 \\ -\varphi_1\psi_2 + \varphi_2\psi_1 + \varphi_2\psi_3 - \varphi_3\psi_2 \end{pmatrix}.$$

For N distinct $\lambda_j (j = 1, 2, ..., N)$ and some fixed integer k $(k = 0, 1, 2, ...)$, bi-symmetric constraint can in general be expressed as

$$J\frac{\delta H_k}{\delta u} = J\beta_k \sum_{j=1}^{N} E_j \frac{\delta\lambda_j}{\delta u}, \tag{11}$$

where β_k is a constant to be determined, E_j are normal constants. As far as Eq.(5) goes, Eq.(11) can be reduced to

$$\begin{pmatrix} 4b_k \\ -4c_k \end{pmatrix} = \beta_k \begin{pmatrix} \langle \Phi_1, \Psi_2 \rangle + \langle \Phi_2, \Psi_1 \rangle + \langle \Phi_2, \Psi_3 \rangle + \langle \Phi_3, \Psi_2 \rangle \\ -\langle \Phi_1, \Psi_2 \rangle + \langle \Phi_2, \Psi_1 \rangle + \langle \Phi_2, \Psi_3 \rangle - \langle \Phi_3, \Psi_2 \rangle \end{pmatrix}. \tag{12}$$

Taking $k = 1$ and $\beta_1 = 2\alpha$, we can reduce Eq.(12) to:

$$u = \frac{1}{2}(\langle \Phi_1, \Psi_2 \rangle + \langle \Phi_2, \Psi_1 \rangle + \langle \Phi_2, \Psi_3 \rangle + \langle \Phi_3, \Psi_2 \rangle),$$
$$v = \frac{1}{2}(\langle \Phi_1, \Psi_2 \rangle - \langle \Phi_2, \Psi_1 \rangle - \langle \Phi_2, \Psi_3 \rangle + \langle \Phi_3, \Psi_2 \rangle). \tag{13}$$

Inserting Eq.(13) into Eqs.(1) and (10) we get the space symmetric constrained flows as follows:

$$\begin{pmatrix} \varphi_{1j} \\ \varphi_{2j} \\ \varphi_{3j} \end{pmatrix}_x = U|_A \begin{pmatrix} \varphi_{1j} \\ \varphi_{2j} \\ \varphi_{3j} \end{pmatrix},$$
$$\begin{pmatrix} \psi_{1j} \\ \psi_{2j} \\ \psi_{3j} \end{pmatrix}_x = U^*|_A \begin{pmatrix} \psi_{1j} \\ \psi_{2j} \\ \psi_{3j} \end{pmatrix}, \qquad (14)$$

where the symbol $|_A$ denotes substitution of Eq.(13) into corresponding isospectral problems.

By direct computation, we have

$$H_0 = \langle \Lambda \Phi_1, \Psi_1 \rangle + (\langle \Phi_2, \Psi_1 \rangle + \langle \Phi_2, \Psi_3 \rangle) \times (\langle \Phi_1, \Psi_2 \rangle + \langle \Phi_3, \Psi_2 \rangle) - 2\langle \Lambda \Phi_2, \Psi_2 \rangle + \langle \Lambda \Phi_3, \Psi_3 \rangle,$$

then we obtain a general Hamiltonian form of the constrained flows Eq.(14) as follows:

$$\Phi_{ix} = \frac{\partial H_0}{\partial \Psi_i}, \quad \Psi_{ix} = -\frac{\partial H_0}{\partial \Phi_i}, \quad (i = 1, 2, 3), \qquad (15)$$

where $\Phi_i = (\varphi_{i1}, ..., \varphi_{iN})^{\mathrm{T}}$, $\Psi_i = (\psi_{\mathrm{i}1}, ..., \psi_{iN})^{\mathrm{T}}$.

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References

[1] Ablowitz M J and Clarkson P A 1991 *Solitons, Nonlinear Evolution Equations and Inverse Scattering* (Cambridge: Cambridge University Press)
[2] Newell A C 1985 *Soliton in Mathematics and Physics* (Philadelphia: SIAM)
[3] Wadati M 1973 *J. Phys. Soc. Jan.* **34** 1289
[4] Wadati M, Sanuki H and Konno K 1975 *Prog. Theor. Phys.* **53** 419
[5] Tu G Z 1989 *J. Math. Phys.* **30** 330
[6] Fan E G 2001 *J. Phys.* A: *Math. Gen.* **34** 513
[7] Yan Z Y and Zhang H Q 2002 *Chaos, Solitons and Fractals* **13** 1439
[8] Yan Z Y 2003 *Chaos, Solitons and Fractals* **15** 639
[9] Fan E G 2001 *Physica* A **301** 105
[10] Fan E G 2000 *J. Math. Phys.* **41** 7769
[11] Ma W X 1992 *J. Math. Ann.* **13** 115
[12] Guo F K and Zhang Y F 2002 *Acta Phys. Chin.* **51** 951 (in Chinese)
[13] Ma W X 1996 *Chaos, Solitons and Fractals* **7** 1227
[14] Zhang Y F 2004 *Chaos, Solitons and Fractals* **21** 305
[15] Zhang Y F and Guo F K 2004 *Acta Phys. Chin.* **53** 1276 (in Chinese)
[16] Cao C W 1990 *Sci. China* Ser. A **33** 528
[17] Zeng Y B 1991 *Phys. Lett.* A **160** 541
[18] Zeng Y B and Hietarinta J 1996 *J. Phys.* A **29** 5241
[19] Qiao Z J 1993 *J. Phys.* A **26** 4407
[20] Fuchstreiner B In 1993 *Applications of Analytic and Geometric Methods to Nonlinear Differential Equations* Clarkson P A ed (Dordrecht: Kluwer) pp125—138
[21] Zhang Y F and Yan Q Y 2003 *Acta Phys. Chin.* **52** 2109 (in Chinese)
[22] Zhang Y F 2003 *Chaos, Solitons and Fractals* **18** 855