Vol 16 No 8, August 2007
1009-1963/2007/16(08)/2428-05
Chinese Physics
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The spatial properties of atomic Raman–Nath diffraction*

Li Li-Ping(李利平)[a)†], Zhang Li-Yan (张利彦)[a)], Song Pei-Jun(宋佩君)[a)], Xie Xiao-Tao(谢小涛)[a)], and Li Wei-Bin(李伟斌) [b)]

[a)] *Department of Physics, Huazhong University of Science and Technology, Wuhan* 430074 *China*
[b)] *State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan* 430071 *China*

(Received 23 October 2006; revised manuscript received 8 December 2006)

This paper obtains the exact analytical solution of atomic Raman–Nath diffraction in the coordinate representation and discusses the influence of different initial conditions and detunings on the atomic spatial population distribution. The phase difference between the dipole matrix element and initial atomic population may influence the atomic spatial population distribution after diffraction, which has never been discussed before as far as we know. It offers a method to measure the phase by the spatial population distribution, which is interesting in the study of quantum optics.

Keywords: Raman–Nath diffraction, spatial population distribution, phase difference, coordinate representation
PACC: 4250, 3280

1. Introduction

Raman–Nath diffraction has attracted much attention for investigating the interaction between atoms and the optical potential,[1−5] which has been discussed extensively.[6−8] This effect is not only a simple example of stimulated momentum transfer between atoms and optical standing wave but also a way to slow, cool and trap atoms.[9,10] Since the optical standing wave can be used to deflect,[11] focus[12] and split[13] an atomic beam, Raman–Nath diffraction also has important applications in isotope separation, beam splitter, and microscopy, etc.[1,6−8,11,12] Recently, it has played an essential role in modulation, localization and atom lasers.[14−16] Furthermore, Raman–Nath diffraction has stimulated the research of various non-classical states[17,18] and some fully quantum phenomena such as the collapse-revivals of atomic inversion,[19] sub-Poisson statistics[20] and the squeezing of light.[21] However, as Ref.[10] mentioned, many of these studies, both experimentally and theoretically, focused only on determining the final momentum distribution, whereas study on the spatial distribution may better illustrate atom diffraction by optical standing wave.

In this paper, we study Raman–Nath diffraction in the coordinate representation and obtain an exact analytical solution of the interaction of atoms and light field. Our display of atomic spatial population distribution under different initial conditions and detunings reveals that the atom diffraction is intensively modulated by the optical standing wave. Besides, we find that the phase difference between the dipole matrix element and initial atomic population may influence the spatial population distribution. Meanwhile, this paper offers a possibility to measure the phase by atomic spatial population distribution, which is interesting in the study of quantum optics. It should be mentioned that our results exactly agree with Raman–Nath diffraction in the momentum representation[6−8,22] and verify the adiabatic elimination results in the far detuning situation.[22]

2. Model and solutions

The solvable Jaynes–Cummings model[23] and its various extensions[24] which have been deeply studied[25] are the basic models in describing the interaction of atoms and light field. The standard Jaynes–Cummings model is used in this paper. The incident

*Project supported by the National Natural Science Foundation of China (Grant Nos 10575740, 90503010, 60478029 and 10634060) and the National Basic Research Program of China (Grant No 2005CB724508).
†E-mail: 11p010910@sohu.com

atomic beam propagating along the x-axis is treated as a two-level system and perpendicular to the optical standing wave which is composed of two counter-propagating plane waves along the z-axis with wave vector $\pm k\hat{z}$ ($\hat{z}$ is the unit vector along the z axis), which is analogous to the scheme used in Ref.[22]. The atomic beam coming from the region $x < 0$ interacts with the optical standing wave existing within the region $x \in [0, L]$, and atoms are diffracted in the $x > L$ region. As other authors have discussed,[5,7] both the atomic spontaneous emission and the longitudinal velocity spread of the incident atoms along the z-axis before entering the interacting region are assumed to be negligible. The optical standing wave can be expressed as $E(z,t) = \varepsilon\cos(\omega t)\cos(kz)$ where ε is the amplitude and $\omega = ck$ is the frequency of the optical field (c is the vacuum optical speed). In the dipole and rotating-wave approximation, the Hamiltonian for the system can be written as[22]

$$H = \hbar\omega_a|a\rangle\langle a| + \hbar\omega_b|b\rangle\langle b| - \hbar\Omega_0\cos(kz)f(t) \times (\mathrm{e}^{\mathrm{i}\omega t}\mathrm{e}^{\mathrm{i}\phi}|a\rangle\langle b| + \mathrm{e}^{\mathrm{i}\omega t}\mathrm{e}^{\mathrm{i}\phi}|b\rangle\langle a|). \quad (1)$$

