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Quantum fluctuations of mesoscopic damped double resonance RLC circuit with mutual capacitance–inductance coupling in thermal excitation state*

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Based on the scheme of damped harmonic oscillator quantization and thermo-field dynamics (TFD), the quantization of mesoscopic damped double resonance RLC circuit with mutual capacitance–inductance coupling is proposed. The quantum fluctuations of charge and current of each loop in a squeezed vacuum state are studied in the thermal excitation case. It is shown that the fluctuations not only depend on circuit inherent parameters, but also rely on excitation quantum number and squeezing parameter. Moreover, due to the finite environmental temperature and damped resistance, the fluctuations increase with the temperature rising, and decay with time.

Keywords: mesoscopic double resonance RLC circuit, linear transformation, thermal excitation state, quantum fluctuation

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1. Introduction

In the 1960s, when Josephson studied the weak juncture of superconductivity, he brought forward the Josephson junction and predicted the Josephson effect[1] which has been employed widely today in the superconducting quantum interference device (SQUID),[2] superfluidity phenomenon,[3,4] Bose–Einstein condensate (BEC),[5] etc. In recent years, the Josephson junction has become a main research subject because it can be equivalent to an RLC (resistance–inductance–capacitance) resonance circuit. In the 1970s, Louisell[6] first discussed the quantum effects of an LC circuit and gave its quantum noise in a vacuum state. With the rapid development of nanometre techniques and microelectronics, the trend of miniaturizing integrated circuits and electronic components becomes stronger and stronger. When the phase coherence length of the charge-carrier reaches the Fermi wavelength, the quantum effects should be taken into account. Afterward, the quantum effects of circuit and device have become one of the hotspots in mesoscopic physics for its potential applications to electronic and optoelectronic devices. In a lot of articles,[7−38] quantum fluctuations of the charge, voltage and current are widely studied in mesoscopic circuits. A lot of research has been done on quantum effects in non-dissipative mesoscopic circuits. Recently, some research has been done on quantum effects under the influence of a resistance and temperature in the dissipative mesoscopic circuits. We also discussed the quantum effects of charge current and voltage in a damped resonance RLC circuit.[31−38] It is well known that the harmonic oscillator model is more significant in quantum mechanics. If an actual physics problem can be equivalent to a quantized harmonic oscillator, it can be conveniently solved. Furthermore, because of the current Joule heat in circuit and environmental temperature, practical circuits are always working in damped states at finite temperatures. The temperature effects must be taken into

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account. But using the scheme of damped harmonic oscillator quantization,[39] thrice linear transformation and TFD theory,[40−44] the articles of the quantum fluctuations of mesoscopic damped double resonance RLC circuit with mutual capacitance–inductance coupling at finite temperatures have not been found yet. As the vacuum state, thermal vacuum state, thermal squeezed vacuum state can be regarded as special cases of a thermal excitation state, it is more general and significant to study quantum fluctuations in a thermal excitation state. In this paper, a mesoscopic damped double resonance RLC circuit with mutual capacitance–inductance coupling is quantized by the scheme of damped harmonic oscillator quantization, thrice linear transformation and TFD theory. Moreover, the quantum fluctuations of charge and current of each loop are researched in a thermal excitation state, thermal squeezed vacuum state and thermal vacuum state.

2. Quantization of the mesoscopic damped double resonance RLC circuit with mutual capacitance-inductance coupling

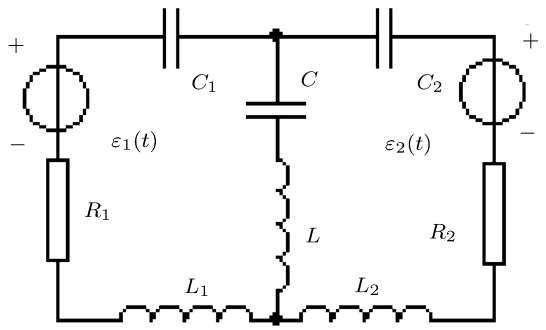


Fig.1. The damped double resonance RLC circuit with mutual capacitance–inductance coupling.

Figure 1 shows a damped double resonance RLC circuit with mutual capacitance–inductance coupling. According to Kirchhoff's law, the classical equations of motion of the system are

$$(L+L_1)\frac{\mathrm{d}^2q_1}{\mathrm{d}t^2}+L\frac{\mathrm{d}^2q_2}{\mathrm{d}t^2}+R_1\frac{\mathrm{d}q_1}{\mathrm{d}t}+\frac{1}{C}(q_1+q_2)+\frac{1}{C_1}q_1=\varepsilon_1(t),$$

$$(L+L_2)\frac{\mathrm{d}^2q_2}{dt^2}+L\frac{\mathrm{d}^2q_1}{dt^2}+R_2\frac{\mathrm{d}q_2}{\mathrm{d}t}+\frac{1}{C}(q_1+q_2)+\frac{1}{C_2}q_2=\varepsilon_2(t), \tag{1}$$

where q_u and $\varepsilon_u(t)$ $(u=1,2)$ denote the charge of capacitance and the voltage of message source, respectively. Defining $p_u=L_u\dot{q}_u$, we obtain

$$\dot{p}_1=-\lambda_1p_1+B_1p_2+B_2q_1+B_3q_2+f_1(t),$$

$$\dot{p}_2=-\lambda_2p_2+D_1p_1+D_2q_1+D_3q_2+f_2(t), \tag{2}$$

where

$$\lambda_1=\frac{R_1(L+L_2)}{LL_1+LL_2+L_1L_2},$$

$$\lambda_2=\frac{R_2(L+L_1)}{LL_1+LL_2+L_1L_2};$$

$$B_1=\frac{R_2LL_1}{L_2(LL_1+LL_2+L_1L_2)},$$

$$D_1=\frac{R_1LL_2}{L_1(LL_1+LL_2+L_1L_2)};$$

$$B_2=\frac{-C_1^{-1}L_1(L+L_2)-C^{-1}L_1L_2}{LL_1+LL_2+L_1L_2},$$

$$D_2=\frac{C_1^{-1}LL_2-C^{-1}L_1L_2}{LL_1+LL_2+L_1L_2};$$

$$B_3=\frac{C_2^{-1}LL_1-C^{-1}L_1L_2}{LL_1+LL_2+L_1L_2},$$

$$D_3=\frac{-C_2^{-1}L_2(L+L_1)-C^{-1}L_1L_2}{LL_1+LL_2+L_1L_2};$$

$$f_1(t)=\frac{L_1(L+L_2)}{LL_1+LL_2+L_1L_2}\varepsilon_1(t)-\frac{LL_1}{LL_1+LL_2+L_1L_2}\varepsilon_2(t),$$

$$f_2(t)=\frac{L_2(L+L_1)}{LL_1+LL_2+L_1L_2}\varepsilon_2(t)-\frac{LL_2}{LL_1+LL_2+L_1L_2}\varepsilon_1(t). \tag{3}$$

