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Atomic coherence control on the entanglement of two atoms in two-photon processes*

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Considering two identical two-level atoms interacting with a single-mode thermal field through two-photon processes, this paper studies the atomic coherence control on the entanglement between two two-level atoms, and finds that the entanglement is greatly enhanced due to the initial atomic coherence. The results show that the entanglement can be manipulated by changing the initial parameters of the system, such as the superposition coefficients and the relative phases of the initial atomic coherent state and the mean photon number of the cavity field.

Keywords: entanglement, single-mode thermal field, atomic coherence
PACC: 4250

1. Introduction

Quantum entanglement is one of the most striking features of quantum mechanics and plays a key role in quantum information and quantum computation,[1] quantum cryptography,[2] quantum teleportation,[3] quantum dense coding,[4] and spin squeezing,[5] etc. Thus the issue of creating and controlling two-particle entanglement in various composite systems has great practical importance in actual quantum information processing and has attracted much attention in theory and experiment technology.[6−21] Quite recently, it has been shown that the existence of relative phase between two Rabi frequencies can be used to control entanglement of the two-qubit system at will.[22] Schemes for controlling the entanglement of a two-qubit system by a local magnetic pulse have also been proposed.[23]

As is well known, entanglement is intrinsically related to the superposition principle of quantum mechanics. Thus the problem of creating or controlling the entanglement is greatly related to the atomic coherence of population between different levels. Atomic coherence,[24] which results from a coherent superposition of different states of a single atom, can lead to many different quantum optical phenomena, such as correlated spontaneous emission laser,[25,26] spontaneous emission cancellation,[27,28] and electromagnetically induced transparency,[29] etc. In recent years, atomic coherence has been shown to play an important role in inducing, enhancing and controlling entanglement. For example, Li *et al* showed that the atomic coherence can always induce the entanglement between two thermal fields, no matter how high the temperature of the field is.[30] In Ref.[31], Wang *et al* discovered that the two important concepts, entanglement and the atomic coherence, are closely related. On the other hand, many authors investigated the entanglement between two two-level atoms interacting with a single-mode thermal field[32,33,35] or with two-mode thermal field,[34] but they did not involve the influence of the atomic coherence. In this paper, we study the atomic coherence control on the entanglement of the two atoms (ETA) interacting with a single-mode thermal field through two-photon processes. Our results show that ETA is greatly enhanced due to the initial atomic coherence, and can be manipulated by changing the initial parameters of the system, such as the superposition coefficients and the relative phases of the initial atomic coherent state and the mean photon number of the cavity field.

2. The model

We consider two identical two-level atoms simultaneously interacting with a single-mode thermal field through two-photon processes. For simplicity, we ignore the stark shift in the two-photon process. The

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cavity mode is assumed to be resonant with the atomic transition frequency. Under the rotating wave approximation, the Hamiltonian of this system in the interaction picture can be expressed as (assuming $\hbar = 1$)

$$H_{\rm I} = g \sum_{i=1,2} (a^{+2}\sigma_i^- + a^2\sigma_i^+), \tag{1}$$

where $a^+(a)$ is the photon creation (annihilation) operator for the cavity field and g is the atoms-field coupling constant, $\sigma_i^- = |g\rangle_i\langle e|(i = 1, 2)$ and $\sigma_i^+ = |e\rangle_i\langle g|(i = 1, 2)$ are the atomic transition operators, while $|g\rangle_i$ and $|e\rangle_i$ denote the ground and excited states of the ith atom respectively.

Using the above Hamiltonian and Taylor expansions of sine and cosine functions in the atomic basis $|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle$ (In this paper, matrices are all in this basis), we can obtain the exact time evolution operator $U_{\rm I}(t)$ for this system:

$$U_{\rm I}(t) = \begin{pmatrix} a^2C_na^{+2} + 1 & -{\rm i}a^2S_n & -{\rm i}a^2S_n & a^2C_na^2 \\ -{\rm i}S_na^{+2} & \cos^2\dfrac{\Omega_nt}{2} & -\sin^2\dfrac{\Omega_nt}{2} & -{\rm i}S_na^2 \\ -{\rm i}S_na^{+2} & -\sin^2\dfrac{\Omega_nt}{2} & \cos^2\dfrac{\Omega_nt}{2} & -{\rm i}S_na^2 \\ a^{+2}C_na^{+2} & -{\rm i}a^{+2}S_n & -ia^{+2}S_n & a^{+2}C_na^2 + 1 \end{pmatrix}, \tag{2}$$

where $\Omega_n = 2g\sqrt{a^+aa^+a + a^+a + 1}$ and the time-dependent operators $C_n = 2g^2\Omega_n^{-2}(\cos\Omega_nt - 1)$ and $S_n = g\Omega_n^{-1}\sin\Omega_nt$.

The calculation of the density operator follows the traditional route in quantum optics, using the evolution operator, we can generate the density operator evolution of the system at time t:

$$\rho(t) = U_{\rm I}(t)[\rho_{\rm f}(0) \otimes \rho_s(0)]U_{\rm I}(t), \tag{3}$$

where $\rho_{\rm f}(0) = \sum_n P_n|n\rangle\langle n|$ denotes the single-mode thermal radiation field with its mean photon number $\bar{n} = ({\rm e}^{\beta\hbar\omega} - 1)^{-1}$, $\beta = 1/k_{\rm B}T$, $k_{\rm B}$ is the Boltzmann constant and ω is the frequency of the field.

