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The nonlinear squeezed one-photon states and their nonclassical properties*

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By virtue of the technique of integration within an ordered product (IWOP) of operators and the properties of the inverses of annihilation and creation operators of f-oscillator, this paper obtains two new types of squeezed operators and f-analogues of squeezed one-photon states, which are quite different from ones constructed by Song and Fan (*Phys. Lett.* A **294** (2002) 66). Subsequently, some nonclassical properties of the states are investigated in detail.

Keywords: nonlinear squeezed one-photon state, amplitude-squared squeezing, sub-Poissonian behaviour, phase probability distribution

PACC: 4250, 0365

1. Introduction

In the past years much attention has been paid to the study of nonlinear coherent states (NLCSs)[1−4] because they are useful in the description of the motion of a trapped ion and exhibit some useful nonclassical features such as quadrature squeezing, amplitude-squared squeezing and antibunching properties etc. In Refs.[5–8] by using two different ways, the notion of NLCSs has been generalized to even and odd NLCSs and by examining their nonclassical properties it was found that these even and odd NLCSs have rather different statistical features from those of the usual even and odd coherent states. Recently, owing to squeezed states exhibiting the property that one of the uncertainties is smaller than that in coherent state and have promising applications in the interferometric detection of gravitational radiation and sub-shot-noise phase measurements, many physicists have done a great deal of work about various squeezed states.[9−11] In this paper in the same way as in Ref.[10] we shall introduce two normalizable f-analogues of the squeezed one-photon states by virtue of the technique of integration within an ordered product (IWOP) of operators and the properties of the inverses of the annihilation operator $A_{\rm f} = [1/f(N)]a$ and the creation operator $A_f^+ = a^+[1/f(N)]$ of f-oscillators, and then some nonclassical properties of the states are examined such as amplitude-squared squeezing, sub-Poissonian behaviour and phase probability distribution, etc.

2. Nonlinear squeezed one-photon states

We first introduce the generalized annihilation operator $A_{\rm f}$ and creation operator $A_{\rm f}^+$ associated with nonlinear coherent states as follows:

$$A_{\rm f} = \frac{1}{f(N)}a, \quad A_{\rm f}^+ = a^+\frac{1}{f(N)}, \tag{1}$$

where $f(N)$ is an operator-valued function of the number operators $N = a^+a$, a and a^+ are the bosonic annihilation and creation operators respectively. Therefore, we can easily obtain the inverses of the operators $A_{\rm f}$ and $A_{\rm f}^+$ as follows:

$$\begin{aligned} A_{\rm f}^{-1} &= \sum_{n=0}^{\infty}\frac{f(n)}{\sqrt{n+1}}|n+1\rangle\langle n|, \\ (A_{\rm f}^+)^{-1} &= \sum_{n=0}^{\infty}\frac{f(n)}{\sqrt{n+1}}|n\rangle\langle n+1| \\ &= (A_{\rm f}^{-1})^+. \end{aligned} \tag{2}$$

By virtue of the inverse operator $A_{\rm f}^{-1}$, $(A_{\rm f}^+)^{-1}$ and the number operator N, we construct two new operators:

$$B_{\rm f}^+ = NA_{\rm f}^{-1}, \qquad B_{\rm f} = (A_{\rm f}^+)^{-1}N. \tag{3}$$

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Using Eqs.(1)–(3), we can prove that

$$[A_{\rm f}, B_{\rm f}^{+}] = A_{\rm f} N A_{\rm f}^{-1} - N A_{\rm f}^{-1} A_{\rm f} = (N+1) A_{\rm f} A_{\rm f}^{-1} - N(1-|0\rangle\langle 0|) = 1, \quad (4)$$

$$[B_{\rm f}, A_{\rm f}^{+}] = (A_{\rm f}^{+})^{-1} N A_{\rm f}^{+} - A_{\rm f}^{+}(A_{\rm f}^{+})^{-1} N = (A_{\rm f}^{+})^{-1} A_{\rm f}^{+}(N+1) - (1-|0\rangle\langle 0|)N = 1, \quad (5)$$

$$[B_{\rm f}^{+}, N] = -B_{\rm f}^{+}, [B_{\rm f}, N] = B_{\rm f}. \quad (6)$$

Therefore $B_{\rm f}^{+}$ (or $B_{\rm f}$) and $A_{\rm f}$ (or $A_{\rm f}^{+}$) make up fundamental canonical commutative relations.

