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Effect of spontaneously generated coherence on inversionless lasing gain in an atomic system with Doppler broadening*

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We have studied the effect of the spontaneously generated coherence (SGC) on gain of lasing without inversion (LWI) in a closed three-level Λ-type atomic system with Doppler broadening. It is shown that, regardless of the driving and probe fields being co- or counter-propagating, at a suitable value of the Doppler width, we can obtain a much larger LWI gain with SGC than that without SGC; and the region of the LWI gain spectrum with SGC is obviously larger than that without SGC. When the Doppler width takes a constant value, the gain does not monotonically decrease or increase with increasing strength of SGC, the largest LWI gain can be obtained by adjusting strength of SGC. Generally speaking, the co-propagating probe and driving fields is favourable to obtain a larger LWI gain.

Keywords: spontaneously generated coherence, Doppler broadening, lasing without inversion, gain
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1. Introduction

Quantum coherence and interference in an atomic system have shown a number of important phenomena, such as lasing without inversion (LWI),[1,2] cancellation of spontaneous emission,[3] subluminal and superluminal light,[4] the enhancement of the refractive index,[5] coherent population trapping (CPT),[6] the electromagnetically induced transparency (EIT),[7,8] and so on. In particular, LWI has attracted much attention. There are many ways to generate quantum coherence and interference in an atomic system. Generally, they are produced by a coherent driving field or by initial coherence. A kind of coherence can be created by interference of spontaneous emission, usually called spontaneously generated coherence (SGC). SGC can be created from two close-lying atomic levels to a common atomic level coherence (V-type atom) as well as a single excited level to two close-lying atomic levels (Λ-type atom), and it can also be created in the near-spaced atomic levels case (ladder-type atom). The study of SGC has attracted considerable attention.[9−21] At present the effects of SGC on gain, dispersion and populations in an atomic system have been intensively investigated.[22−26] However, these studies for the effect of SGC on LWI gain just considered the static atomic systems. Under the action of the laser field, the motion of atoms will lead to the Doppler broadening. Some papers have discussed the influence of the Doppler broadening on LWI gain in the atomic systems without SGC.[27−35] We also have analysed the effect of Doppler broadening on LWI gain in a three-level closed Λ-type atomic system without SGC.[36] In this paper, we explore the main effect of SGC on LWI gain in a closed three-level Λ-type atomic system with Doppler broadening. We also consider the combined effects of SGC and Doppler broadening on LWI gain in an atomic system, for the first time, to our best knowledge.

2. Model and equations of motion

Consider a closed three-level Λ-type atomic system with an upper level $|1\rangle$ and two lower close-lying levels $|2\rangle$ and $|3\rangle$, as illustrated in Fig.1. The transi-

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tion between levels $|1\rangle$ and $|2\rangle$ is driven by a strong coherent driving field $\boldsymbol{E}_{\rm d}$ with the frequency $\omega_{\rm d}$ and Rabi frequency Ω. An incoherent pump with a pumping rate 2Λ and a weak coherent probe field $\boldsymbol{E}_{\rm p}$ with the frequency $\omega_{\rm p}$ and Rabi frequency α are applied to levels $|1\rangle$ and $|3\rangle$. The spontaneous emission rates from the level $|1\rangle$ to levels $|3\rangle$ and $|2\rangle$ are denoted by $2\gamma_1$ and $2\gamma_2$, respectively.

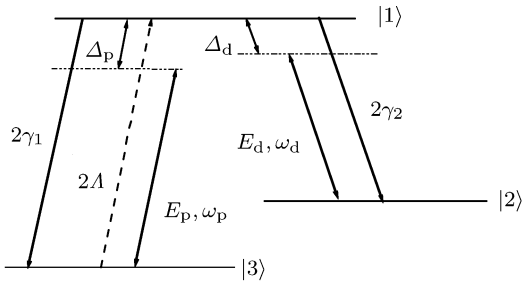


Fig.1. A closed three-level Λ-type atomic system.