We are only interested in the evolution of the atoms. Hence the system density operator (atom–atom density operator) can be obtained by tracing over the field variables:

$$\begin{aligned}\rho_{\rm S}(t) &= {\rm Tr}_{\rm f}[\rho(t)] \\ &= \sum_{m,n} P_n\langle m|U_{\rm I}(t)|n\rangle\rho_{\rm S}(0)\langle n|U_{\rm I}(t)|m\rangle. \end{aligned}\tag{4}$$

We use the negativity (ε) to measure ETA: ε is defined in terms of the negative eigenvalues of the partial transposition of the density matrix[36,37]

$$\varepsilon = -2\sum_i \mu_i^-. \tag{5}$$

Where μ_i^- is the ith negative eigenvalue of the partial transposition of $\rho_{\rm S}(t)$. When $\varepsilon = 1$ the two atoms are maximally entangled and when $\varepsilon = 0$ the two atoms are separable or do not become entangled.

3. The results

We investigate three kinds of initial coherent states for the two atoms. The first one is $|\Psi_1(0)\rangle = A_1|e\rangle + \sqrt{(1 - A_1^2)}{\rm e}^{{\rm i}\varphi_1}|g\rangle$ and $|\Psi_2(0)\rangle = A_2|e\rangle + \sqrt{(1 - A_2^2)}{\rm e}^{{\rm i}\varphi_2}|g\rangle$, the latter two of which are $|\Psi(0)\rangle = A|eg\rangle + \sqrt{(1 - A^2)}{\rm e}^{{\rm i}\varphi}|ge\rangle$ and $|\Psi(0)\rangle = B|ee\rangle + \sqrt{(1 - B^2)}{\rm e}^{{\rm i}\varphi'}|gg\rangle$. Here, A_1, A_2, A and B are all real superposition coefficients and φ_1, φ_2, φ and φ' are the relative phases.

3.1. The entanglement between the atom 1 in $|\Psi_1(0)\rangle = A_1|e\rangle + \sqrt{(1 - A_1^2)}\,{\rm e}^{{\rm i}\varphi_1}|g\rangle$ and the atom 2 in $|\Psi_2(0)\rangle = A_2|e\rangle + \sqrt{(1 - A_2^2)}{\rm e}^{{\rm i}\varphi_2}|g\rangle$

In this case, the initial density matrix for the two atoms can be written as

$$\rho_s(0) = \begin{pmatrix} \rho_{11}(0) & \rho_{12}(0) & \rho_{13}(0) & \rho_{14}(0) \\ \rho_{12}^*(0) & \rho_{22}(0) & \rho_{23}(0) & \rho_{24}(0) \\ \rho_{13}^*(0) & \rho_{23}^*(0) & \rho_{33}(0) & \rho_{34}(0) \\ \rho_{14}^*(0) & \rho_{24}^*(0) & \rho_{34}^*(0) & \rho_{44}(0) \end{pmatrix}, \tag{6}$$

where the matrix elements are expressed as follows:

$$\begin{aligned} \rho_{11}(0) &= A_1^2A_2^2, \\ \rho_{13}(0) &= A_1A_2^2\sqrt{1 - A_1^2}\,{\rm e}^{-{\rm i}\varphi_1}, \\ \rho_{12}(0) &= A_1^2A_2\sqrt{1 - A_2^2}{\rm e}^{-{\rm i}\varphi_2}, \\ \rho_{14}(0) &= A_1A_2\sqrt{1 - A_1^2}\sqrt{1 - A_2^2}\,{\rm e}^{-{\rm i}(\varphi_1+\varphi_2)}, \end{aligned}$$

$$
\begin{aligned}
&\rho_{22}(0) = A_1^2(1-A_2^2),\\
&\rho_{23}(0) = A_1A_2\sqrt{1-A_1^2}\sqrt{1-A_2^2}\,\mathrm{e}^{-\mathrm{i}(\varphi_1-\varphi_2)},\\
&\rho_{33}(0) = A_2^2(1-A_1^2),\\
&\rho_{24}(0) = A_1\sqrt{1-A_1^2}(1-A_2^2)\,\mathrm{e}^{-\mathrm{i}\varphi_1},\\
&\rho_{34}(0) = A_2(1-A_1^2)\sqrt{1-A_2^2}\,\mathrm{e}^{-\mathrm{i}\varphi_2},\\
&\rho_{44}(0) = 1-\rho_{11}(0)-\rho_{22}(0)-\rho_{33}(0). \qquad (7)
\end{aligned}
$$

