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Noise-induced striped trajectories of Rossler systems*

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In this paper, the generation of striped trajectories in phase space by noise-injection is considered. With suitable amplitudes of noise, the steady-state system orbits appear in rectangular striped shape. The relationship between the shape (including the range and the number of stripes) and some parameters is discussed. The result shows that noise can also generate the striped shape effectively and simply, which is similar to the newly-discovered striped pattern generated by controlled Rossler systems.

Keywords: multiple stripes，controlled Rossler systems, noise
PACC: 0547, 0545

1. Introduction

Nonlinear dynamics and complex phenomena have received much attention, and different viewpoints and methods based on mathematics or physics or control theory have been proposed to investigate the problems in the past several years.[1−5] After knowing the underlying mechanism of some complex behaviours such as chaos, one begins to consider the control and synchronization to reduce the complexity if possible;[6−10] moreover, one tends to consider the generation and employment of complex behaviours. In recent years, many effective methods have been proposed to generate specific chaotic attractors such as ring or spherical attractors,[11,12] multi-scroll attractors,[13] and striped attractors.[14,15]

On the other hand, noise has also been taken into consideration to eliminate or change chaotic behaviours in the last decade. Particularly, it has been suggested that noise-induced chaos may indeed exist in real systems. For example, it is shown in Ref.[16] that the chaos is induced by an spontaneous-emission noise in a semiconductor laser and it can do the job at an experimentally measured level. Moreover, in Ref.[17] how the additive noise affects near-integral second-order dynamical systems is studied. In addition, numerical simulations of a water wheel have been performed to investigate the influence of stochastic noise on the chaotic transients in Ref.[18].

The purpose of this paper is to create noise-induced 'chaos' in rectangular striped shape, and show how the noise amplitude influences the generation of these phenomena. The striped attractors were first generated and studied in Refs.[14,15]. In Ref.[14] it is shown that a striped chaotic attractor is generated by a simple controlled Rossler attractor with a non-smooth feedback, and then in Ref.[15] it is found that striped attractors can be easily generated by a modified Rossler system even with smooth feedback terms. Here we show that the noise can play almost the same role in creating the striped 'attractors'.

The rest of this paper is organized as follows. In Section 2, we introduce the striped 'chaotic' orbits generated by Rossler systems with an additive noise. Then in Section 3, some analyses for their generation are given, especially for different parameters. Finally, some concluding remarks are given in Section 4.

2. Striped shape

As has been done in Refs.[14,15], we also use the Rossler system. However, we will modify the system into the following form:

$$\begin{aligned} x' &= -y - z + l_1(\alpha N_1 + f(e_1 x)), \\ y' &= x + ay + l_2(\alpha N_2 + f(e_2 y)), \\ z' &= z(x - c) + b, \end{aligned} \tag{1}$$

where a, b and c are the parameters of the Rossler system, $N_i (i = 1, 2)$ denote the noise uniformly dis-

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tributed in an interval [0,1], α is a parameter, f is a periodic function, $l_i, (i = 1, 2)$ is the control gain, and parameter $e_i, (i = 1, 2)$ represents the frequency of the periodic function. The nonnegative and periodic function f can be taken in many forms such as $f(x) = 1 + \sin(x)$ or $f(x) = \arccos(\cos(x))$ or $f(x) = \left|\sin\left(\frac{x}{2}\right)\right|$.

In this paper, we take $f(x) = 1 + \sin(x)$, for the other non-negative and periodic functions show similar phenomena.

Then system (1) can be rewritten as

$$\begin{aligned} x' &= -y - z + l_1(\alpha N_1 + 1 + \sin(e_1 x)), \\ y' &= x + ay + l_2(\alpha N_2 + 1 + \sin(e_2 y)), \\ z' &= z(x - c) + b. \end{aligned} \tag{2}$$

In what follows, we discuss how to select suitable parameter α to generate the steady-state trajectory in striped rectangular form. When $\alpha = 0$, that is, in the noise-free case, we can see that the trajectory tends to a fixed point in the phase space.

In fact, we find that there exists a critical value $\alpha_c \approx 0.201$. When $0 < \alpha < \alpha_c$, the system state converges to a fixed point, and when $1 > \alpha > \alpha_c$, the system orbit appears in striped shape in the phase space.

Figure 1 shows that the noise-induced trajectory in striped rectangular shape is generated by system (2) with $a = 0.05$, $b = 0.20$, $c = 20.0$, $l_1 = l_2 = 20\pi$, and $e_1 = e_2 = 1$. As indicated in Fig.2, the numeral simulation results show the stable striped shapes when the parameter α varies from 0.216 to 0.6.