In Ref.[12] Fan and Jing have proved that the normal ordering form of the vacuum projector is

$$|0\rangle\langle 0| = {:}\ \exp(-a^{+}a)\ {:}\ . \quad (7)$$

In the same way we can also prove that the normal ordering form of the vacuum projector is

$$|0\rangle\langle 0| = \overset{\circ}{\underset{\circ}{}} \exp(-B_{\rm f}^{+}A_{\rm f}) \overset{\circ}{\underset{\circ}{}} = \vdots \exp(-A_{\rm f}^{+}B_{\rm f}) \vdots , \quad (8)$$

where the normal ordering $\overset{\circ}{\underset{\circ}{}}\ \overset{\circ}{\underset{\circ}{}}$ is for the operators $B_{\rm f}^{+}$ and $A_{\rm f}$, the normal ordering $\vdots\ \vdots$ is for the operators $A_{\rm f}^{+}$ and $B_{\rm f}$. They distinct from the normal ordering form of the vacuum projector $|0\rangle\langle 0|$ in Eq.(7).

Subsequently, we construct the following eigenstates:

$$\|x_1\rangle = \pi^{-1/4} \exp\Big[-\frac{1}{2}x_1^2 + \sqrt{2}x_1 B_{\rm f}^{+} - \frac{1}{2}B_{\rm f}^{+2}\Big]|0\rangle, \quad (9)$$

$$\langle x_1| = \pi^{-1/4}\langle 0|\exp\Big[-\frac{1}{2}x_1^2 + \sqrt{2}x_1 A_{\rm f} - \frac{1}{2}A_{\rm f}^{2}\Big], \quad (10)$$

$$\|x_2\rangle = \pi^{-1/4} \exp\Big[-\frac{1}{2}x_2^2 + \sqrt{2}x_2 A_{\rm f}^{+} - \frac{1}{2}A_{\rm f}^{+2}\Big]|0\rangle, \quad (11)$$

$$\langle x_2| = \pi^{-1/4}\langle 0|\exp\Big[-\frac{1}{2}x_2^2 + \sqrt{2}x_2 B_{\rm f} - \frac{1}{2}B_{\rm f}^{2}\Big], \quad (12)$$

where

$$x_1 = (A_{\rm f} + B_{\rm f}^{+})/\sqrt{2}, \qquad x_2 = (B_{\rm f} + A_{\rm f}^{+})/\sqrt{2}. \quad (13)$$

By virtue of Eqs.(8)–(12) and the IWOP technique, then performing the following integration we can obtain two completeness relations:

$$\int_{-\infty}^{\infty} {\rm d}x_1 \|x_1\rangle\langle x_1| = \pi^{-1/2} \int_{-\infty}^{\infty} {\rm d}x_1 \overset{\circ}{\underset{\circ}{}} \exp\Big[-x_1^2 + \sqrt{2}x_1(B_{\rm f}^{+} + A_{\rm f}) - \frac{1}{2}(B_{\rm f}^{+2} + A_{\rm f}^{2}) - B_{\rm f}^{+}A_{\rm f}\Big] \overset{\circ}{\underset{\circ}{}} = 1, \quad (14)$$

$$\int_{-\infty}^{\infty} {\rm d}x_2 \|x_2\rangle\langle x_2| = \pi^{-1/2} \int_{-\infty}^{\infty} {\rm d}x_2 \vdots \exp\Big[-x_2^2 + \sqrt{2}x_2(A_{\rm f}^{+} + B_{\rm f}) - \frac{1}{2}(A_{\rm f}^{+2} + B_{\rm f}^{2}) - A_{\rm f}^{+}B_{\rm f}\Big] \vdots = 1. \quad (15)$$

From the above discussion we find that the completeness relations are made up of the bra and ket which are not mutually Hermitian conjugate. This is rather different from the usual completeness relations. By means of simple calculation, we can prove that the eigenvalue equations of the coordinate operator $\hat{x}_1$ and $\hat{x}_2$ may be written as

$$\hat{x}_1\|x_1\rangle = x_1\|x_1\rangle, \qquad \langle x_1|\hat{x}_1 = \langle x_1|x_1, \quad (16a)$$

$$\hat{x}_2\|x_2\rangle = x_2\|x_2\rangle, \qquad \langle x_2|\hat{x}_2 = \langle x_2|x_2. \quad (16b)$$