First, we consider the following transformations:

$$q_1'=q_1+\delta_1\frac{L_2}{L_1}q_2,\quad q_2'=q_2+\delta_2\frac{L_1}{L_2}q_1;$$

$$p_1'=p_1+\delta_1p_2,\qquad p_2'=p_2+\delta_2p_1, \tag{4}$$

where δ_1 and δ_2 satisfy the following equation: respectively:

$$D_1\delta_1^2-(\lambda_1-\lambda_2)\delta_1-B_1=0,$$

$$B_1\delta_2^2-(\lambda_2-\lambda_1)\delta_2-D_1=0. \tag{5}$$

We obtain

$$\delta_1=\frac{\lambda_1-\lambda_2}{2D_1}+\sqrt{\Big(\frac{\lambda_1-\lambda_2}{2D_1}\Big)^2+\frac{B_1}{D_1}}\,,$$

$$\delta_2=\frac{\lambda_2-\lambda_1}{2B_1}-\sqrt{\Big(\frac{\lambda_1-\lambda_2}{2B_1}\Big)^2+\frac{D_1}{B_1}}\,. \tag{6}$$

From Eqs.(2) and (4), we obtain

$$\dot{q}'_1 = \frac{p'_1}{L_1}, \quad \dot{q}'_2 = \frac{p'_2}{L_2};$$
$$\dot{p}'_1 = -\beta_1 p'_1 - L_1\omega_1^2 q'_1 + \rho_1 q'_2 + f'_1(t),$$
$$\dot{p'}_2 = -\beta_2 p'_2 - L_2\omega_2^2 q'_2 + \rho_2 q'_1 + f'_2(t), \qquad (7)$$

where

$$\beta_1 = \lambda_1 - \delta_1 D_1, \qquad \beta_2 = \lambda_2 - \delta_2 B_1;$$
$$\omega_1 = \sqrt{\frac{1}{1-\delta_1\delta_2}\Big[\frac{\delta_2}{L_2}(B_3+\delta_1 D_3) - \frac{1}{L_1}(B_2+\delta_1 D_2)\Big]},$$
$$\omega_2 = \sqrt{\frac{1}{1-\delta_1\delta_2}\Big[\frac{\delta_1}{L_1}(D_2+\delta_2 B_2) - \frac{1}{L_2}(D_3+\delta_2 B_3)\Big]};$$
$$\rho_1 = \frac{1}{1-\delta_1\delta_2}\Big[(B_3+\delta_1 D_3) - \delta_1\frac{L_2}{L_1}(B_2+\delta_1 D_2)\Big],$$
$$\rho_2 = \frac{1}{1-\delta_1\delta_2}\Big[(D_2+\delta_2 B_2) - \delta_2\frac{L_1}{L_2}(D_3+\delta_2 B_3)\Big];$$
$$f'_1(t) = f_1(t) + \delta_1 f_2(t), \quad f'_2(t) = f_2(t) + \delta_2 f_1(t). \qquad (8)$$

Second, we consider the following commutations:

$$q'_u = X_u\tau^{-1/2}\exp(-\beta_u t/2),$$
$$p'_u = L_u\dot{q}'_u = (Y_u - X_u\beta_u L_u/2)\tau^{-1/2}$$
$$\times \exp(-\beta_u t/2), \quad (u=1,2). \qquad (9)$$

From Eqs.(7) and (9), we obtain

$$\dot{X}_1 = \frac{Y_1}{L_1}, \quad \dot{X}_2 = \frac{Y_2}{L_2};$$
$$\dot{Y}_1 = -L_1\omega_1'^2 X_1 + \rho_1 X_2\exp\Big(\frac{\beta_1-\beta_2}{2}t\Big)$$
$$+ f'_1(t)\exp\Big(\frac{\beta_1}{2}t\Big),$$
$$\dot{Y}_2 = -L_2\omega_2'^2 X_2 + \rho_2 X_1\exp\Big(\frac{\beta_2-\beta_1}{2}t\Big)$$
$$+ f'_2(t)\exp\Big(\frac{\beta_2}{2}t\Big), \qquad (10)$$

where

$$\omega'_1 = \sqrt{\omega_1^2 - \frac{\beta_1^2}{4}}\,, \quad \omega'_2 = \sqrt{\omega_2^2 - \frac{\beta_2^2}{4}}\,. \qquad (11)$$

Third, we consider the following transformations:

$$Q_1 = X_1 + \gamma_1\frac{L_2}{L_1}X_2, \quad Q_2 = \gamma_2\frac{L_1}{L_2}X_1 + X_2;$$
$$P_1 = Y_1 + \gamma_1 Y_2, \quad P_2 = \gamma_2 Y_1 + Y_2, \qquad (12)$$

where γ_1 and γ_2 satisfy the following equation, respectively:

$$L_2\rho_2\exp\Big(\frac{\beta_2-\beta_1}{2}t\Big)\gamma_1^2 - L_1L_2(\omega_1'^2 - \omega_2'^2)\gamma_1$$
$$-L_1\rho_1\exp\Big(\frac{\beta_1-\beta_2}{2}t\Big) = 0,$$
$$L_1\rho_1\exp\Big(\frac{\beta_1-\beta_2}{2}t\Big)\gamma_2^2 - L_1L_2(\omega_2'^2 - \omega_1'^2)\gamma_1$$
$$-L_2\rho_2\exp\Big(\frac{\beta_2-\beta_1}{2}t\Big) = 0. \qquad (13)$$

We obtain

$$\gamma_1 = L_1\frac{(\omega_1'^2-\omega_2'^2) \mp \sqrt{(\omega_1'^2-\omega_2'^2)^2 + \dfrac{4\rho_1\rho_2}{L_1L_2}}}{2\rho_2\exp\Big(\dfrac{\beta_2-\beta_1}{2}t\Big)},$$
$$\gamma_2 = L_2\frac{(\omega_2'^2-\omega_1'^2) \pm \sqrt{(\omega_1'^2-\omega_2'^2)^2 + \dfrac{4\rho_1\rho_2}{L_1L_2}}}{2\rho_1\exp\Big(\dfrac{\beta_1-\beta_2}{2}t\Big)}. \qquad (14)$$