In the interaction picture, using the rotating-wave and electric dipole approximations, the density matrix equations of motion of this system can be written as

$$\dot{\rho}_{11} = -2(\gamma_1+\gamma_2)\rho_{11} + {\rm i}\alpha\rho_{31} - {\rm i}\alpha^*\rho_{13} + 2\Lambda\rho_{33} + {\rm i}\Omega\rho_{21} - {\rm i}\Omega^*\rho_{12}, \tag{1a}$$

$$\dot{\rho}_{22} = 2\gamma_2\rho_{11} + {\rm i}\Omega^*\rho_{12} - {\rm i}\Omega\rho_{21}, \tag{1b}$$

$$\dot{\rho}_{12} = -(\gamma_1+\gamma_2+{\rm i}\Delta_{\rm d})\rho_{12} + {\rm i}\alpha\rho_{32} - {\rm i}\Omega(\rho_{11}-\rho_{22}), \tag{1c}$$

$$\dot{\rho}_{13} = -(\gamma_1+\gamma_2+\Lambda+{\rm i}\Delta_{\rm p})\rho_{13} + {\rm i}\Omega\rho_{23} - {\rm i}\alpha(\rho_{11}-\rho_{33}), \tag{1d}$$

$$\dot{\rho}_{23} = -(\Lambda+{\rm i}\Delta_{\rm p}-{\rm i}\Delta_{\rm d})\rho_{23} + 2p\sqrt{\gamma_1\gamma_2}\rho_{11} + {\rm i}\Omega^*\rho_{13} - {\rm i}\alpha\rho_{21}, \tag{1e}$$

along with the equations of their complex conjugates, $\rho_{ij} = \rho^*_{ji}(i \neq j, i, j = 1, 2, 3)$ and the closure relation $\rho_{11}+\rho_{22}+\rho_{33} = 1$. $\Delta_{\rm p}$ $(= \omega_{13} - \omega_{\rm p})$ and $\Delta_{\rm d}$ $(= \omega_{12} - \omega_{\rm d})$ are the detunings of the probe and driving fields from their relevant atomic transitions, respectively. And $2p\sqrt{\gamma_1\gamma_2}$ represents the quantum interference effect from the cross-coupling between the spontaneous emissions $|1\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |3\rangle$, that is, the SGC effect. The parameter p denotes the alignment of the dipole matrix elements ($\boldsymbol{\mu_{12}}$ and $\boldsymbol{\mu_{13}}$) and is defined as $p \equiv \boldsymbol{\mu}_{13}\cdot\boldsymbol{\mu}_{12}/|\boldsymbol{\mu}_{13}||\boldsymbol{\mu}_{12}| = \cos\theta$, and θ represents the angle between the two dipole moments $\boldsymbol{\mu}_{13}$ and $\boldsymbol{\mu}_{12}$. If we consider the linearly polarized electric fields with the restriction that $\boldsymbol{\mu}_{12}\cdot\boldsymbol{E}_{\rm p} = 0$ and $\boldsymbol{\mu}_{13}\cdot\boldsymbol{E}_{\rm d} = 0$, then the Rabi frequencies are connected with the parameter p by the relation $\alpha = \alpha_0\sqrt{1-p^2} = \alpha_0\sin\theta$ and $\Omega = \Omega_0\sqrt{1-p^2} = \Omega_0\sin\theta$, with $\alpha_0 = |\boldsymbol{\mu}_{13}||\boldsymbol{E}_{\rm p}|/\hbar$, $\Omega_0 = |\boldsymbol{\mu}_{12}||\boldsymbol{E}_{\rm d}|/\hbar$.[18] If $\boldsymbol{\mu}_{13}$ and $\boldsymbol{\mu}_{12}$ is orthogonal to each other, then $p = 0$ and Eqs.(1a)–(1e) reduce to those for a closed Λ-type three-level atomic system without SGC.[36]