Then using Eqs.(8)–(12) we can have the following squeezing operators:

$$S_1(r) = \int_{-\infty}^{\infty} \frac{{\rm d}x_1}{\sqrt{\mu}} \|x_1\rangle\langle x_1| = \exp\Big(-\frac{1}{2}B_{\rm f}^{+2}\tanh r\Big) \exp\Big[(N+\frac{1}{2})\ln{\rm sech}\, r\Big] \exp\Big(\frac{1}{2}A_{\rm f}^{2}\tanh r\Big), \quad (17)$$

$$S_2(r) = \int_{-\infty}^{\infty} \frac{{\rm d}x_2}{\sqrt{\mu}} \|x_2\rangle\langle x_2| = \exp\Big(-\frac{1}{2}A_{\rm f}^{+2}\tanh r\Big) \exp\Big[(N+\frac{1}{2})\ln{\rm sech}\, r\Big] \exp\Big(\frac{1}{2}B_{\rm f}^{2}\tanh r\Big), \quad (18)$$

where $\mu = \exp(r)$, $\tanh r = (\mu^2-1)/(\mu^2+1)$ and the squeezed operators $S_1(r)$ and $S_2(r)$ are quite different from ones constructed in Ref.[10]. Using the two new squeezing operators we can derive the following transformations:

$$S_1(r) A_{\rm f} S_1^{-1}(r) = A_{\rm f}\cosh r + B_{\rm f}^{+}\sinh r, \qquad S_1(r) B_{\rm f}^{+} S_1^{-1}(r) = B_{\rm f}^{+}\cosh r + A_{\rm f}\sinh r, \quad (19)$$

$$S_2(r) B_{\rm f} S_2^{-1}(r) = B_{\rm f}\cosh r + A_{\rm f}^{+}\sinh r, \qquad S_2(r) A_{\rm f}^{+} S_2^{-1}(r) = A_{\rm f}^{+}\cosh r + B_{\rm f}\sinh r. \quad (20)$$

Operating the new squeezed operators $S_1(r)$ and $S_2(r)$ on the one-photon state $|1\rangle$ respectively, we obtain two nonlinear squeezed one-photon states

$$|r,f\rangle_1 = C_1 \exp\Big(-\frac{1}{2}B_{\rm f}^{+2}\tanh r\Big)|1\rangle = C_1\sum_{n=0}^{\infty}\Big(-\frac{1}{2}\tanh r\Big)^n\frac{\sqrt{(2n+1)!}}{n!}f(2n)!|2n+1\rangle, \tag{21}$$

$$|r,f\rangle_2 = C_2 \exp\Big(-\frac{1}{2}A_{\rm f}^{+2}\tanh r\Big)|1\rangle = C_2\sum_{n=0}^{\infty}\Big(-\frac{1}{2}\tanh r\Big)^n\frac{\sqrt{(2n+1)!}}{n!f(2n)!}|2n+1\rangle, \tag{22}$$

where C_1 and C_2 are the normalization constants of the states $|r,f\rangle_1$ and $|r,f\rangle_2$ respectively, and are given by

$$C_1^2 = \Big\{\sum_{n=0}^{\infty}\Big(\frac{1}{2}\tanh r\Big)^{2n}\frac{(2n+1)!}{(n!)^2}[f(2n)!]^2\Big\}^{-1},$$

$$C_2^2 = \Big\{\sum_{n=0}^{\infty}(\frac{1}{2}\tanh r\Big)^{2n}\frac{(2n+1)!}{(n!)^2[f(2n)!]^2}\Big\}^{-1}, \tag{23}$$

$$f(n)! = f(0)!f(1)!f(2)!\cdots f(n)!, \quad f(0)! = 1. \tag{24}$$

In the following sections we shall examine various nonclassical properties for the nonlinear squeezed one-photon states. However, we have different nonlinear squeezed one-photon states for different nonlinearity functions $f(n)$, so it is necessary to specify a nonlinearity function $f(n)$ before we study these states. In this paper we shall choose the nonlinearity function $f(n)$ given by[1]

$$f(n) = L_n^1(\eta^2)[(n+1)L_n^0(\eta^2)]^{-1}, \tag{25}$$

which is usually used in the description of the motion of a trapped ion, where η is the Lamb–Dicke parameter and $L_n^m(\eta^2)$ is an associated Laguerre polynomials. By virtue of the nonlinearity function $f(n)$, it is very meaningful to study some nonclassical properties of the nonlinear squeezed one-photon states.