From Eqs.(10) and (12), we obtain

$$\dot{Q}_1 = \frac{P_1}{L_1}, \qquad \dot{Q}_2 = \frac{P_2}{L_2},$$
$$\dot{P}_1 = -L_1\Omega_1^2 Q_1 + F_1(t);$$
$$\dot{P}_2 = -L_2\Omega_2^2 Q_2 + F_2(t), \qquad (15)$$

where

$$\Omega_1 = \sqrt{\omega_1'^2 - \frac{\gamma_1}{L_1}\rho_2\exp\Big(\frac{\beta_2-\beta_1}{2}t\Big)}\,,$$
$$\Omega_2 = \sqrt{\omega_2'^2 - \frac{\gamma_2}{L_2}\rho_1\exp\Big(\frac{\beta_1-\beta_2}{2}t\Big)}\,;$$
$$F_1(t) = f'_1(t)\exp\Big(\frac{\beta_1}{2}t\Big) + \gamma_1 f'_2(t)\exp\Big(\frac{\beta_2}{2}t\Big),$$
$$F_2(t) = \gamma_2 f'_1(t)\exp\Big(\frac{\beta_2}{2}t\Big) + f'_2(t)\exp\Big(\frac{\beta_2}{2}t\Big). \qquad (16)$$

The Q_u and P_u denote coordinate operator and momentum operator in the Heisenberg picture, respectively, which satisfy the following commutation relations:

$$[Q_u, P_v] = j\hbar\delta_{uv},$$
$$[Q_u, Q_v] = [P_u, P_v] = 0, \quad (u,v=1,2). \qquad (17)$$

According to Eq.(15) and canonical Hamiltonian equations

$$\dot{Q}_u = \frac{\partial H}{\partial P_u}, \quad \dot{P}_u = -\frac{\partial H}{\partial Q_u},$$

we obtain

$$H = \frac{P_1^2}{2L_1} + \frac{P_2^2}{2L_2} + \frac{1}{2}L_1\Omega_1^2 Q_1^2$$
$$+ \frac{1}{2}L_2\Omega_2^2 Q_2^2 - F_1(t)Q_1 - F_2(t)Q_2. \qquad (18)$$

We introduce generalized annihilation and creation operators for the two independent oscillators

$$a_u = \frac{1}{\sqrt{2L_u\Omega_u\hbar}}(L_u\Omega_u Q_u + \mathrm{j}P_u),$$

$$a_u^+ = \frac{1}{\sqrt{2L_u\Omega_u\hbar}}(L_u\Omega_u Q_u - \mathrm{j}P_u). \tag{19}$$

Substituting Eq.(19) into Eq.(18), the Hamiltonian of this system can be written as

$$\begin{aligned} H = {} & \hbar\Omega_1\Big(a_1^+a_1 + \frac{1}{2}\Big) + \hbar\Omega_2\Big(a_2^+a_2 + \frac{1}{2}\Big) \\ & - \sqrt{\frac{\hbar}{2L_1\Omega_1}}(a_1 + a_1^+)F_1(t) \\ & - \sqrt{\frac{\hbar}{2L_2\Omega_2}}(a_2 + a_2^+)F_2(t). \end{aligned} \tag{20}$$

When $\varepsilon_u(t) = 0$, the Hamiltonian of this system is

$$H = \hbar\Omega_1\Big(a_1^+a_1 + \frac{1}{2}\Big) + \hbar\Omega_2\Big(a_2^+a_2 + \frac{1}{2}\Big), \tag{21}$$

which shows that when a mesoscopic damped double resonance RLC circuit with mutual capacitance–inductance coupling is quantized, it is equivalent to quantized coupled double harmonic oscillators. Equation (21) has reduced to a diagonalized non-coupled boson systematic Hamiltonian. Therefore, the energy spectrum of mesoscopic damped double resonance RLC circuit with mutual capacitance–inductance coupling can be written as

$$\begin{aligned} & E_{n_1,n_2} = \hbar\Omega_1 n_1 + \hbar\Omega_2 n_2 + \frac{1}{2}(\hbar\Omega_1 + \hbar\Omega_2), \\ & (n_1, n_2 = 0, 1, 2, \cdots). \end{aligned} \tag{22}$$

3. Thermal excitation state

Vacuum state $|0\rangle$ ($a|0\rangle = 0$) can be defined by the nature of boson harmonic oscillator operators. The reciprocal of the quality factor $1/Q$ ($Q = (1/R)\sqrt{L/C}$) of resonance circuit can be regard as the squeezed coefficient of squeezed operator $S(z) = \exp[\frac{1}{2}z^*a^2 - \frac{1}{2}z(a^+)^2]$ to prepare a squeezed vacuum state. $z = r\mathrm{e}^{\mathrm{j}\theta}$, r ($0 \le r \le \infty$), and θ represent squeezed coefficient and squeezed angle, respectively. The formula used to describe the squeezed vacuum state can take the form[45]

$$\begin{aligned} |0\rangle_r &= S(z)|0\rangle \\ &= \mathrm{sech}^{1/2}r \sum_{n=0}^{\infty} \frac{(-\mathrm{e}^{\mathrm{j}\theta}\tanh r)^n[(2n)!]^{1/2}}{n!2^n}|2n\rangle. \end{aligned} \tag{23}$$

The inverses of boson harmonic oscillator annihilation and creation operators are introduced by their actions of Fock state $|n\rangle$[46]

$$\begin{aligned} & a^{-1}|n\rangle = \frac{1}{\sqrt{n+1}}|n+1\rangle, \\ & (a^+)^{-1}|n\rangle = \begin{cases} \dfrac{1}{\sqrt{n}}|n-1\rangle, & (n \ne 0), \\ 0, & (n = 0). \end{cases} \end{aligned} \tag{24}$$

We can find that a^{-1} is the right inverse of a and $(a^+)^{-1}$ is the left inverse of a^+. According to the nature of inverse boson harmonic oscillator operators, excitation state of squeezed vacuum state $|0\rangle_{\tilde{r}} = a^{-k}|0\rangle_r$ can be defined as

$$\begin{aligned} |0\rangle_{\tilde{r}} &= a^{-k}|0\rangle_r \\ &= D\mathrm{sech}^{1/2}r \sum_{n=0}^{\infty} \frac{(-\mathrm{e}^{\mathrm{j}\theta}\tanh r)^n[(2n)!]^{1/2}}{n!2^n} \\ &\quad \times \sqrt{\frac{(2n)!}{(2n+k)!}}|2n+k\rangle, \end{aligned} \tag{25}$$

where $k (k = 1, 2, 3, ...)$ is the excitation quantum number, and the normalization coefficient is

$$|D|^2 = \Big\{\mathrm{sech}r \sum_{n=0}^{\infty} \frac{(\tanh r)^{2n}[(2n)!]^2}{(n!)^2 2^{2n}(2n+k)!}\Big\}^{-1}.$$

For the circuit in Fig.1, it is equivalent to two mutual independent boson harmonic oscillators via linear transformations of Eqs.(4), (9), (12) and its quantization. And the Hamiltonian is in the form of Eq.(21).