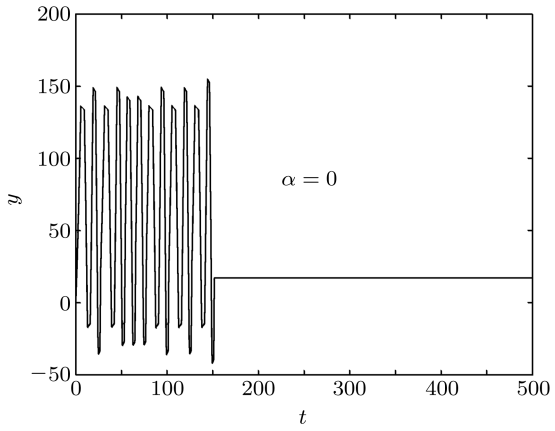


Fig.1. The trajectory in the case of $\alpha = 0$.

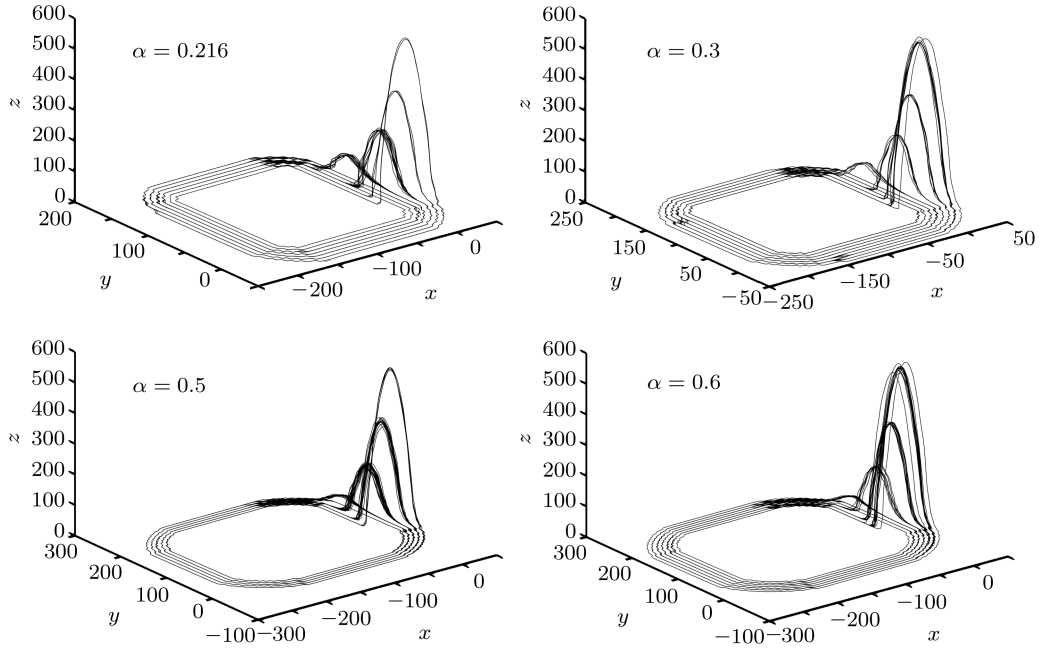


Fig.2. Trajectories with different values of parameter α: 0.216, 0.3, 0.5 and 0.6 respectively.

Furthermore, trajectory z looks like an 'intermittent' function which is almost zero sometimes and jumps up irregularly. In fact, due to the multiple stripes, bursting values of z usually are taken around four different positive levels: near 110, 240, 420, and 600, respectively, as shown in Fig.3. Note that this intermittent phenomenon is different from (noise-induced) on-off intermittency studied in Ref.[19].