3. Some nonclassical properties

3.1. Amplitude-squared squeezing

In order to examine whether or not two new nonlinear squeezed one-photon states exhibit amplitude-squared squeezing, let us consider the following Hermitian quadrature operators:

$$Y_1 = (a^2 + a^{+2})/2, \qquad Y_2 = (a^2 - a^{+2})/2{\rm i}. \tag{26}$$

Then the operators Y_1 and Y_2 yield the commutation relation

$$[Y_1, Y_2] = \frac{\rm i}{2}[a^2, a^{+2}] = {\rm i}(2N+1), \tag{27}$$

and obey the uncertainty relation

$$(\Delta Y_1)^2(\Delta Y_2)^2 \geq \frac{1}{4}|\langle[Y_1,Y_2]\rangle|^2 = |\langle N+1/2\rangle|^2. \tag{28}$$

The field is said to be in an amplitude-squared squeezing state if

$$(\Delta Y_j)^2 \leq |\langle N+1/2\rangle|, (j=1,2). \tag{29}$$

In order to examine the squeezing degree of the light field, we define the squeezing degree in the following form:

$$D(Y_1) = \frac{2\langle a^{+2}a^2\rangle + \langle a^4 + a^{+4}\rangle - \langle a^{+2} + a^2\rangle^2}{\langle a^2a^{+2}\rangle - \langle a^{+2}a^2\rangle}, \tag{30}$$

$$D(Y_2) = \frac{2\langle a^{+2}a^2\rangle - \langle a^4 + a^{+4}\rangle + \langle a^{+2} - a^2\rangle^2}{\langle a^2a^{+2}\rangle - \langle a^{+2}a^2\rangle}. \tag{31}$$

If the squeezing degree $D(Y_j)(j=1,2)$ satisfies the conditions $-1 \leq D(Y_j) < 0$, it means that the field shows the amplitude-squared squeezing effect in the direction $Y_j(j=1,2)$, and the maximum squeezing (100%) is obtained while for $D_j = -1$.

Using two new nonlinear squeezed one-photon states given by Eqs.(21) and (22), we have the following expectation values of some operators in the state $|r,f\rangle_1$:

$$\langle a^+a\rangle_1 = C_1^2\sum_{n=0}^{\infty}\Big(\frac{1}{2}\tanh r\Big)^{2n}\frac{(2n+1)!(2n+1)}{(n!)^2}[f(2n)!]^2, \tag{32}$$

$$\langle a^{+2}a^2\rangle_1 = C_1^2\sum_{n=0}^{\infty}\Big(\frac{1}{2}\tanh r\Big)^{2n}\frac{2n(2n+1)(2n+1)!}{(n!)^2}[f(2n)!]^2, \tag{33}$$

$$\langle a^2 a^{+2}\rangle_1 = C_1^2 \sum_{n=0}^{\infty} \left(\frac{1}{2}\tanh r\right)^{2n} \frac{(2n+3)!}{(n!)^2} [f(2n)!]^2, \tag{34}$$

$$\langle a^2\rangle_1 = \langle a^{+2}\rangle_1 = C_1^2 \sum_{n=0}^{\infty} \left(-\frac{1}{2}\tanh r\right)^{2n+1} \frac{2(2n+3)(2n+1)!}{(n!)^2} f(2n)! f(2n+2)!, \tag{35}$$

$$\langle a^4\rangle_1 = \langle a^{+4}\rangle_1 = C_1^2 \sum_{n=0}^{\infty} \left(\frac{1}{2}\tanh r\right)^{2n+2} \frac{4(2n+3)(2n+5)(2n+1)!}{(n!)^2} f(2n)! f(2n+4)!. \tag{36}$$

Similarly, under the states $|r, f\rangle_2$ the expectation values of the above operators are easily obtained via Eqs.(21)–(25) and (32)–(36). We evaluate the squeezing functions in Eqs.(30) and (31) and the numerical calculation results are shown in Fig.1. In Fig.1 we plot the graphs of the variations of the squeezing functions $D_i(Y_j)(i, j = 1, 2)$ with respect to the real squeeze parameter r for fixed $\eta^2 = 0.7$. From Fig.1 it can be seen that, in a certain range of r, the squeezing functions $D_1(Y_1)$ of the states $|r, f\rangle_1$ and the squeezing functions $D_2(Y_2)$ of the states $|r, f\rangle_2$ are negative and satisfy the conditions $-1 \le D_i(Y_j) < 0 (i = j = 1, 2)$, so it means that the states $|r, f\rangle_1$ and $|r, f\rangle_2$ exhibit amplitude-squared squeezing in the direction Y_1 and Y_2 respectively. The states start unsqueezed for $r = 0$ and the squeezing sets in for $r > 0$. However, the variations of the depth of squeezing with increasing r in both directions are rather similar. To be concrete, with increasing r the depth of squeezing always increases and subsequently decreases in the directions Y_1 and Y_2. But the range of the squeeze parameter r which the states $|r, f\rangle_1$ show amplitude-squared squeezing is smaller than that of the states $|r, f\rangle_2$.