Here $|a\rangle, |b\rangle$ denote the excited and ground state of the atoms with eigenfrequencies ω_a, ω_b respectively. $\Omega_0 = (|D_{ab}|\varepsilon)/2\hbar$ (ϕ is the phase of the dipole matrix element $D_{ab} = e\langle a|x|b\rangle = D^*_{ba} = \mathrm{e}^{\mathrm{i}\phi}|D_{ab}|$) is the Rabi frequency. $f(t) = \Theta(t) - \Theta(t + L/v_x)$ ($\Theta(t)$ is the Heaviside function, v_x is the atomic transverse velocity) describes the interaction time. The wave function of the two-level atoms can be written as $|\psi(t)\rangle = C_a(t)|a\rangle + C_b(t)|b\rangle$ ($C_a(t)$, $C_b(t)$ are the probability amplitudes of finding the atoms in states $|a\rangle$, $|b\rangle$ respectively). Using the Schrödinger equation, we can obtain the following dynamical equations of $C_a(t)$ and $C_b(t)$:

$$\dot{C}_a(t) = -\mathrm{i}\omega_a(t)C_a(t) + \mathrm{i}\,\Omega_0\mathrm{e}^{\mathrm{i}\omega t}\mathrm{e}^{\mathrm{i}\phi}C_b(t)\cos(kz), \quad (2)$$

$$\dot{C}_b(t) = -\mathrm{i}\omega_b(t)C_b(t) + \mathrm{i}\,\Omega_0\mathrm{e}^{\mathrm{i}\omega t}\mathrm{e}^{\mathrm{i}\phi}C_a(t)\cos(kz). \quad (3)$$

Introducing the transformation $C_a(t) = c_a(t)\mathrm{e}^{\mathrm{i}\omega_a t}$ and $C_b(t) = c_b(t)\mathrm{e}^{\mathrm{i}\omega_b t}$, the dynamical equations can be easily solved and the solutions for $c_a(t)$, $c_b(t)$ can be written as

$$c_a(t) = \Big(a_1\mathrm{e}^{\mathrm{i}\,\Omega t/2} + a_2\mathrm{e}^{-\mathrm{i}\,\Omega t/2}\Big)\mathrm{e}^{\mathrm{i}\,\Delta t/2}, \quad (4)$$

$$c_b(t) = \Big(b_1\mathrm{e}^{\mathrm{i}\,\Omega t/2} + b_2\mathrm{e}^{-\mathrm{i}\,\Omega t/2})\mathrm{e}^{-\mathrm{i}\,\Delta t/2}. \quad (5)$$

where $\Delta = \omega_a - \omega_b - \omega$ is the detuning between the atomic transition frequency and the optical standing wave. $\Omega = \sqrt{4\Omega_0^2\cos^2(kz) + \Delta^2}$ and a_1, a_2, b_1, b_2 are integration constants which can be determined by the initial conditions $c_a(0)$ and $c_b(0)$,

$$a_1 = \frac{1}{2\Omega}[(\Omega - \Delta)c_a(0) + 2\Omega_0 c_b(0)\mathrm{e}^{\mathrm{i}\phi}\cos(kz)], \quad (6)$$

$$a_2 = \frac{1}{2\Omega}[(\Omega + \Delta)c_a(0) - 2\Omega_0 c_b(0)\mathrm{e}^{\mathrm{i}\phi}\cos(kz)], \quad (7)$$

$$b_1 = \frac{1}{2\Omega}[(\Omega + \Delta)c_b(0) + 2\Omega_0 c_a(0)\mathrm{e}^{\mathrm{i}\phi}\cos(kz)], \quad (8)$$

$$b_2 = \frac{1}{2\Omega}[(\Omega - \Delta)c_b(0) - 2\Omega_0 c_a(0)\mathrm{e}^{\mathrm{i}\phi}\cos(kz)]. \quad (9)$$

These solutions make it possible to research into the evolution of the atomic probability amplitudes with any value of detuning, outrunning previous works[6−8,26] which just discussed the properties in the resonant or far-off resonant situations.