Substituting Eqs.(2), (6) and (7) into Eq.(4), we can write down the density matrix of the two atoms at time t:

$$
\rho_s(t) = \begin{pmatrix}
\rho_{11} & \rho_{12} & \rho_{13} & \rho_{14}\\
\rho_{12}^* & \rho_{22} & \rho_{23} & \rho_{24}\\
\rho_{13}^* & \rho_{23}^* & \rho_{33} & \rho_{34}\\
\rho_{14}^* & \rho_{24}^* & \rho_{34}^* & \rho_{44}
\end{pmatrix}, \qquad (8)
$$

where

$$
\begin{aligned}
\rho_{11} =& \sum_n P_n\{[\rho_{22}(0)+\rho_{23}(0)+\rho_{32}^*(0)+\rho_{33}(0)]n(n-1)S_n^2\\
&+\rho_{11}(0)[(n+1)(n+2)C_{n+2}+1]^2+\rho_{44}(0)n(n-1)(n-2)(n-3)C_{n-2}^2\},\\
\rho_{12} =& \sum_n P_n\Big\{\rho_{12}(0)[(n+1)(n+2)C_{n+2}+1]\cos^2\Big(\frac{1}{2}\Omega_n t\Big)\\
&-\rho_{13}(0)[(n+1)(n+2)C_{n+2}+1]\sin^2\Big(\frac{1}{2}\Omega_n t\Big)+[\rho_{24}(0)+\rho_{34}(0)]n(n-1)S_nS_{n-2}\Big\},\\
\rho_{13} =& \sum_n P_n\Big\{-\rho_{12}(0)[(n+1)(n+2)C_{n+2}+1]\sin^2\Big(\frac{1}{2}\Omega_n t\Big)\\
&+\rho_{13}(0)[(n+1)(n+2)C_{n+2}+1]\cos^2\Big(\frac{1}{2}\Omega_n t\Big)+[\rho_{24}(0)+\rho_{34}(0)]n(n-1)S_nS_{n-2}\Big\},\\
\rho_{14} =& \sum_n P_n\{\rho_{14}(0)[(n+1)(n+2)C_{n+2}+1][n(n-1)C_{n-2}+1]\},\\
\rho_{22} =& \sum_n P_n\Big\{\rho_{22}(0)\cos^4\Big(\frac{1}{2}\Omega_n t\Big)-[\rho_{23}(0)+\rho_{23}^*(0)]\cos^2\Big(\frac{1}{2}\Omega_n t\Big)\sin^2\Big(\frac{1}{2}\Omega_n t\Big)\\
&+\rho_{33}(0)\sin^4\Big(\frac{1}{2}\Omega_n t\Big)+\rho_{11}(0)(n+1)(n+2)S_{n+2}^2+\rho_{44}(0)n(n-1)S_{n-2}^2\Big\},\\
\rho_{23} =& \sum_n P_n\Big\{\rho_{23}(0)\cos^4\Big(\frac{1}{2}\Omega_n t\Big)-[\rho_{22}(0)+\rho_{33}(0)]\cos^2(\frac{1}{2}\Omega_n t\Big)\sin^2\Big(\frac{1}{2}\Omega_n t\Big)\\
&+\rho_{23}^*(0)\sin^4\Big(\frac{1}{2}\Omega_n t\Big)+\rho_{44}(0)n(n-1)S_{n-2}^2+\rho_{11}(0)(n+1)(n+2)S_{n+2}^2\Big\},\\
\rho_{24} =& \sum_n P_n\Big\{\rho_{24}(0)[n(n-1)C_{n-2}+1]\cos^2\Big(\frac{1}{2}\Omega_n t\Big)+\rho_{12}(0)(n+1)(n+2)S_nS_{n+2}\\
&-\rho_{34}(0)[n(n-1)C_{n-2}+1]\sin^2\Big(\frac{1}{2}\Omega_n t\Big)+\rho_{13}(0)(n+1)(n+2)S_nS_{n+2}\Big\},\\
\rho_{33} =& \sum_n P_n\Big\{-\rho_{23}^*(0)\cos^2\Big(\frac{1}{2}\Omega_n t\Big)\sin^2\Big(\frac{1}{2}\Omega_n t\Big)\\
&-\rho_{23}(0)\cos^2\Big(\frac{1}{2}\Omega_n t\Big)\sin^2\Big(\frac{1}{2}\Omega_n t\Big)+\rho_{33}(0)\cos^4\Big(\frac{1}{2}\Omega_n t\Big)\\
&+\rho_{22}(0)\sin^4\Big(\frac{1}{2}\Omega_n t\Big)+\rho_{44}(0)n(n-1)S_{n-2}^2+\rho_{11}(0)(n+1)(n+2)S_{n+2}^2\Big\},\\
\rho_{34} =& \sum_n P_n\Big\{\rho_{34}(0)[n(n-1)C_{n-2}+1]\cos^2\Big(\frac{1}{2}\Omega_n t\Big)\\
&-\rho_{24}(0)[n(n-1)C_{n-2}+1]\sin^2\Big(\frac{1}{2}\Omega_n t\Big)+[\rho_{12}(0)+\rho_{13}(0)](n+1)(n+2)S_{n+2}^2\Big\},\\
\rho_{44} =& 1-\rho_{11}-\rho_{22}-\rho_{33}. \qquad (9)
\end{aligned}
$$