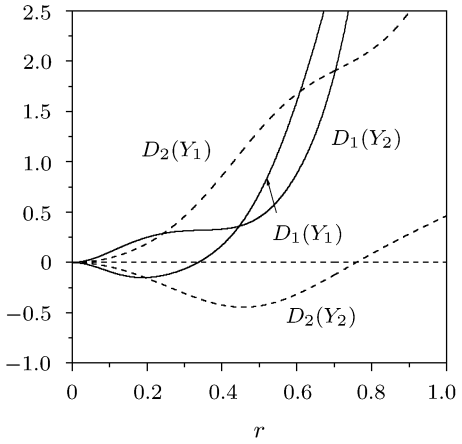


Fig.1. Variations of the amplitude-squared squeezing functions $D_i(Y_j)\,(i, j = 1, 2)$ of the states $|r, f\rangle_{1,2}$ with r for $\eta^2 = 0.7$.

3.2. Sub-Poissonian behaviour

Now we study the sub-Poissonian behaviour of two nonlinear squeezed one-photon states $|r, f\rangle_1$and $|r, f\rangle_2$. At the first step we consider the second-order correlation function $g^{(2)}(0)$ defined as

$$g^{(2)}(0) = \frac{\langle a^{+2} a^2\rangle}{\langle a^+ a\rangle^2}. \tag{37}$$

Then if $g^{(2)}(0) < 1$ (or $g^{(2)}(0) > 1$), the distribution is called sub-Poissonian (or super-Poissonian). If $g^{(2)}(0) = 1$, the distribution is called Poissonian. Here by virtue of the numerical calculation results we plot the second-order correlation function $g^{(2)}(0)$ against r keeping $\eta^2 = 0.7$ in Fig.2.

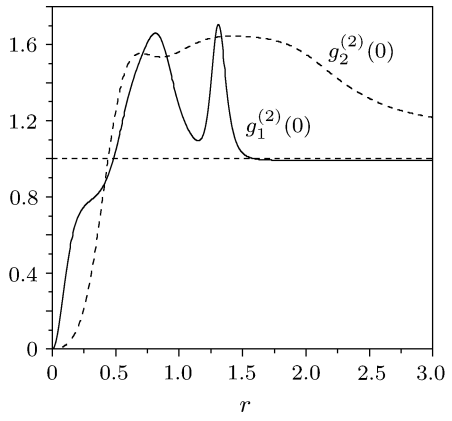


Fig.2. Sub-Poissonian behaviour of the states $|r, f\rangle_{1,2}$ for $\eta^2 = 0.7$.

From Fig.2 it is clearly seen that $g^{(2)}(0) < 1$ in a wide range of r for the nonlinear squeezed one-photon states $|r, f\rangle_{1,2}$. This means that the states $|r, f\rangle_{1,2}$ exhibit sub-Poissonian behaviour. With increasing r the second-order function $g^{(2)}(0)$ is greater than zero and then has a trend to get to one, therefore the states $|r, f\rangle_{1,2}$ shall exhibit super-Poissonian and Poissonian behaviour successively.

3.3. The phase probability distribution

Now we turn to consider the phase probability distributions for two nonlinear squeezed one-photon states given by Eqs.(21) and (22). In terms of the Pegg–Barnett phase operator formalism in a finite-dimensional $(s+1)$ Hilbert space we define a set of complete orthonormal phase states $|\theta_m\rangle$ as

$$|\theta_m\rangle = \frac{1}{\sqrt{s+1}} \sum_{n=0}^{s} \exp(\mathrm{i}n\theta_m)|n\rangle, \tag{38}$$

where θ_m is given by

$$\theta_m = \theta_0 + \frac{2\pi m}{s+1}, \quad m = 0, 1, 2, \cdots, s, \tag{39}$$

where the value of θ_0 is arbitrary and indicates a particular basis in the phase space. In this space the Hermitian phase operator Φ_θ is defined by

$$\Phi_\theta = \sum_{n=0}^{s} \theta_m |\theta_m\rangle\langle\theta_m|. \tag{40}$$