In a steady state case ${\rm d}\rho_{mn}/{\rm d}t = 0$, Eqs.(1a)–(1e) reduce to a set of coupled algebraic equations. After splitting them into real and imaginary parts, we obtain a system of 9 algebraic equations. Then we can obtain the exact nonlinear analytical expressions for ${\rm Im}\rho_{31}$, ρ_{11}, ρ_{22}, ρ_{33} by using the symbolic package Mathematica or Maple. ${\rm Im}\rho_{31}$, ρ_{11}, ρ_{22}, ρ_{33} are complicated functions of Ω, α, $\Delta_{\rm p}$, $\Delta_{\rm d}$ and other parameters of the system. To reduce the length of the article, we do not give the detailed representations here. The probe gain corresponds to the imaginary of ρ_{31}. If ${\rm Im}\rho_{31} > 0$, the system exhibits the gain for the probe field. If ${\rm Im}\rho_{31} < 0$, the probe field will attenuate. When $\rho_{11} - \rho_{33} < 0$ and ${\rm Im}\rho_{31} > 0$, the probe field obtains inversionless gain. Similar to Refs.[32]–[34] and [36], when Doppler broadening is present, the gain and population can be represented by

$$G = \frac{2N\mu_{13}^2}{\hbar\alpha}{\rm Im}\int_{-\infty}^{\infty}\rho_{31}(\delta_{\rm p})\rho(\delta_{\rm p}){\rm d}\delta_{\rm p}, \tag{2}$$

$$\bar{\rho}_{ii} = \int_{-\infty}^{\infty}\rho_{ii}(\delta_{\rm p})\rho(\delta_{\rm p}){\rm d}\delta_{\rm p}, \tag{3}$$

where N is the medium density, $\hbar$ is Planck's constant.

$$\rho(\delta_{\rm p}) = \frac{1}{\sqrt{2\pi D^2}}{\rm e}^{-(\delta_{\rm p}-\Delta_{\rm p})/2D^2} \tag{4}$$

is the Maxwell–Boltzmann distribution for the atomic velocities, with D being the Doppler width, $\delta_{\rm p}(v)$ the detuning of the probe field from the respective transitions, as seen by the moving atom.

$$\delta_{\rm p}(v) = \Delta_{\rm p} \mp \omega_{\rm p}v/c\,. \tag{5}$$

where v is the velocity of the atom, and the negative (positive) sign corresponds to co-propagating (counter-propagating) atom and probe, and the detuning $\delta_{\rm d}(v)$ corresponds to the coherent driving field that has been represented by a function of $\delta_{\rm p}(v)$.

We have already discussed the effect of Doppler broadening on LWI gain in a closed Λ-type three-level system without SGC.[36] It is shown that, regardless of the probe and driving fields being counter- or co-propagating, the gain does not always monotonically

decrease or increase with the increase of the Doppler width; at a suitable Doppler width one can obtain a maximum value of gain much larger than that without Doppler broadening. Now we give a physical explanation for this striking behaviour.

For the Λ-type atomic system without Doppler broadening, the probe amplification may proceed through the stimulated emission between the inverted dressed state transitions.[37] The gain appears due to the population inversion in the dressed state basis when the driving field detunes sufficiently from the resonance. Moreover, the gain reaches a maximum value when the driving detuning takes a suitable value. For moving atoms, the Doppler effect will lead to detunings of the probe and driving frequencies from the resonance in the same direction, then the relative populations of dressed states also change. The detuning from the inversed dressed state transition is dependent, via Doppler shifts, on the velocity for a given velocity group and on the relative magnitudes of driving and generated waves' wave vectors. Thus some of the velocity groups contribute via this mechanism much more strongly to the observed gain than the other groups. At the same time, low velocity groups contribute also to the gain due to the LWI mechanism in Ref.[37]. An amplification mechanism (and lasing) based on inverted dressed state transition has been studied in detail for homogeneously broadened systems.[38−40] The interesting feature of the inhomogeneously broadened Λ-type atomic system considered in Ref.[36] is that the required driving field detuning appears as Doppler shifts.

In the following we will analyse the effect of SGC on the gain of the system with Doppler broadening according to the numerical calculation results from Eqs.(2) and (3).