The exact analytical expressions of the probability amplitudes in the excited and the ground state in the coordinate representation with a random initial condition and detuning are rewritten as

$$c_a(t) = \Big\{c_a(0)\Big[\cos\Big(\frac{\Omega t}{2}\Big) - \frac{\mathrm{i}\,\Delta}{\Omega}\sin\Big(\frac{\Omega t}{2}\Big)\Big] + 2\mathrm{i}\,\frac{\Omega_0}{\Omega}\mathrm{e}^{\mathrm{i}\phi}c_b(0)\sin\Big(\frac{\Omega t}{2}\Big)\cos(kz)\Big\}\mathrm{e}^{\mathrm{i}\,\Delta t/2}, \quad (10)$$

$$c_b(t) = \Big\{c_b(0)\Big[\cos\Big(\frac{\Omega t}{2}\Big) + \frac{\mathrm{i}\,\Delta}{\Omega}\sin\Big(\frac{\Omega t}{2}\Big)\Big] + 2\mathrm{i}\,\frac{\Omega_0}{\Omega}\mathrm{e}^{\mathrm{i}\phi}c_a(0)\sin\Big(\frac{\Omega t}{2}\Big)\cos(kz)\Big\}\mathrm{e}^{-\mathrm{i}\,\Delta t/2}. \quad (11)$$

The results are analogous to the results considering the interaction of a single-mode radiation field and a two-level atom in Ref.[17]. The interaction of a field with varying frequency and a two-level atom without rotating wave approximation has been reported recently.[27] It is worth noting that the atomic spatial population distribution is intensively modulated by the optical standing wave and related to the initial condition, atomic position in the optical standing wave and the detuning. All the spatial properties with a random initial condition and detuning can be discussed by directly using our results. There is also no difficulty in verifying the conservation of atoms for $|c_a(t)|^2 + |c_b(t)|^2 = 1$.

3. Results and analysis

The influence of different initial conditions and detunings on the atomic spatial population distribution of Raman–Nath diffraction will be discussed in detail now. We assume that in the first three parts of the following analysis all the incident atoms are populated in the ground state initially while in the final discussion, half of them are in the ground state.

First of all, when all the atoms are populated in the ground state initially ($c_a(0) = 0$, $c_b(0) = \mathrm{e}^{\mathrm{i}\varphi}$, φ is the initial relative phase of incident atoms population), the population of the ground state in the resonant situation can be written as

$$|c_b(t)|^2 = \cos^2\Big(\frac{\Omega t}{2}\Big) = \cos^2[\Omega_0 t\cos(kz)]. \tag{12}$$

The evolution of the ground state population is shown in Fig.1. In the relatively short interaction time ranging from 0 to $1/2\Omega_0$ as shown in Fig.1.(a), the atomic spatial population distribution is axially symmetric after diffraction. The interaction of atoms and the resonant optical standing wave results in atomic transition from the ground state to the excited state. If the interaction time is enlarged, there exists a characteristic evolution time

$$t_n = (2n+1)\pi/2\Omega_0 \quad (n = 0, 1, 2, \cdots). \tag{13}$$

And the corresponding ground state population can be written as

$$|c_b(t)|^2 = \frac{1}{2}\{1 + \cos[(2n+1)\pi\cos(kz)]\}. \tag{14}$$

The ground state population exhibits a number of properties along $\Omega_0 t$- and kz-axes. Along the kz-axis, the number of peaks increases as the time extends. On the other hand, Fig.1(b) shows the properties when the interaction time ranges from 0 to $5\pi/2\Omega_0$ and indicates that the atomic spatial population distribution after diffraction may become more complex through the increased interaction time. However, at certain positions $z = (2n+1)\pi/2k$, atoms are never kicked to excited states. These positions correspond to the wave nodes, where the atoms are not affected by the optical force.