It is difficult to obtain the analytical results of the eigenvalues of the partial transposition of the system density matrix corresponding to Eq.(8). Therefore, we numerically calculate the corresponding eigenvalues, and illustrate the atom–atom negativity in Fig.1 for different $\bar{n}$ and the relative phases. The time evolution of atom–atom negativity is plotted by the dotted lines (which is the same as Ref.[32]) for the two atoms initially in the incoherent states and by the solid lines for them initially in the coherent states. From these figures we can find that, firstly, ETA is enhanced owing to the atomic coherence comparing the dotted line with the solid line in Figs.1(a)–(c). The maximum of atom–atom negativity reaches 0.5 for small value of the mean photon number $\bar{n} = 0.01$ due to the atomic coherence while it is only 0.2 (the same as that shown in Ref.[32]) when the two atoms are initially in the incoherent state (as shown in Fig.1(a)). Secondly, with increase of the mean photon number (see Figs.1(a) to 1(c)), which means that the weight factor P_n of the single-mode thermal field becomes flat (which washes out the entanglement feature), the value of atom–atom negativity decreases (as shown in Figs.1(b),(c)). Finally, when the mean photon number is small, there are some disentangled states of the two atoms, while for large $\bar{n}$, such as $\bar{n} = 5.0$ (as shown in Fig.1(c)), there are not disentangled states and the two atoms are entangled at all times except at $t = 0$ for the solid line. Our results also show that ETA decreases with increase of the relative phase $\Delta\varphi$ ($\Delta\varphi = |\varphi_2 - \varphi_1|$) of the two initial coherent states, while the case is completely contrary for $\bar{n} = 5.0$, which means that ETA depends not only on the initial atomic states and the mean photon number but also on the relative phase $\Delta\varphi$ of the two initial atomic coherent states. Hence we can control ETA by changing the proper superposition coefficients and relative phases in the initial atomic states.