3. Numerical calculation results and discussions

According to the numerical calculation result, Fig.2 illustrates the gain G as a function of the probe detuning $\Delta_{\rm p}$ for different Doppler widths when the probe and driving fields are co-propagating. It should be noted that here and in the following discussion, the inversionless condition $\bar{\rho}_{11} - \bar{\rho}_{33} < 0$ is always satisfied; if $G > 0$, we always obtain the inversionless gain, and thus LWI can be realized.

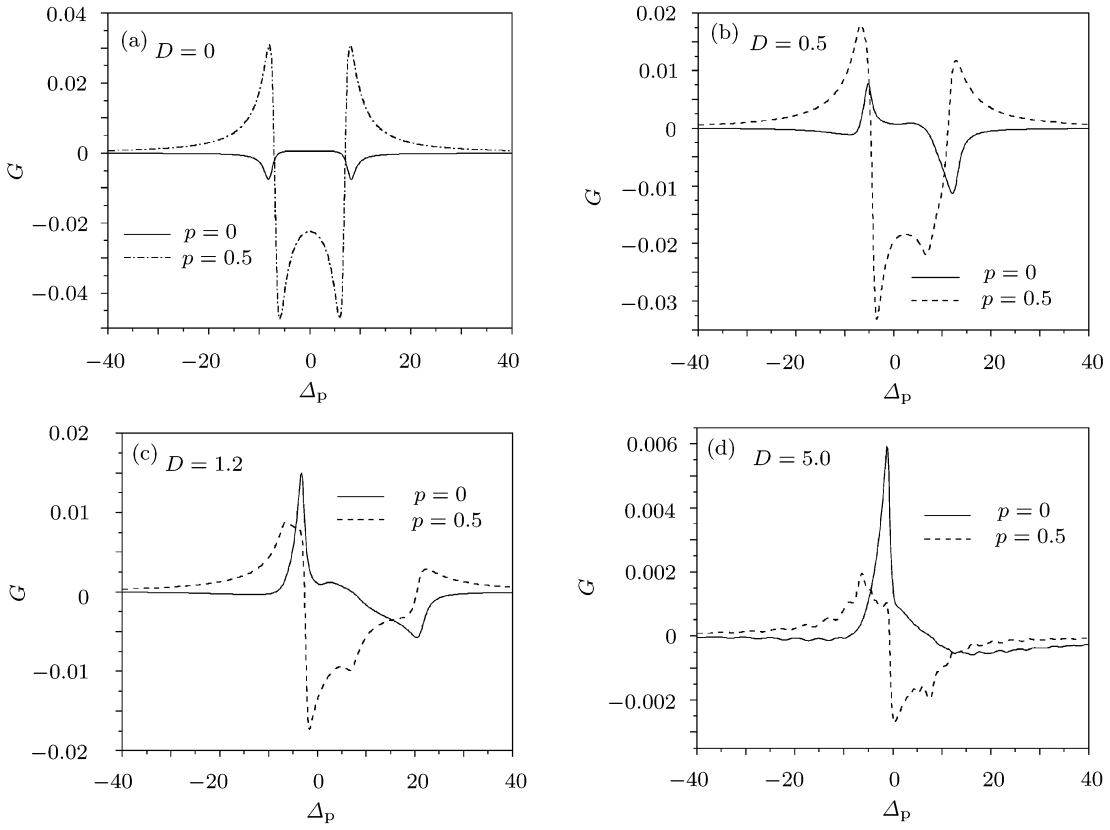


Fig.2. G versus $\Delta_{\rm p}$ for different Doppler widths when the probe and driving fields are co-propagating. The parameter values are $2\gamma_1 = 1.0$, $2\gamma_2 = 1.0$, $2\Lambda = 0.92$, $\alpha = 0.3\sqrt{1-p^2}$, $\Omega = 8.0\sqrt{1-p^2}$, $\Delta_{\rm d} = 0$。

In Fig.2 and the following figures, the values of G are given in the unit of $N\mu_{13}^2/(\varepsilon_0\hbar)$. When the value of D is very small, especially when $D = 0$ (Fig.2(a)), we can have a much larger LWI gain with SGC than that without SGC. When $D < 0.7$, e.g. $D = 0.5$ (Fig.2(b)), the maximum value of LWI gain with SGC is always larger than that without SGC; when $D > 0.7$, (see Figs.2(c) and (d)) the maximum value of LWI gain with SGC is smaller than that without SGC, but in some regions of the gain spectrum, the LWI gain with SGC is still larger than that without SGC. In addition, the region of the gain spectrum with SGC is obviously larger than that without SGC.