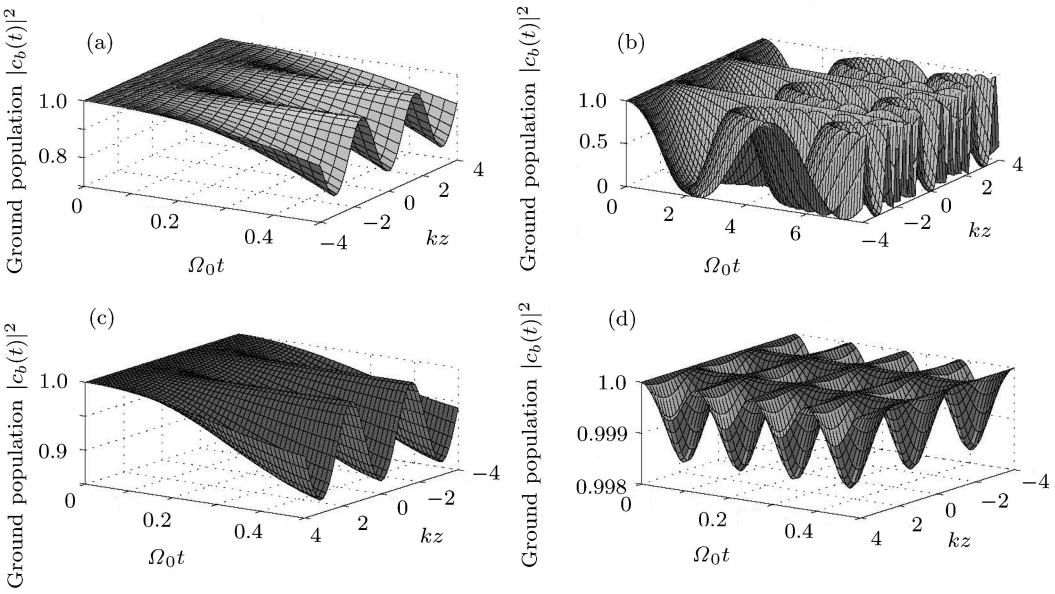


Fig.1. The evolution of the atomic ground state population with kz ranging from -4 to 4. $\Omega_0 t$ ranges from 0 to 0.5 in (a), (c), (d) and from 0 to $5\pi/2$ in (b). (a) and (b) describe the resonant situation while (c) and (d) describe the non-resonant situation with $\Delta = 5$ and $\Delta = 50$ respectively.

Secondly, with the help of some transformations, the evolution of probability amplitudes in the coordinate representation exactly agrees with that in the momentum representation. By using the transformation

$$\mathrm{e}^{\pm\mathrm{i}x\cos\theta} = \sum_{m=-\infty}^{\infty} (\pm\mathrm{i})^m J_m(x)\mathrm{e}^{\mathrm{i}m\theta}$$

($J_m(x)$ is the first Bessel function) and a Fourier series expansion $c_b(z,t) = \sum\limits_m c_m(t)\mathrm{e}^{\mathrm{i}mkz}$ (m is an integer), the evolution equation of the probability amplitude $c_b(t) = \cos(\Omega_0 t\cos kz)\mathrm{e}^{\mathrm{i}\varphi}$ can be expressed as $c_b(t) = \mathrm{i}^m J_m(\Omega_0 t)\mathrm{e}^{\mathrm{i}\varphi}$ (m is even here), which exactly accords with that in the momentum representation of the Raman–Nath diffraction if φ equals zero.[6−8,22]

Thirdly, let us discuss the properties of the atomic ground state population when all the incident atoms are populated in the ground state initially and under the non-resonance situation ($\Delta \neq 0$). The atomic ground state population can be rewritten as

$$|c_b(t)|^2 = \cos^2\Big(\frac{\Omega t}{2}\Big) + \frac{\Delta^2}{\Omega^2}\sin^2\Big(\frac{\Omega t}{2}\Big). \tag{15}$$

The spatial population distributions of the atomic ground state with different detunings are shown in Figs.1(c) and 1(d) with the time ranging from 0 to $1/2\Omega_0$. When Δ is small (near resonance) in comparison with Fig.1(a), more atoms can be observed in the ground state during the same interaction time and the ground state population probability can never be zero even the interaction time is long enough. There is no characteristic evolution time of the ground state population and the period becomes shorter when Δ increases. However, the atomic spatial population distribution at the positions $z = (2n+1)\pi/2k$ (n is an integer) does not change. In the far-off resonant situation, the excited state population only has a very small change, which verifies the adiabatic elimination results in far detuning situation.[22] As a result, it is worth mentioning that detuning is an important factor in the evolution of the atomic spatial population distribution.