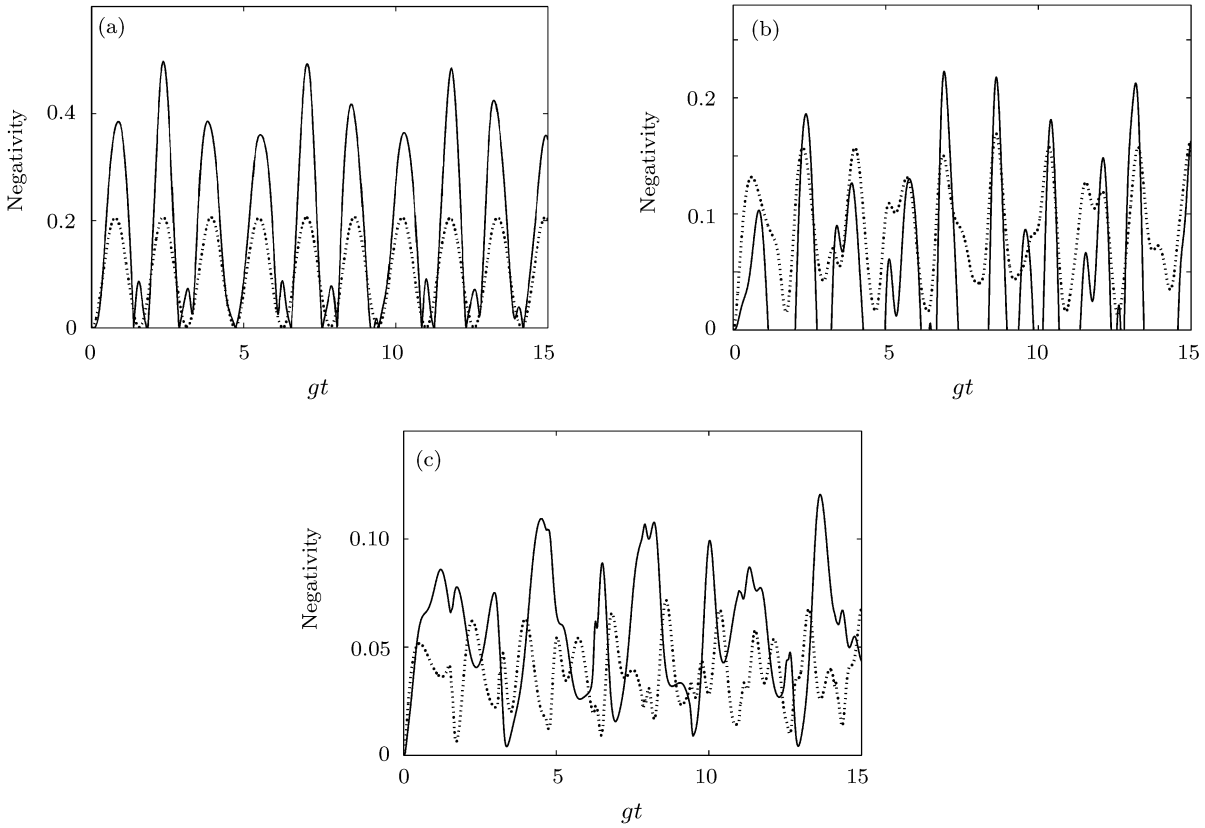


Fig.1. The time evolution of atom–atom negativity. (a) $\bar{n} = 0.01$, $|\Psi_1(0)\rangle = |e\rangle$ and $|\Psi_2(0)\rangle = |g\rangle$ (dotted), $|\Psi_1(0)\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$ and $|\Psi_2(0)\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$(solid). (b) $\bar{n} = 1.0$, $|\Psi_1(0)\rangle = |e\rangle$ and $|\Psi_2(0)\rangle = |g\rangle$ (dotted), $|\Psi_1(0)\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$ and $|\Psi_2(0)\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$ (solid). (c) $\bar{n} = 5.0$, $|\Psi_1(0)\rangle = |e\rangle$ and $|\Psi_2(0)\rangle = |g\rangle$ (dotted), $|\Psi_1(0)\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$ and $|\Psi_2(0)\rangle = \frac{1}{\sqrt{2}}(|e\rangle - |g\rangle)$ (solid).

3.2. The entanglement between the two atoms in $|\Psi(0)\rangle = A|eg\rangle + \sqrt{(1-A^2)}\mathrm{e}^{\mathrm{i}\varphi}|ge\rangle$

Here A is the real coefficient and φ is the relative phase. It should be noticed that the two atoms in this initial state are initially entangled when $A \neq 1$ or 0 and maximally entangled when $A = 1/\sqrt{2}$. The initial density matrix for the two atoms can be expressed as

$$\rho_s(0) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A^2 & A\sqrt{1-A^2}\mathrm{e}^{-\mathrm{i}\varphi} & 0 \\ 0 & A\sqrt{1-A^2}\mathrm{e}^{\mathrm{i}\varphi} & 1-A^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (10)$$

We can obtain the density matrix of the two atoms at time t when we substitute Eqs.(2), (10) into Eq.(4):

$$\rho_s(t) = \begin{pmatrix} \rho_{11} & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23}^* & \rho_{33} & 0 \\ 0 & 0 & 0 & \rho_{44} \end{pmatrix}, \quad (11)$$

where the corresponding matrix elements are defined as follows:

$$\begin{aligned}
\rho_{11} =& \sum_n P_n[1 + A\sqrt{1-A^2}\mathrm{e}^{-\mathrm{i}\varphi} \\
&+ A\sqrt{1-A^2}\mathrm{e}^{\mathrm{i}\varphi}]n(n-1)S_n^2, \\
\rho_{22} =& \sum_n P_n\Big\{A^2\cos^4\Big(\frac{1}{2}\Omega_n t\Big) \\
&+ (1-A^2)\sin^4\Big(\frac{1}{2}\Omega_n t\Big) \\
&- (A\sqrt{1-A^2}\mathrm{e}^{-\mathrm{i}\varphi} + A\sqrt{1-A^2}\mathrm{e}^{\mathrm{i}\varphi}) \\
&\times \cos^2\Big(\frac{1}{2}\Omega_n t\Big)\sin^2\Big(\frac{1}{2}\Omega_n t\Big)\Big\}, \\
\rho_{23} =& \sum_n P_n\{-\cos^2\Big(\frac{1}{2}\Omega_n t\Big)\sin^2\Big(\frac{1}{2}\Omega_n t\Big) \\
&+ A\sqrt{1-A^2}\mathrm{e}^{-\mathrm{i}\varphi}\cos^4\Big(\frac{1}{2}\Omega_n t\Big) \\
&+ A\sqrt{1-A^2}\mathrm{e}^{\mathrm{i}\varphi}\sin^4\Big(\frac{1}{2}\Omega_n t\Big)\}, \\
\rho_{33} =& \sum_n P_n\Big\{(1-A^2)\cos^4\Big(\frac{1}{2}\Omega_n t\Big) \\
&+ A^2\sin^4\Big(\frac{1}{2}\Omega_n t\Big) \\
&- (A\sqrt{1-A^2}\mathrm{e}^{-\mathrm{i}\varphi} + A\sqrt{1-A^2}\mathrm{e}^{\mathrm{i}\varphi}) \\
&\times \cos^2\Big(\frac{1}{2}\Omega_n t\Big)\sin^2\Big(\frac{1}{2}\Omega_n t\Big)\Big\}, \\
\rho_{44} =& 1 - \rho_{11} - \rho_{22} - \rho_{33}. \quad (12)
\end{aligned}$$