Taking same parameter values as those in Fig.2, Fig.3 shows G as a function of $\Delta_{\rm p}$ for different Doppler widths when the probe and drive fields are counter-propagating. From Fig.3 we can see that, the maximum value of LWI gain with SGC is always larger than that without SGC for different Doppler widths including $D = 0.8$, which is the value of Doppler width corresponding to the largest LWI gain without SGC;[36] and the region of the LWI gain spectrum with SGC is obviously larger than that without SGC. When D is large enough, gain oscillation occurs, and the oscillation amplitude and the region with SGC are much larger than that without SGC.

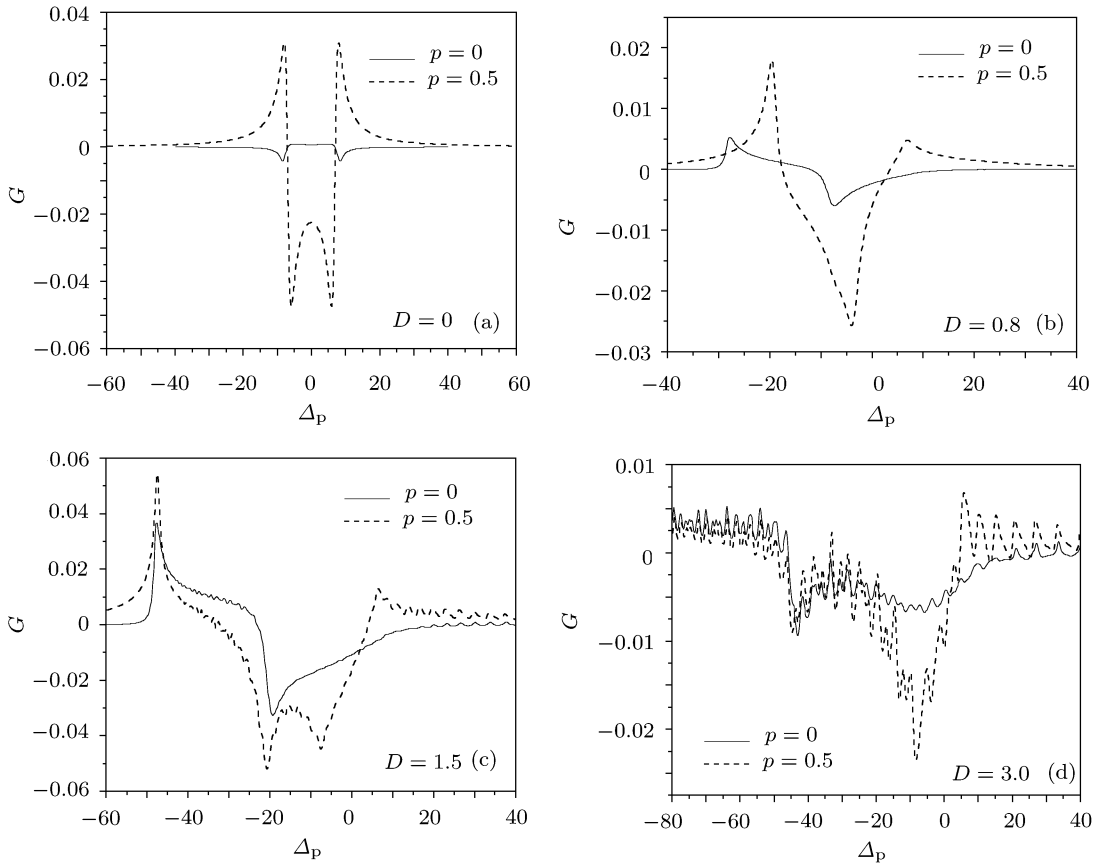


Fig.3. G versus $\Delta_{\rm p}$ when the probe and driving fields are counter-propagation, with the same parameter values as in Fig.2.