For the nonlinear squeezed one-photon states the expectation value of the phase operator Φ_θ is given by

$${}_{1,2}\langle r, f|\, \Phi_\theta |r, f\rangle_{1,2} = \sum_{n=0}^{s} \theta_m |\langle\theta_m|r, f\rangle_{1,2}|^2, \tag{41}$$

where $|\langle\theta_m|r, f\rangle_{1,2}|^2$ denotes the probability of the states $|r, f\rangle_{1,2}$ in the phase states $|\theta_m\rangle$. By virtue of simple calculation the phase probability distribution is expressed as

$$|\langle\theta_m|r, f\rangle_{1,2}|^2 = \frac{1}{s+1} + \frac{2}{s+1} \sum_{n>l} \sum_{l} (b_{1,2})_n (b_{1,2})_l \times \cos[2(n-l)\theta_m]. \tag{42}$$

If set θ_0 as

$$\theta_0 = \varphi - \frac{\pi s}{s+1}, \tag{43}$$

then we obtain

$$|\langle\theta_m|r, f\rangle_{1,2}|^2 = \frac{1}{s+1} + \frac{2}{s+1} \sum_{n>l} \sum_{l} (b_{1,2})_n (b_{1,2})_l \times \cos\Big[(n-l)\frac{4\pi\mu}{s+1}\Big], \tag{44}$$

where $\mu = m - s/2$. Since the density of phase states is $(s+1)/2\pi$, thus if set $s \to \infty$ in Eq.(25), we can get the continuous phase probability distribution

$$P(\theta) = \lim_{s\to\infty} \frac{s+1}{2\pi} |\langle\theta_m|r, f\rangle_{1,2}|^2 = \frac{1}{2\pi}\Big(1 + 2\sum_{n>l} \sum_{l} (b_{1,2})_n (b_{1,2})_l \cos[2(n-l)\theta]\Big), \quad -\pi \le \theta \le \pi. \tag{45}$$

On the other hand, we find that the phase probability distribution $P(\theta)$ in Eq.(45) can be written as a sum of the mixture part

$$P^M(\theta) = \frac{1}{2\pi}, \tag{46}$$

and the quantum interference part

$$P^I(\theta) = \frac{1}{\pi} \sum_{n>l} \sum_{l} (b_{1,2})_n (b_{1,2})_l \cos[2(n-l)\theta], \tag{47}$$

where

$$(b_1)_n = C_1 \sum_{n=0}^{\infty} \Big(-\frac{1}{2}\tanh r\Big)^n \frac{\sqrt{(2n+1)!}}{n!} f(2n)!, \tag{48}$$

$$(b_2)_n = C_2 \sum_{n=0}^{\infty} \Big(-\frac{1}{2}\tanh r\Big)^n \frac{\sqrt{(2n+1)!}}{n! f(2n)!}, \tag{49}$$

where the constants C_1 and C_2 are given by Eq.(23).

In Fig.3 we plot the phase probability distributions for the states $|r, f\rangle_{1,2}$ with respect to θ for different values of $\eta(\eta = 0.1, 0.6$ and $1.1)$ when $r = 0.5$. From Fig.3 for the state $|r, f\rangle_1$ we find that, for a wide range of η, there are always two well developed peaks at $\theta = \pm\frac{\pi}{2}$, but the values of two peaks decrease as η increases. For a larger value of η, the peak begins to stand up at $\theta = 0$. The appearance of the peaks is an indication of quantum interference. However, for the state $|r, f\rangle_2$ the phase distributions have two peaks at $\theta = \pm\frac{\pi}{2}$ for small value of η and with the increase of η the multiple peak structure of the phase distribution develops. This implies that the quantum interference of the states $|r, f\rangle_2$ increases as η increases. Therefore, for relatively larger values of η the quantum interference effects of the state $|r, f\rangle_2$ become more prominent than those of the state $|r, f\rangle_1$.