Therefore, the excitation state of the squeezed vacuum state $|00\rangle_{\tilde{r}_1,\tilde{r}_2}$ takes the following form in the particle number representation:

$$\begin{aligned} |00\rangle_{\tilde{r}_1\tilde{r}_2} = {} & D_1\mathrm{sech}^{1/2}r_1 \sum_{n=0}^{\infty} \frac{(-\mathrm{e}^{\mathrm{j}\theta_1}\tanh r_1)^n[(2n)!]^{1/2}}{n!2^n} \sqrt{\frac{(2n)!}{(2n+k_1)!}}|2n+k_1\rangle \\ & \otimes D_2\mathrm{sech}^{1/2}r_2 \sum_{n=0}^{\infty} \frac{(-\mathrm{e}^{\mathrm{j}\theta_2}\tanh r_2)^n[(2n)!]^{1/2}}{n!2^n} \sqrt{\frac{(2n)!}{(2n+k_2)!}}|2n+k_2\rangle, \end{aligned} \tag{26}$$

where k_u, r_u, and θ_u ($u = 1, 2$) denote the excitation quantum number, squeezed coefficient and squeezed angle, respectively. Normalized coefficient D_u is

$$|D_u|^2 = \left\{ \mathrm{sech} r_u \sum_{n=0}^{\infty} \frac{(\tanh r_u)^{2n}[(2n)!]^2}{(n!)^2 2^{2n}(2n+k_u)!} \right\}^{-1}, \quad (u=1,2). \tag{27}$$

Taking into account the effect of current joule heat and environmental temperature, then the circuit will evolve into a thermal excitation state from its excitation state of the squeezed vacuum state. We introduce a relative tilde space besides the Hilbert space in TFD theory, and the direct product space is made up of above two spaces. Operators and states in the Hilbert space each have their corresponding operator and state in the tilde space. The a_u and a_u^+ have corresponding operators $\tilde{a}_u$ and $\tilde{a}_u^+$ which obey

$$[\tilde{a}_u, \tilde{a}_v^+] = \delta_{uv}, [\tilde{a}_u, a_v] = [\tilde{a}_u, a_v^+] = [a_u, \tilde{a}_v^+] = [a_u^+, \tilde{a}_v^+] = 0, \quad (u,v=1,2). \tag{28}$$

In the direct product space of TFD, the vacuum state $|00\tilde{0}\tilde{0}\rangle$ of two-mode is zero temperature in H-T direct product space. The thermal vacuum state $|00\tilde{0}\tilde{0}\rangle_T$ can be obtained by the following definition at a finite temperature T:

$$|00\tilde{0}\tilde{0}\rangle_T = T(\eta_1)T(\eta_2)|00\tilde{0}\tilde{0}\rangle\,, \tag{29}$$

where

$$T(\eta_u) = \exp[-\eta_u(a_u\tilde{a}_u - a_u^+\tilde{a}_u^+)]\,. \tag{30}$$

Here $T(\eta_u)$ denotes the thermal Bogoliubov transformation. The η_u is related to counted average thermal particle number n_{0u} by $n_{0u} = \sinh^2\eta_u$. We obtain the relation between n_{0u} and temperature T by the Bose–Einstein distribution

$$n_{0u} = [\exp(\hbar\Omega_u/k_{\mathrm{B}}T) - 1]^{-1}, \tag{31}$$

where Ω_u is the resonance frequency of the field, k_{B} is the Boltzmann constant. Definition of thermal excitation state is

$$|00\tilde{0}\tilde{0}\rangle_{T,\tilde{r}_1\tilde{r}_2} = T(\eta_1)T(\eta_2)|00\tilde{0}\tilde{0}\rangle_{\tilde{r}_1\tilde{r}_2}. \tag{32}$$

4. The quantum fluctuations of charge and current in thermal excitation state

Now, we study the quantum fluctuations of the charge and current in a thermal excitation state, thermal squeezed vacuum state, thermal vacuum state and vacuum state separately.

From Eqs.(28) and (30), we obtain

$$T^+(\eta_u)a_uT(\eta_u) = a_u\cosh\eta_u + \tilde{a}_u^+\sinh\eta_u, \quad \text{and} \quad T^+(\eta_u)a_u^+T(\eta_u) = a_u^+\cosh\eta_u + \tilde{a}_u\sinh\eta_u, \tag{33}$$

where we have used the relation

$$e^{\mathrm{A}}Be^{-\mathrm{A}} = B + [A,B] + \frac{1}{2!}[A,[A,B]] + \frac{1}{3!}[A,[A,[A,B]]] + \cdots\,. \tag{34}$$

From Eqs.(4), (9), (12) and (23), we obtain

$$\begin{aligned}
q_1 &= \sqrt{\frac{\hbar}{2L_1\Omega_1}}(\sigma_1 + \delta_1\gamma_2\sigma_2)m_1 - \frac{L_2}{L_1}\sqrt{\frac{\hbar}{2L_2\Omega_2}}(\gamma_1\sigma_1 + \delta_1\sigma_2)m_2,\\
q_2 &= \sqrt{\frac{\hbar}{2L_2\Omega_2}}(\sigma_2 + \delta_2\gamma_1\sigma_1)m_2 - \frac{L_1}{L_2}\sqrt{\frac{\hbar}{2L_1\Omega_1}}(\gamma_2\sigma_2 + \delta_2\sigma_1)m_1;\\
p_1 &= \mathrm{j}\sqrt{\frac{L_2\Omega_2\hbar}{2}}(\gamma_1\sigma_1 + \delta_1\sigma_2)m_2' - \mathrm{j}\sqrt{\frac{L_1\Omega_1\hbar}{2}}(\sigma_1 + \delta_1\gamma_2\sigma_2)m_1'\\
&\quad - \frac{L_1}{2}\sqrt{\frac{\hbar}{2L_1\Omega_1}}(\beta_1\sigma_1 + \delta_1\beta_2\gamma_2\sigma_2)m_1 + \frac{L_2}{2}\sqrt{\frac{\hbar}{2L_2\Omega_2}}(\beta_1\gamma_1\sigma_1 + \delta_1\beta_2\sigma_2)m_2,\\
p_2 &= \mathrm{j}\sqrt{\frac{L_1\Omega_1\hbar}{2}}(\gamma_2\sigma_2 + \delta_2\sigma_1)m_1' - \mathrm{j}\sqrt{\frac{L_2\Omega_2\hbar}{2}}(\sigma_2 + \delta_2\gamma_1\sigma_1)m_2'
\end{aligned}$$