The numerical results of atom–atom negativity according to Eqs.(11) and (12) are displayed in Figs.2 and 3. Figure 2 shows how the mean photon number $\bar{n}$ and the coefficient A affect ETA, while Fig.3 describes the dependence of the atom–atom negativity on the coefficient A and the relative phase φ. From these figures, we can conclude the following: (1) Comparing the solid lines and the dashed lines with the dotted lines in every figure, we find that the atom–atom negativity of the two atoms in the initial coherent states is always larger than that of the two atoms initially in the incoherent states, which means that it is possible to get strong entanglement between the two atoms by means of the initial atomic coherence. (2) With increase of the coefficient A from 0 to $1/\sqrt{2}$, ETA is enhanced and becomes largest when $A = 1/\sqrt{2}$, which indicates that, the stronger the coherence of the initial states, the larger ETA. (3) The influence of the relative phase on the entanglement can be observed from Fig.3. It is clear that, with increase of the relative phase from 0 to π (corresponding to the dotted, dashed, dash-dotted and solid lines in the figures), the ETA increased. When $\varphi = 0$ the two atoms are periodically disentangled, while when $\varphi \neq 0$ the two atoms are entangled at all time. In addition, from Fig.3(a), we find that the atom–atom negativity becomes larger than its initial value for $\varphi = 2\pi/3$ and $\varphi = \pi$, which means that ETA can be amplified by changing appropriate initial parameters of the system. (4) The entanglement dynamics between the two atoms tends to harmonic oscillation when $\bar{n} = 0.1$ (as shown in Figs.2(a) and 3), this stems from the fact that the nonlinear interaction behaviour is suppressed when the mean photon number is very small. (5) For $\bar{n} = 40$, corresponding to the high temperature, ETA is still strong and its time evolution displays approximately period with the period π (as shown in Fig.2(c)). Similar dynamics appears for the other values of the mean photon number if only $\bar{n} \geq 5$. This denotes that one can prepare atom–atom entanglement under high temperature by use of the atomic coherence.

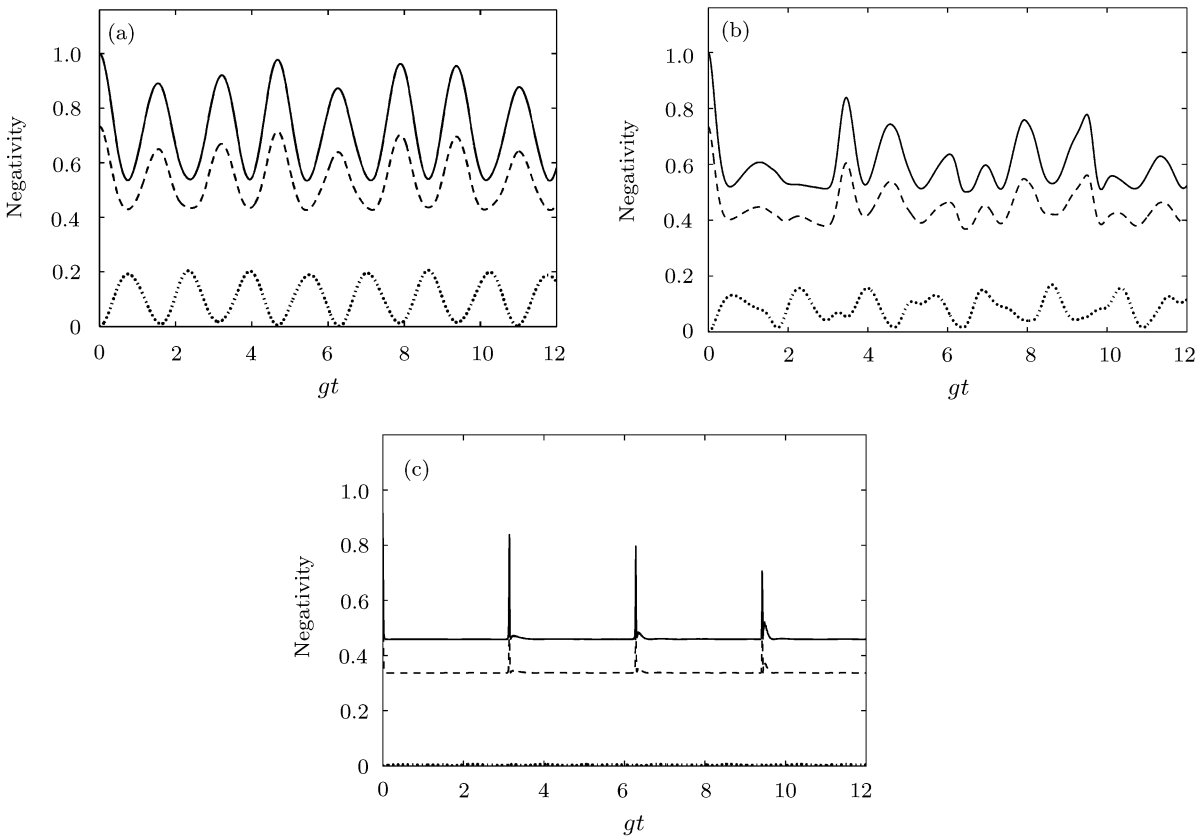


Fig.2. The time evolution of atom–atom negativity induced by interacting with a thermal field for (a) $\bar{n} = 0.1$, (b) $\bar{n} = 1.0$ and (c) $\bar{n} = 40$ when the atoms are initially prepared in $|\Psi(0)\rangle = |eg\rangle$ (dotted), $|\Psi(0)\rangle = 0.4|eg\rangle + 0.9165\mathrm{e}^{2\pi\mathrm{i}/3}|ge\rangle$ (dashed) and $|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|eg\rangle + \mathrm{e}^{2\pi\mathrm{i}/3}|ge\rangle)$ (solid).