In Fig.4 we plot the phase probability distributions of the states $|r, f\rangle_{1,2}$ against θ keeping η fixed at 0.7 for different values of $r(r = 0.2, 0.4$ and $0.6)$. From Fig.4 for small value of r the phase probability distribution of the state $|r, f\rangle_1$ has a central peak at $\theta = 0$. With the increase of r the multiple peak structure of the phase distribution begins to occur and becomes more and more prominent, but the number and the value of the peaks increase and then decrease. As r increases the phase distributions of the states $|r, f\rangle_2$ have three peaks and also the values of the three peaks increase and subsequently decrease. Therefore, we conclude that as r increases the quantum interference effects of the states $|r, f\rangle_{1,2}$ always increase and subsequently decrease. On the other hand, in Figs.3 and 4 the phase distributions clearly reflect the different features of quantum interference of the states $|r, f\rangle_{1,2}$. This is the result of the different interference parts in Eqs.(47)–(49).

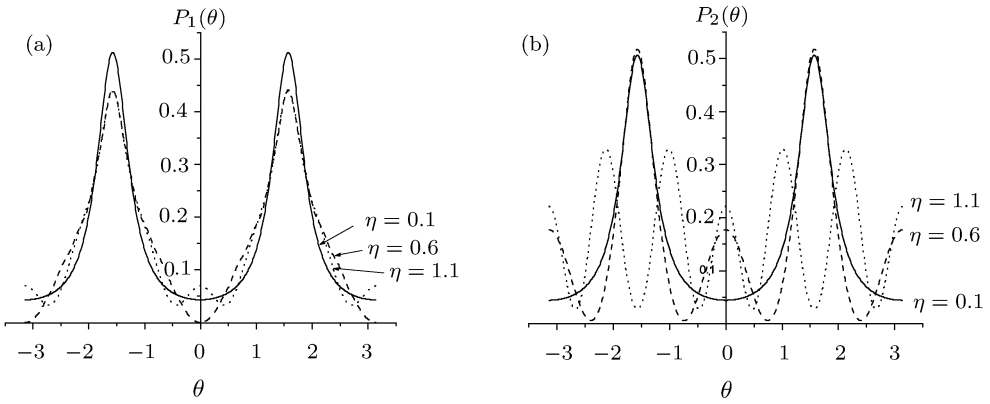


Fig.3. The phase probability distributions of (a) the states $|r, f\rangle_1$ and (b) the states $|r, f\rangle_2$ for $r = 0.5$.

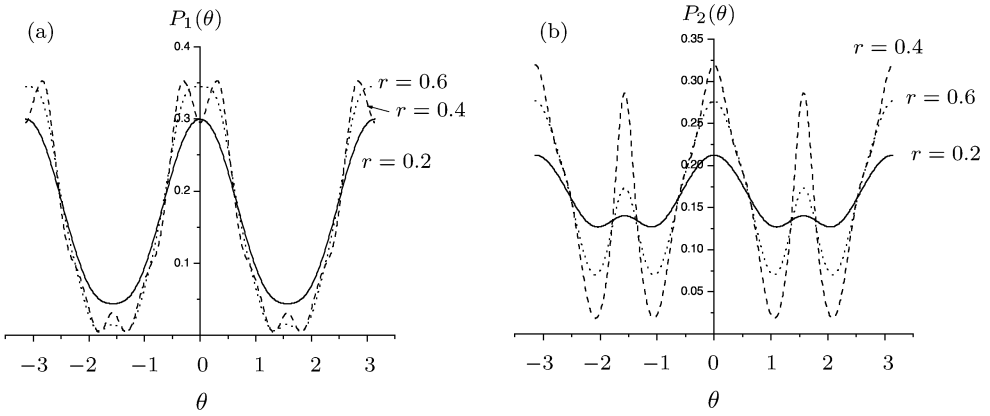


Fig.4. The phase probability distributions of (a) the states $|r, f\rangle_1$ and (b) the states $|r, f\rangle_2$ for $\eta = 0.7$.

4. Conclusions

In summary, using the IWOP technique and the properties of the inverses of annihilation and creation operators of f-oscillator we have obtained two new types of squeezed operators and f-analogues of squeezed one-photon states, which are quite different from ones constructed in Ref.[10]. Subsequently, by virtue of the numerical calculation results some nonclassical properties of the states are examined in detail. It shows that both the state $|r, f\rangle_1$ and $|r, f\rangle_2$ can exhibit amplitude-squared squeezing and sub-Poissonian behaviour in different ranges of r. However, the phase distributions of the states $|r, f\rangle_{1,2}$ clearly reflect more remarkable different quantum interference effects.

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