$$-\frac{L_2}{2}\sqrt{\frac{\hbar}{2L_2\Omega_2}}(\beta_2\sigma_2+\delta_2\beta_1\gamma_1\sigma_1)m_2+\frac{L_1}{2}\sqrt{\frac{\hbar}{2L_1\Omega_1}}(\beta_2\gamma_2\sigma_2+\delta_2\beta_1\sigma_1)m_1, \tag{35}$$

where

$$\sigma_u=\frac{\tau^{-1/2}}{(1-\delta_1\delta_2)(1-\gamma_1\gamma_2)}\exp\Big(-\frac{\beta_u}{2}t\Big),\quad m_u=(a_u+a_u^+)\cosh\eta_u+(\tilde{a}_u+\tilde{a}_u^+)\sinh\eta_u,$$
$$m'_u=(a_u-a_u^+)\cosh\eta_u-(\tilde{a}_u-\tilde{a}_u^+)\sinh\eta_u,\quad (u=1,2).$$

From Eqs.(26), (28), (32) and (35), we can obtain that the mean values and mean-square values of charge and current in a thermal excitation state as follows:

$$\langle q_1\rangle_{T,\tilde{r}_1\tilde{r}_2}=0,\qquad \langle q_2\rangle_{T,\tilde{r}_1\tilde{r}_2}=0;\qquad \langle p_1\rangle_{T,\tilde{r}_1\tilde{r}_2}=0,\qquad \langle p_2\rangle_{T,\tilde{r}_1\tilde{r}_2}=0;$$
$$\langle q_1^2\rangle_{T,\tilde{r}_1\tilde{r}_2}=\frac{\hbar}{2L_1\Omega_1}(\sigma_1+\delta_1\gamma_2\sigma_2)^2x_1+\frac{\hbar}{2L_2\Omega_2}\Big(\frac{L_2}{L_1}\Big)^2(\gamma_1\sigma_1+\delta_1\sigma_2)^2x_2,$$
$$\langle q_2^2\rangle_{T,\tilde{r}_1\tilde{r}_2}=\frac{\hbar}{2L_2\Omega_2}(\sigma_2+\delta_2\gamma_1\sigma_1)^2x_2+\Big(\frac{L_1}{L_2}\Big)^2\frac{\hbar}{2L_1\Omega_1}(\gamma_2\sigma_2+\delta_2\sigma_1)^2x_1;$$
$$\begin{aligned}\langle p_1^2\rangle_{T,\tilde{r}_1\tilde{r}_2}=&\frac{L_1\Omega_1\hbar}{2}(\sigma_1+\delta_1\gamma_2\sigma_2)^2y_1+\frac{\hbar}{2L_1\Omega_1}\Big(\frac{L_1}{2}\Big)^2(\beta_1\sigma_1+\delta_1\beta_2\gamma_2\sigma_2)^2x_1\\&+\frac{L_2\Omega_2\hbar}{2}(\gamma_1\sigma_1+\delta_1\sigma_2)^2y_2+\frac{\hbar}{2L_2\Omega_2}\Big(\frac{L_2}{2}\Big)^2(\beta_1\gamma_1\sigma_1+\delta_1\beta_2\sigma_2)^2x_2\\&+\frac{L_1\hbar}{2}(\sigma_1+\delta_1\gamma_2\sigma_2)(\beta_1\sigma_1+\delta_1\beta_2\gamma_2\sigma_2)z_1+\frac{L_2\hbar}{2}(\gamma_1\sigma_1+\delta_1\sigma_2)(\beta_1\gamma_1\sigma_1+\delta_1\beta_2\sigma_2)z_2,\end{aligned}$$
$$\begin{aligned}\langle p_2^2\rangle_{T,\tilde{r}_1\tilde{r}_2}=&\frac{L_1\Omega_1\hbar}{2}(\gamma_2\sigma_2+\delta_2\sigma_1)^2y_1+\frac{\hbar}{2L_1\Omega_1}\Big(\frac{L_1}{2}\Big)^2(\beta_2\gamma_2\sigma_2+\delta_2\beta_1\sigma_1)^2x_1\\&+\frac{L_2\Omega_2\hbar}{2}(\sigma_2+\delta_2\gamma_1\sigma_1)^2y_2+\frac{\hbar}{2L_2\Omega_2}\Big(\frac{L_2}{2}\Big)^2(\beta_2\sigma_2+\delta_2\beta_1\gamma_1\sigma_1)^2x_2\\&+\frac{L_1\hbar}{2}(\gamma_2\sigma_2+\delta_2\sigma_1)(\beta_2\gamma_2\sigma_2+\delta_2\beta_1\sigma_1)z_1+\frac{L_2\hbar}{2}(\sigma_2+\delta_2\gamma_1\sigma_1)(\beta_2\sigma_2+\delta_2\beta_1\gamma_1\sigma_1)z_2,\end{aligned} \tag{36}$$

where

$$x_u=|D_u|^2\coth\Big(\frac{\hbar\Omega_u}{2k_{\rm B}T}\Big)\mathrm{sech}r_u\Big\{\sum_{n=0}^{\infty}\frac{[\tanh r_u]^{2n}[(2n)!]^2}{(n!)^2 2^{2n}(2n+k_u)!}[4n+2k_u+1-2(2n+1)\cos\theta_u\tanh r_u]\Big\},$$
$$y_u=|D_u|^2\coth\Big(\frac{\hbar\Omega_u}{2k_{\rm B}T}\Big)\mathrm{sech}r_u\Big\{\sum_{n=0}^{\infty}\frac{[\tanh r_u]^{2n}[(2n)!]^2}{(n!)^2 2^{2n}(2n+k_u)!}[4n+2k_u+1+2(2n+1)\cos\theta_u\tanh r_u]\Big\},$$
$$z_u=|D_u|^2\coth\Big(\frac{\hbar\Omega_u}{2k_{\rm B}T}\Big)\mathrm{sech}r_u\Big\{\sum_{n=0}^{\infty}\frac{(\tanh r_u)^{2n}[(2n)!]^2}{(n!)^2 2^{2n}(2n+k_u)!}2(2n+1)\sin\theta_u\tanh r_u\Big\},\quad (u=1,2). \tag{37}$$

We see that the mean values of charge and current are not related to temperature. But the mean-square values of charge and current are related to environmental temperature.