Figure 4 gives $G_{\max}$ as a function of Doppler width for different values of p, i.e. the strength of SGC, when the probe and drive fields are co-propagating. We can see that: 1) for any value of p, when D is very small, $G_{\max}$ with SGC is always much larger than that without SGC; however, when D is larger, $G_{\max}$ in the case with SGC is considerably smaller than that in the case without SGC. 2) $G_{\max}$ corresponding to a larger value of p decreases monotonically with increasing D but $G_{\max}$ corresponding to a smaller value of p does not monotonically decrease or increase with increasing D.

When the probe and driving fields are counter-propagating, with increasing D the variation of $G_{\max}$ is somewhat different from that in co-propagating case, i.e. when D is larger, $G_{\max}$ with SGC is nearly

the same as that without SGC. Figure 5 gives a comparison for the co- and counter- propagating cases when $p = 0.5$.

Figures 5(a) and 5(b) show again the variation trend of $G_{\max}$ versus D mentioned above, and Fig.5(c) displays the trend similar to the case without SGC;[36] when SGC is present, the co-propagating probe and driving fields is generally favourable to a larger LWI gain.

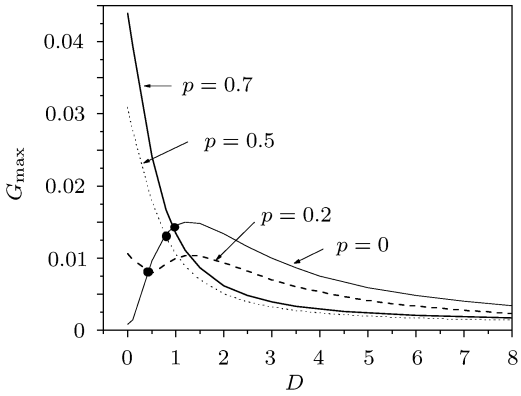


Fig.4. $G_{\max}$ versus D for different values of p when the probe and driving fields are co-propagating with the same parameter values as those in Fig.2.