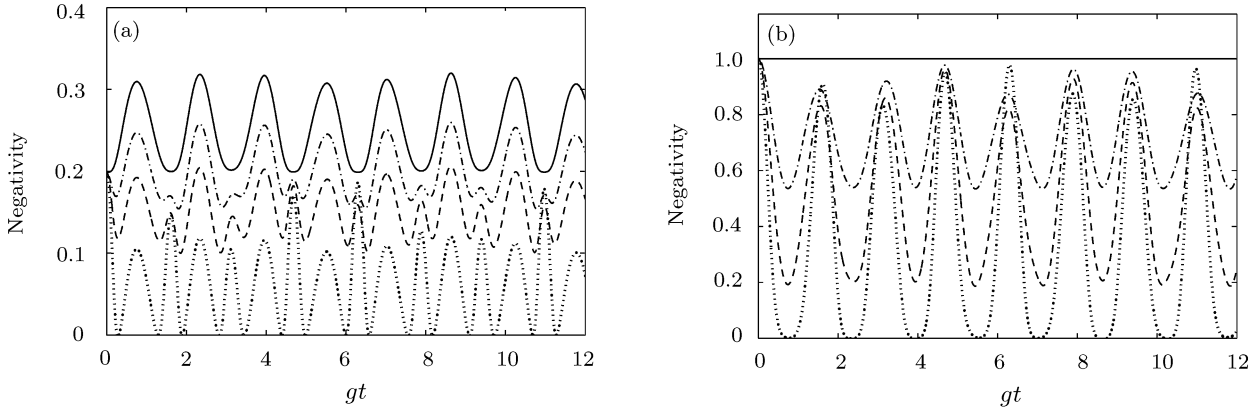


Fig.3. The time evolution of atom–atom negativity induced by interacting with a thermal field for (a) $\bar{n} = 0.1$, $A = 0.1$ and (b) $\bar{n} = 0.1$, $A = 1/\sqrt{2}$ and the phases are $\varphi = 0$ (dotted), $\varphi = \pi/2$ (dashed), $\varphi = 2\pi/3$ (dash-dotted) and $\varphi = \pi$ (solid).

3.3. The entanglement between the two atoms in $|\Psi(0)\rangle = B|ee\rangle + \sqrt{(1-B^2)}\mathrm{e}^{\mathrm{i}\varphi'}|gg\rangle$

In this section, the two atoms are initially in $|\Psi(0)\rangle = B|ee\rangle + \sqrt{(1-B^2)}\mathrm{e}^{\mathrm{i}\varphi'}|gg\rangle$, where B is the real coefficients and φ' is the phase. By using the same method in section 3.2, the density matrix of the two atoms at time t can be obtained as follows:

$$\rho_s(t) = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23}^* & \rho_{33} & 0 \\ \rho_{14}^* & 0 & 0 & \rho_{44} \end{pmatrix}, \tag{13}$$

where

$$\rho_{11} = \sum_n P_n\{B^2[(n+1)(n+2)C_{n+2}+1]^2 + \sqrt{1-B^2}n(n-1)(n-2)(n-3)C_{n-2}^2\},$$
$$\rho_{14} = \sum_n P_n B\sqrt{1-B^2}\mathrm{e}^{-\mathrm{i}\varphi'}[(n+1)(n+2)C_{n+2}+1][n(n-1)C_{n-2}+1],$$
$$\rho_{22} = \sum_n P_n\{(1-B^2)n(n-1)S_{n-2}^2 + B^2(n+1)(n+2)S_{n+2}^2\},$$
$$\rho_{23} = \sum_n P_n\{(1-B^2)n(n-1)S_{n-2}^2 + B^2(n+1)(n+2)S_{n+2}^2\},$$
$$\rho_{33} = \sum_n P_n\{(1-B^2)n(n-1)S_{n-2}^2 + B^2(n+1)(n+2)S_{n+2}^2\},$$
$$\rho_{44} = 1-\rho_{11}-\rho_{22}-\rho_{33}. \tag{14}$$