Therefore, from Eq.(36), we obtain the quantum fluctuations of the charge and current in a thermal excitation state, and they are

$$\langle(\Delta q_1)^2\rangle_{T,\tilde{r}_1\tilde{r}_2}=\frac{\hbar}{2L_1\Omega_1}(\sigma_1+\delta_1\gamma_2\sigma_2)^2x_1+\frac{\hbar}{2L_2\Omega_2}\Big(\frac{L_2}{L_1}\Big)^2(\gamma_1\sigma_1+\delta_1\sigma_2)^2x_2,$$
$$\langle(\Delta q_2)^2\rangle_{T,\tilde{r}_1\tilde{r}_2}=\frac{\hbar}{2L_2\Omega_2}(\sigma_2+\delta_2\gamma_1\sigma_1)^2x_2+\Big(\frac{L_1}{L_2}\Big)^2\frac{\hbar}{2L_1\Omega_1}(\gamma_2\sigma_2+\delta_2\sigma_1)^2x_1;$$
$$\begin{aligned}\langle(\Delta p_1)^2\rangle_{T,\tilde{r}_1\tilde{r}_2}=&\frac{L_1\Omega_1\hbar}{2}(\sigma_1+\delta_1\gamma_2\sigma_2)^2y_1+\frac{\hbar}{2L_1\Omega_1}\Big(\frac{L_1}{2}\Big)^2(\beta_1\sigma_1+\delta_1\beta_2\gamma_2\sigma_2)^2x_1\\&+\frac{L_2\Omega_2\hbar}{2}(\gamma_1\sigma_1+\delta_1\sigma_2)^2y_2+\frac{\hbar}{2L_2\Omega_2}\Big(\frac{L_2}{2}\Big)^2(\beta_1\gamma_1\sigma_1+\delta_1\beta_2\sigma_2)^2x_2\\&+\frac{L_1\hbar}{2}(\sigma_1+\delta_1\gamma_2\sigma_2)(\beta_1\sigma_1+\delta_1\beta_2\gamma_2\sigma_2)z_1+\frac{L_2\hbar}{2}(\gamma_1\sigma_1+\delta_1\sigma_2)(\beta_1\gamma_1\sigma_1+\delta_1\beta_2\sigma_2)z_2,\end{aligned}$$

$$\langle(\Delta p_2)^2\rangle_{T,\tilde{r}_1\tilde{r}_2} = \frac{L_1\Omega_1\hbar}{2}(\gamma_2\sigma_2+\delta_2\sigma_1)^2y_1 + \frac{\hbar}{2L_1\Omega_1}\Big(\frac{L_1}{2}\Big)^2(\beta_2\gamma_2\sigma_2+\delta_2\beta_1\sigma_1)^2x_1$$
$$+ \frac{L_2\Omega_2\hbar}{2}(\sigma_2+\delta_2\gamma_1\sigma_1)^2y_2 + \frac{\hbar}{2L_2\Omega_2}\Big(\frac{L_2}{2}\Big)^2(\beta_2\sigma_2+\delta_2\beta_1\gamma_1\sigma_1)^2x_2$$
$$+ \frac{L_1\hbar}{2}(\gamma_2\sigma_2+\delta_2\sigma_1)(\beta_2\gamma_2\sigma_2+\delta_2\beta_1\sigma_1)z_1 + \frac{L_2\hbar}{2}(\sigma_2+\delta_2\gamma_1\sigma_1)(\beta_2\sigma_2+\delta_2\beta_1\gamma_1\sigma_1)z_2. \tag{38}$$

We see that the quantum fluctuations not only depend on the circuit inherent parameters, but also rely on the excitation quantum number and squeezing parameter. Moreover, due to the existence of finite environmental temperature and damped resistance, the fluctuations increase with the temperature rising, and decay with time.

When $k_u \to 0$, The circuit will evolve into a thermal squeezed vacuum state from its thermal excitation state. From Eq.(38), we obtain the quantum fluctuations of the charge and current in a thermal squeezed vacuum state as follows:

$$\langle(\Delta q_1)^2\rangle_{T,r_1r_2} = \frac{\hbar}{2L_1\Omega_1}(\sigma_1+\delta_1\gamma_2\sigma_2)^2x_1' + \frac{\hbar}{2L_2\Omega_2}\Big(\frac{L_2}{L_1}\Big)^2(\gamma_1\sigma_1+\delta_1\sigma_2)^2x_2',$$
$$\langle(\Delta q_2)^2\rangle_{T,r_1r_2} = \frac{\hbar}{2L_2\Omega_2}(\sigma_2+\delta_2\gamma_1\sigma_1)^2x_2' + \Big(\frac{L_1}{L_2}\Big)^2\frac{\hbar}{2L_1\Omega_1}(\gamma_2\sigma_2+\delta_2\sigma_1)^2x_1';$$
$$\langle(\Delta p_1)^2\rangle_{T,r_1r_2} = \frac{L_1\Omega_1\hbar}{2}(\sigma_1+\delta_1\gamma_2\sigma_2)^2y_1' + \frac{\hbar}{2L_1\Omega_1}\Big(\frac{L_1}{2}\Big)^2(\beta_1\sigma_1+\delta_1\beta_2\gamma_2\sigma_2)^2x_1'$$
$$+ \frac{L_2\Omega_2\hbar}{2}(\gamma_1\sigma_1+\delta_1\sigma_2)^2y_2' + \frac{\hbar}{2L_2\Omega_2}\Big(\frac{L_2}{2}\Big)^2(\beta_1\gamma_1\sigma_1+\delta_1\beta_2\sigma_2)^2x_2'$$
$$+ \frac{L_1\hbar}{2}(\sigma_1+\delta_1\gamma_2\sigma_2)(\beta_1\sigma_1+\delta_1\beta_2\gamma_2\sigma_2)z_1' + \frac{L_2\hbar}{2}(\gamma_1\sigma_1+\delta_1\sigma_2)(\beta_1\gamma_1\sigma_1+\delta_1\beta_2\sigma_2)z_2',$$
$$\langle(\Delta p_2)^2\rangle_{T,r_1r_2} = \frac{L_1\Omega_1\hbar}{2}(\gamma_2\sigma_2+\delta_2\sigma_1)^2y_1' + \frac{\hbar}{2L_1\Omega_1}\Big(\frac{L_1}{2}\Big)^2(\beta_2\gamma_2\sigma_2+\delta_2\beta_1\sigma_1)^2x_1'$$
$$+ \frac{L_2\Omega_2\hbar}{2}(\sigma_2+\delta_2\gamma_1\sigma_1)^2y_2' + \frac{\hbar}{2L_2\Omega_2}\Big(\frac{L_2}{2}\Big)^2(\beta_2\sigma_2+\delta_2\beta_1\gamma_1\sigma_1)^2x_2'$$
$$+ \frac{L_1\hbar}{2}(\gamma_2\sigma_2+\delta_2\sigma_1)(\beta_2\gamma_2\sigma_2+\delta_2\beta_1\sigma_1)z_1' + \frac{L_2\hbar}{2}(\sigma_2+\delta_2\gamma_1\sigma_1)(\beta_2\sigma_2+\delta_2\beta_1\gamma_1\sigma_1)z_2', \tag{39}$$