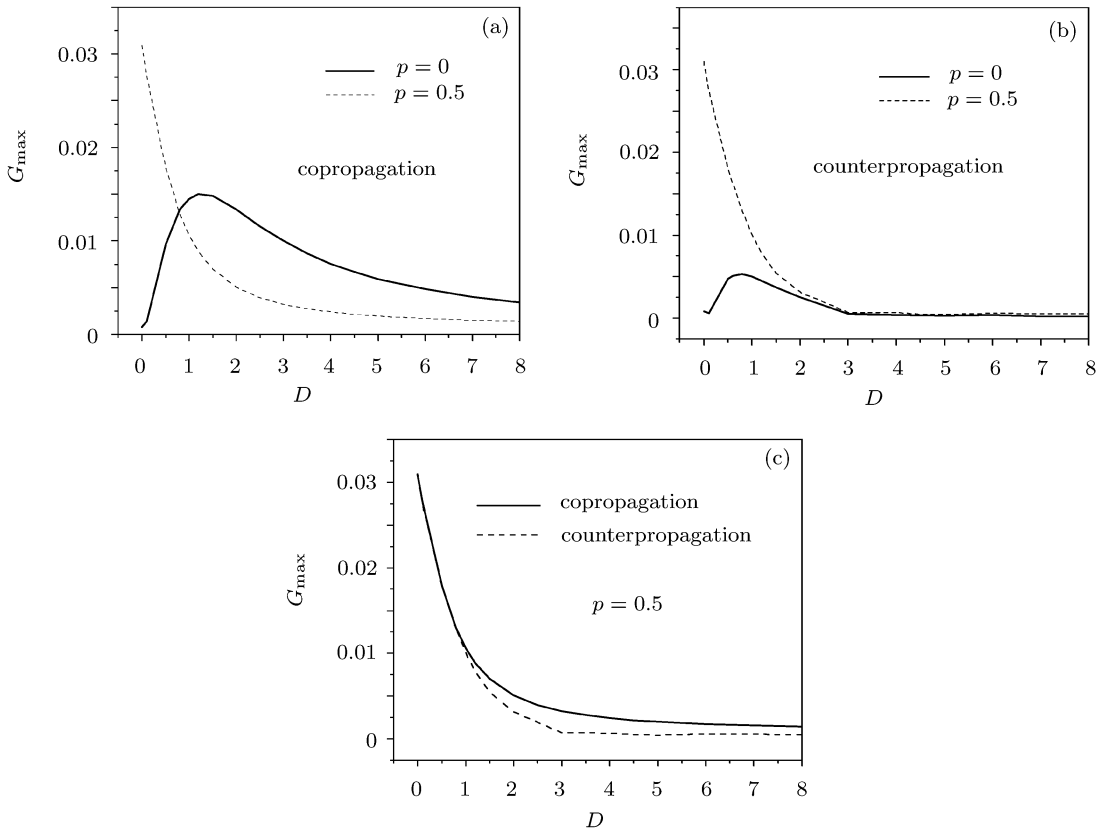


Fig.5. $G_{\max}$ versus D for the co- and counter-propagating probe and driving fields; the parameter values are same as those in Fig.2.

Figure 6 presents $G_{\max}$ as a function of p for the co- propagating probe and driving fields. We can see that when D takes a fixed value (here $D = 0.5$), $G_{\max}$ does not monotonically decrease or increase with increasing p; when $p = 0.82$, one can obtain the largest $G_{\max}(= 0.02634)$.

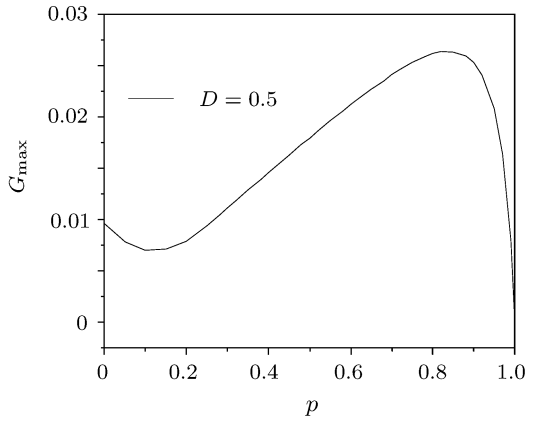


Fig.6. $G_{\max}$ versus p when the probe and driving fields are co-propagating, the parameter values are same as those in Fig.2.

4. Conclusions

In the paper, we discuss the effect of SGC on LWI gain in a closed three-level Λ-type atomic system with Doppler broadening by using the numerical calculation results from the analytical solution of the system. We find that: (i) Regardless of the driving and probe fields being co- or counter-propagating, when the Doppler width is small, the maximum value of LWI gain with SGC is always larger than that without SGC; especially when the Doppler width is very small, we can always obtain a much larger LWI gain with SGC than that without SGC; moreover, the region of LWI gain spectrum increases considerably when SGC is present. (ii) In the case of the co-propagating probe and driving fields, when the Doppler width is larger, although the largest LWI gain with SGC is smaller than that without SGC, we can still realize that the LWI gain with SGC is larger than that without SGC in some regions of the LWI gain spectrum. In the case of counter-propagating probe and driving fields, when the Doppler width is large enough, gain oscillation occurs, and the oscillation amplitude and the region with SGC are much larger than those without SGC. (iii) When the Doppler width is fixed, the maximum gain $G_{\max}$ does not decrease or increase monotonically with increasing p, i.e. increasing strength of SGC, and one can obtain the largest $G_{\max}$ at a suitable value of p. (iv) Similar to the case without SGC,[36] when SGC is present, the co-propagating probe and driving fields are generally favourable to obtain a larger LWI gain.

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