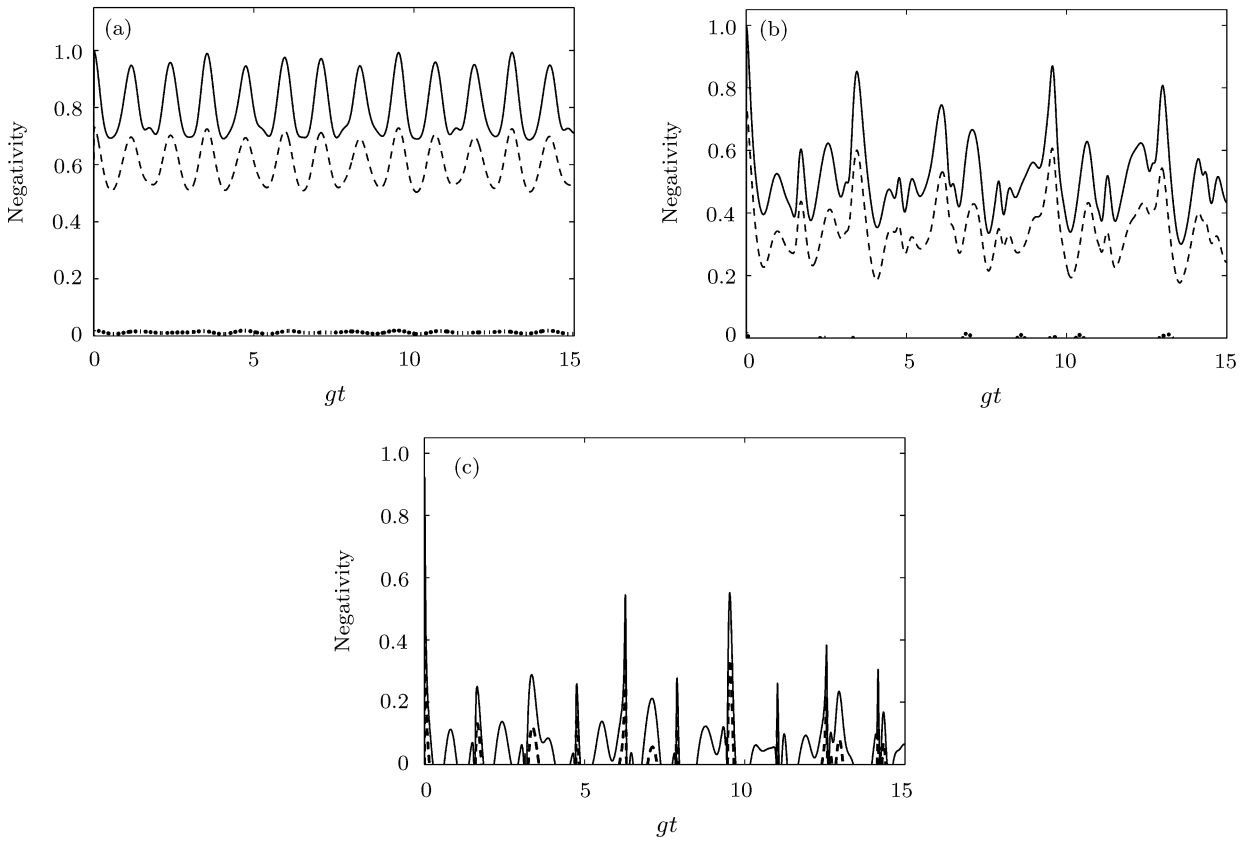


Fig.4. The time evolution of atom–atom negativity induced by interacting with a thermal field for (a) $\bar{n}=0.1$, (b) $\bar{n}=1.0$ and (c) $\bar{n}=10$, the real coefficients are $B=0$ (dotted), $B=0.4$ (dashed) and $B=1/\sqrt{2}$ (solid).

The time evolution of the atom–atom negativity according to Eqs.(13) and (14) is plotted in Fig.4 for different $\bar{n}$ and different values of the coefficient B. Similarly, it is observed that the initial atomic coherence greatly enhances ETA. It is noted that, for $\bar{n}=1.0$, the maximum of the atom–atom negativity is less than 0.016 when the atoms are initially prepared in $|gg\rangle$ in Ref.[32], however, it is more than 0.8 when the state of the two atoms is initially $\Psi(0)=\frac{1}{\sqrt{2}}(|ee\rangle+|gg\rangle)$ (as shown in Fig.4(b)). In addition, the two atoms are not entangled for $\bar{n}=10$ when they are initially prepared in the incoherent states, while they are entangled and the atom–atom negativity can almost reach 0.6 when they are initially in the coherent states (as shown in Fig.3(c)). Hence the two atoms can possibly be entangled in the both cases when they are initially coherent and in the entangled state. It should be noticed that, here, the atom–atom

negativity only depends on the mean photon number of the cavity field and the superposition coefficient of the initial atomic coherent state, and does not depend on the phase φ'. This can be easily explained as follows. Only ρ_{14} of the corresponding matrix elements defined by Eq.(14) is related to the relative phase φ', and from Eq.(13), we obtain the negative eigenvalue $\mu^- = (\rho_{11} + \rho_{44}) - \sqrt{(\rho_{11} + \rho_{44})^2 - 4\rho_{11}\rho_{44} + 4|\rho_{23}|^2}$ if and only if $|\rho_{23}|^2 > \rho_{11}\rho_{44}$, or $\mu^- = (\rho_{22} + \rho_{33}) - \sqrt{(\rho_{22} + \rho_{33})^2 - 4\rho_{22}\rho_{33} + 4|\rho_{14}|^2}$ if and only if $|\rho_{14}|^2 > \rho_{22}\rho_{33}$. It is clear that the two cases of the negative eigenvalue do not depend on the relative phase φ'.

4. Conclusion

In this paper, we have studied the atomic coherence control on the entanglement between two identical two-level atoms interacting with a single-mode thermal field through two-photon processes. Our results show that ETA is greatly enhanced due to initial atomic coherence in comparison with the case when the atoms are initially prepared in incoherent states,[32] and it can be manipulated by changing the system parameters, such as the superposition coefficients and the relative phases of the initial atomic coherent state and the mean photon number of the cavity field. Our results are important for entanglement control in quantum information processing.

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