where

$$x_u' = |D_u'|^2\coth\Big(\frac{\hbar\Omega_u}{2k_{\rm B}T}\Big)\mathrm{sech}r_u\Big\{\sum_{n=0}^{\infty}\frac{(\tanh r_u)^{2n}(2n)!}{(n!)^2 2^{2n}}[4n+1-2(2n+1)\cos\theta_u\tanh r_u]\Big\},$$
$$y_u' = |D_u'|^2\coth\Big(\frac{\hbar\Omega_u}{2k_{\rm B}T}\Big)\mathrm{sech}r_u\Big\{\sum_{n=0}^{\infty}\frac{(\tanh r_u)^{2n}(2n)!}{(n!)^2 2^{2n}}[4n+1+2(2n+1)\cos\theta_u\tanh r_u]\Big\},$$
$$z_u' = |D_u'|^2\coth\Big(\frac{\hbar\Omega_u}{2k_{\rm B}T}\Big)\mathrm{sech}r_u\Big\{\sum_{n=0}^{\infty}\frac{(\tanh r_u)^{2n}(2n)!}{(n!)^2 2^{2n}}2(2n+1)\sin\theta_u\tanh r_u\Big\},\quad (u=1,2). \tag{40}$$

Here

$$|D_u'|^2 = \Big\{\mathrm{sech}r_u\sum_{n=0}^{\infty}\frac{(\tanh r_u)^{2n}(2n)!}{(n!)^2 2^{2n}}\Big\}^{-1}. \tag{41}$$

When $r_u \to 0$ and $\theta_u \to 0$, thus $\cosh 2r_u \to 1$, $\sinh 2r_u \to 0$, $\cos\theta_u \to 1$, $\sin\theta_u \to 0$, then $x_u' = y_u' = \coth\big(\frac{\hbar\Omega_u}{2k_{\rm B}T}\big)$, and $z_u' = 0$. The circuit will evolve into a thermal vacuum state from its thermal squeezed vacuum state. From Eq.(39), we obtain the quantum fluctuations of the charge and current in thermal vacuum state, namely

$$\langle(\Delta q_1)^2\rangle_T = \frac{\hbar}{2L_1\Omega_1}(\sigma_1+\delta_1\gamma_2\sigma_2)^2\coth\Big(\frac{\hbar\Omega_1}{2k_{\rm B}T}\Big) + \frac{\hbar}{2L_2\Omega_2}\Big(\frac{L_2}{L_1}\Big)^2(\gamma_1\sigma_1+\delta_1\sigma_2)^2\coth\Big(\frac{\hbar\Omega_2}{2k_{\rm B}T}\Big),$$
$$\langle(\Delta q_2)^2\rangle_T = \frac{\hbar}{2L_2\Omega_2}(\sigma_2+\delta_2\gamma_1\sigma_1)^2\coth\Big(\frac{\hbar\Omega_2}{2k_{\rm B}T}\Big) + \Big(\frac{L_1}{L_2}\Big)^2\frac{\hbar}{2L_1\Omega_1}(\gamma_2\sigma_2+\delta_2\sigma_1)^2\coth\Big(\frac{\hbar\Omega_1}{2k_{\rm B}T}\Big);$$
$$\langle(\Delta p_1)^2\rangle_T = \Big[\frac{L_1\Omega_1\hbar}{2}(\sigma_1+\delta_1\gamma_2\sigma_2)^2 + \frac{\hbar}{2L_1\Omega_1}\Big(\frac{L_1}{2}\Big)^2(\beta_1\sigma_1+\delta_1\beta_2\gamma_2\sigma_2)^2\Big]\coth\Big(\frac{\hbar\Omega_1}{2k_{\rm B}T}\Big)$$

$$+\Big[\frac{L_2\Omega_2\hbar}{2}(\gamma_1\sigma_1+\delta_1\sigma_2)^2+\frac{\hbar}{2L_2\Omega_2}\Big(\frac{L_2}{2}\Big)^2(\beta_1\gamma_1\sigma_1+\delta_1\beta_2\sigma_2)^2\Big]\coth\Big(\frac{\hbar\Omega_2}{2k_{\rm B}T}\Big),$$

$$\langle(\Delta p_2)^2\rangle_T=\Big[\frac{L_1\Omega_1\hbar}{2}(\gamma_2\sigma_2+\delta_2\sigma_1)^2+\frac{\hbar}{2L_1\Omega_1}\Big(\frac{L_1}{2}\Big)^2(\beta_2\gamma_2\sigma_2+\delta_2\beta_1\sigma_1)^2\Big]\coth\Big(\frac{\hbar\Omega_1}{2k_{\rm B}T}\Big)$$
$$+\Big[\frac{L_2\Omega_2\hbar}{2}(\sigma_2+\delta_2\gamma_1\sigma_1)^2+\frac{\hbar}{2L_2\Omega_2}\Big(\frac{L_2}{2}\Big)^2(\beta_2\sigma_2+\delta_2\beta_1\gamma_1\sigma_1)^2\Big]\coth\Big(\frac{\hbar\Omega_2}{2k_{\rm B}T}\Big). \quad (42)$$