Finally, we shall discuss the properties of the atomic spatial population distribution when the incident atoms are half in the ground state ($c_a(0) = \sqrt{2}/2$, $c_b(0) = (\sqrt{2}/2)\mathrm{e}^{\mathrm{i}\varphi}$). The population difference of the excited state and the ground state $W(t) = |c_a(t)|^2 - |c_b(t)|^2$ can be written as

$$W(t) = \frac{-4\Omega_0\Delta}{\Omega^2}\sin^2\Big(\frac{\Omega t}{2}\Big)\cos(kz)\cos\xi + \frac{2\Omega_0}{\Omega}\sin(\Omega t)\cos(kz)\sin\xi\,. \tag{16}$$

Here $\xi = \phi - \varphi$ is the phase difference between the dipole matrix element and initial atomic population. The population difference obviously relates to the detuning and the phase difference. The cases of dissimilar detuning and phase difference are shown in Fig.2. When the detuning is small, as shown in Figs.2(a) and 2(b), the maximum value of the population difference is nearly 1 which means that almost all the incident atoms can transit between the ground state and the excited state. When the detuning becomes large, as shown in Figs.2(c) and 2(d), the evolution period along kz axis becomes shorter and the atomic transition fluctuates just in a small region. Comparing Fig.2(a) with Fig.2(b) or Fig.2(c) with Fig.2(d), we conclude that the phase difference can change the atomic spatial population distribution. In the resonance situation, the population difference can be simplified as

$$W(t) = \sin[2\Omega_0 t\cos(kz)]\sin\xi. \tag{17}$$

The phase difference and the atomic spatial population distribution obviously have very close relationship, rendering the possibility to get the phase information through the atomic spatial population distribution.

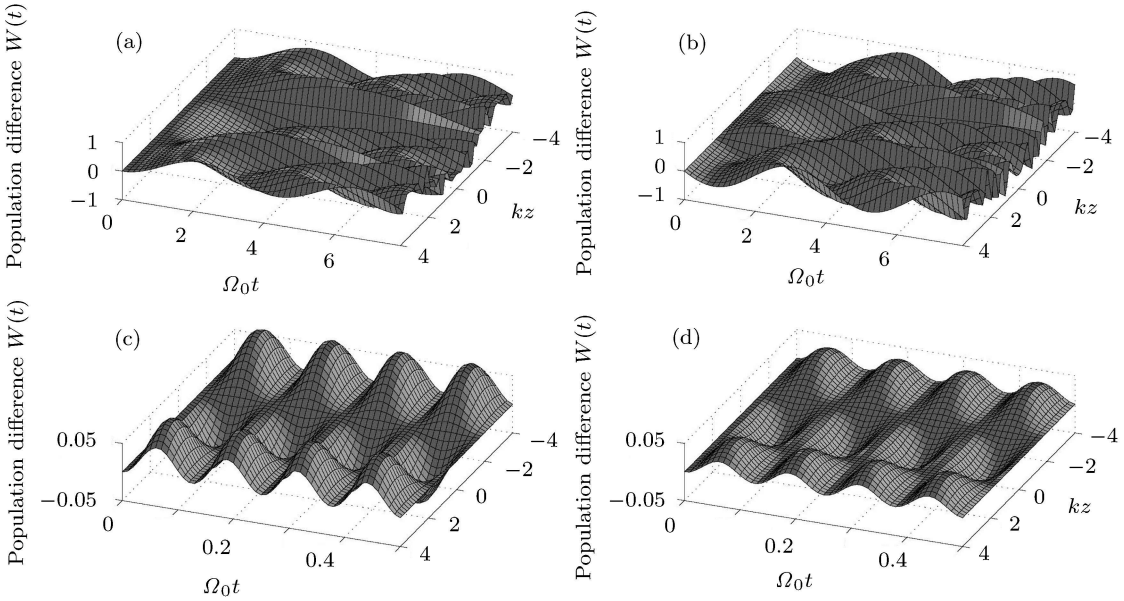


Fig.2. The evolution of the population difference with half the atoms in the ground state initially. (a) $\Delta = 0.6$, $\xi = 0.001$; (b) $\Delta = 0.6$, $\xi = 1$; (c) $\Delta = 50$, $\xi = 0.001$; (d) $\Delta = 50$, $\xi = 1$. The time ranges from 0 to $5\pi/2\Omega_0$ in (a) and (b) and ranges from 0 to $1/2\Omega_0$ in (c) and (d).

4. Conclusion

To sum up, we have worked out an exact analytic solution of the Raman–Nath diffraction in the coordinate representation. The atomic spatial population distribution under different initial conditions and detunings was displayed and discussed in detail. Besides, we find that the phase difference between the dipole matrix element and initial atomic population is related to atomic spatial population distribution, which may offer a method to measure the phase by the spatial population distribution.

It should be pointed out that the spontaneous emission, the atomic longitudinal velocity spread[28] and atomic interactions[29] were not considered in this paper and more subtle phenomena are expected in future investigation.

Acknowledgment

The authors would like to thank Professor Wu Ying for stimulating discussion and encouragement.

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