When $T\to 0$, and $\coth(\frac{\hbar\Omega_u}{2k_{\rm B}T})\to 1$, the circuit will evolve into a vacuum state from its thermal vacuum state. From Eq.(42), we obtain the following quantum fluctuations of the charge and current in vacuum state:

$$\langle(\Delta q_1)^2\rangle=\frac{\hbar}{2L_1\Omega_1}(\sigma_1+\delta_1\gamma_2\sigma_2)^2+\frac{\hbar}{2L_2\Omega_2}\Big(\frac{L_2}{L_1}\Big)^2(\gamma_1\sigma_1+\delta_1\sigma_2)^2,$$

$$\langle(\Delta q_2)^2\rangle=\frac{\hbar}{2L_2\Omega_2}(\sigma_2+\delta_2\gamma_1\sigma_1)^2+\Big(\frac{L_1}{L_2}\Big)^2\frac{\hbar}{2L_1\Omega_1}(\gamma_2\sigma_2+\delta_2\sigma_1)^2;$$

$$\langle(\Delta p_1)^2\rangle=\frac{L_1\Omega_1\hbar}{2}(\sigma_1+\delta_1\gamma_2\sigma_2)^2+\frac{\hbar}{2L_1\Omega_1}\Big(\frac{L_1}{2}\Big)^2(\beta_1\sigma_1+\delta_1\beta_2\gamma_2\sigma_2)^2$$
$$+\frac{L_2\Omega_2\hbar}{2}(\gamma_1\sigma_1+\delta_1\sigma_2)^2+\frac{\hbar}{2L_2\Omega_2}\Big(\frac{L_2}{2}\Big)^2(\beta_1\gamma_1\sigma_1+\delta_1\beta_2\sigma_2)^2,$$

$$\langle(\Delta p_2)^2\rangle=\frac{L_1\Omega_1\hbar}{2}(\gamma_2\sigma_2+\delta_2\sigma_1)^2+\frac{\hbar}{2L_1\Omega_1}\Big(\frac{L_1}{2}\Big)^2(\beta_2\gamma_2\sigma_2+\delta_2\beta_1\sigma_1)^2$$
$$+\frac{L_2\Omega_2\hbar}{2}(\sigma_2+\delta_2\gamma_1\sigma_1)^2+\frac{\hbar}{2L_2\Omega_2}\Big(\frac{L_2}{2}\Big)^2(\beta_2\sigma_2+\delta_2\beta_1\gamma_1\sigma_1)^2. \quad (43)$$

We see that the quantum fluctuations only depend on the circuit inherent parameters. So the thermal squeezed vacuum state, thermal vacuum state and vacuum state can be regarded as special cases of a thermal excitation state.

When $C\to\infty$, $L\to 0$ and $T\to 0$, $r_u\to 0$, then $\delta_u\to 0$, $\gamma_u\to 0$, $\beta_u=\lambda_u=\lambda'_u=R_u/L_u$, $\Omega'_u=\sqrt{1/L_uC_u-\lambda'^2_u/4}$, we can obtain quantum fluctuations of the charge and current in mutual independency damped harmonic oscillator as expressed below:

$$\langle(\Delta q_1)^2\rangle=\frac{\hbar}{2L_1\Omega'_1}\tau^{-1}\exp(-\lambda'_1t),$$

$$\langle(\Delta q_2)^2\rangle=\frac{\hbar}{2L_2\Omega'_2}\tau^{-1}\exp(-\lambda'_2t);$$

$$\langle(\Delta p_1)^2\rangle=\frac{L_1\hbar}{2\Omega'_1}\Big(\Omega'^2_1+\frac{\lambda'^2_1}{4}\Big)\tau^{-1}\exp(-\lambda'_1t),$$

$$\langle(\Delta p_2)^2\rangle=\frac{L_2\hbar}{2\Omega'_2}\Big(\Omega'^2_2+\frac{\lambda'^2_2}{4}\Big)\tau^{-1}\exp(-\lambda'_2t). \quad (44)$$

So the uncertainty relation is

$$\langle(\Delta q_u)^2\rangle\langle(\Delta p_u)^2\rangle=\frac{\hbar^2}{4\Omega'^2_u}\tau^{-2}\omega^2_{0u}\exp(-2\lambda'_ut), \quad (45)$$

where $\omega_{0u}=\sqrt{1/L_uC_u}$, $u=1,2$.

Set $R_1\to 0$, and $R_2\to 0$, then $\lambda'_u=0$, we have that the following quantum fluctuations of two independent non-damped harmonic oscillators:

$$\langle(\Delta q_1)^2\rangle=\frac{\hbar}{2L_1\omega_{01}}\tau^{-1},\quad \langle(\Delta q_2)^2\rangle=\frac{\hbar}{2L_2\omega_{02}}\tau^{-1};$$
$$\langle(\Delta p_1)^2\rangle=\frac{L_1\hbar\omega_{01}}{2}\tau^{-1},\quad \langle(\Delta p_2)^2\rangle=\frac{L_2\hbar\omega_{02}}{2}\tau^{-1}. \quad (46)$$

The quantum fluctuations are related to inherent resonance frequencies and are consistent with the results in Ref.[35,36].

5. Conclusions

Starting with the classical equations of motion, the quantization of mesoscopic damped double resonance RLC circuit with mutual capacitance–inductance coupling is proposed by the scheme of damped harmonic oscillator quantization and thermofield dynamics. Furthermore, the quantum fluctuations of charge and current of each loop are studied in a thermal excitation state, thermal squeezed vacuum state and thermal vacuum state separately. The results indicate as follows. (i) When a mesoscopic damped double resonance RLC circuit with mutual capacitance–inductance coupling is quantized, it is equivalent to quantized double coupled harmonic oscillators. (ii) The mean values of charge and current are not related to environmental temperature. The mean-square values are related to environmental temperature. The quantum fluctuations not only

depend on the circuit inherent parameters, but also rely on the excitation quantum number and squeezing parameter. Moreover, due to the existence of finite environmental temperature and damped resistance, the fluctuations increase with temperature rising, and decay with time. The vacuum state, thermal vacuum state and thermal squeezed vacuum state can be regarded as special cases of a thermal excitation state. (iii) When coupled capacitance $C \to \infty$ and coupled inductance $L \to 0$, the circuit will evolve to two mutual independent damped harmonic oscillators, while $C \to \infty$, $L \to 0$ and $R_1 \to 0$, $R_2 \to 0$, it will evolve into two non-coupled and non-damped harmonic oscillators. The quantum fluctuations are related to inherent resonance frequencies. The circuit quantum noise can be controlled by adjusting device parameters and quantum states. For a mass circuit, it is always workable in a damped thermal state at a finite temperature. Therefore, studying quantum effects of the mesoscopic damped double resonance RLC circuit with mutual capacitance–inductance coupling in thermal excitation state at a finite temperature is more significant for the design of practical mesoscopic circuit and academic guidance.

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