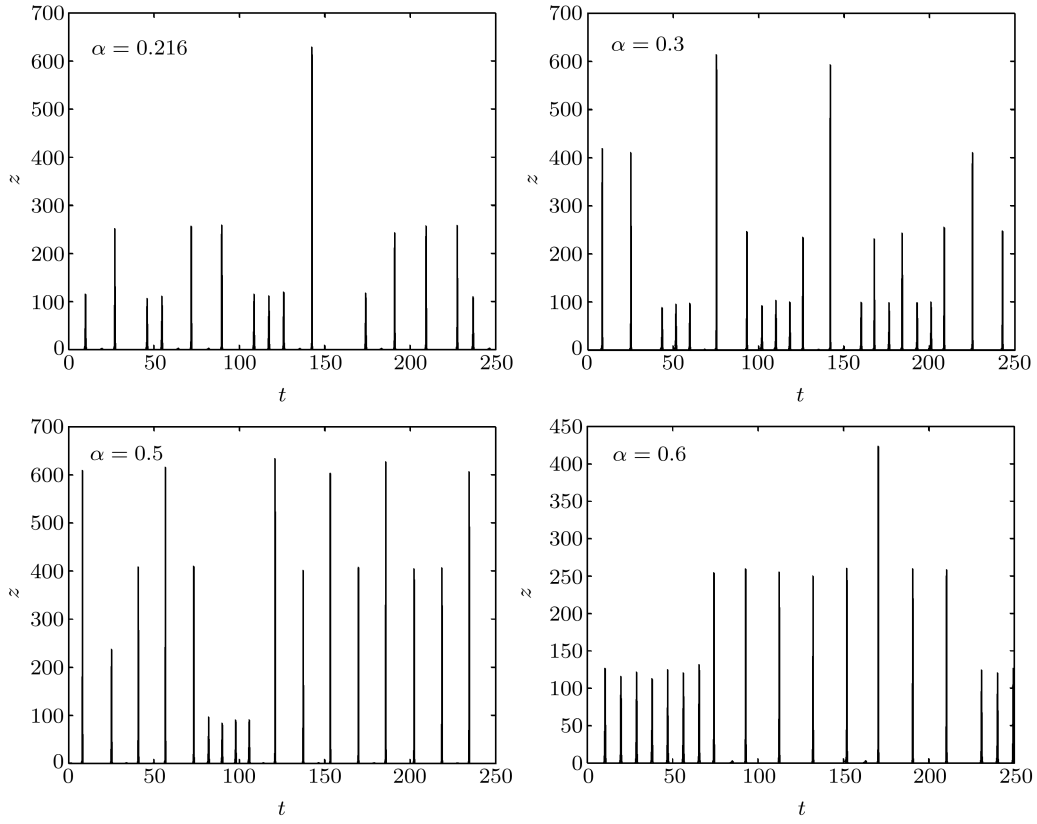


Fig.3. t-z projections of the system orbits with different values of α: 0.216, 0.3, 0.5 and 0.6 respectively.

3. Dynamics analysis

In this section, we will show how the system behaviours change based on the noise level α.

Figure 4 demonstrates that the lengths and the widths of the rectangles increase with the value of α increasing. To be specific, we take the length for example. In a way similar to that given in Ref.[14], we can take it that x is roughly in $[-ay-l_2(\alpha+2), -ay-l_2]$, and further as a is relatively small x may reduce to $[-l_2(\alpha+2), -l_2\alpha]$. In the same way, y may reduce to $[l_1\alpha, (l_1\alpha+\pi)]$. Take $\alpha = 0.216$ and α=0.6 for example. When $\alpha = 0.216$, the value of x for the length parallel to the axis is $[-139, -13]$, and the value of y for the length parallel to the axis is [13, 139]; when α = 0.6, the values of x and y are $[-163, -37]$ and [37, 163], respectively. These results are accordant with the above conclusion.

Figure 5 shows an interesting phenomenon: z will almost always take its peak values when $x \approx c = 20$ (notice that b is very small relatively). Although the peak values of attractors are the same (with the same values of $x \approx c = 20$), y will follow the variation of α.

When the trajectory shifts along the stripes parallel to the y-axis, $x' \approx 0$ for a period of time. Assuming $x = \bar{x}$, with $\bar{x}$ being a constant for the case $x' \approx 0$, system (2) can be written as

$$\begin{aligned} 0 &\approx -y - z + l_1(\alpha N_1 + 1 + \sin(e_1\bar{x})), \\ y' &= \bar{x} + 0.05y + l_2(\alpha N_2 + 1 + \sin(e_2 y)), \\ z' &= z(\bar{x} - 20) + 0.2. \end{aligned} \tag{3}$$

From the first equation in system (3), it is easy to see that as α increases, the value of y increases due to the peak value of z being almost unchanged.

On the other hand, although the length and width of the striped 'attractor' will be longer with the value of α increasing, the distance between the rectangle stripes are not changed with the value of α.

Note that $f(x) = 1 + \sin(x)$ is a function with a period of 2π, which will keep the right-hand side of the first equation unchanged when x moves to $x+2k\pi$ with some positive integer k. Similar discussion can be given to $1+\sin(y)$. As a result, if there exist these parallel stripes, we have the following consequences (D denotes the separation between strips):

1. When $e_1 = e_2$, we have $D_{\text{above}} = D_{\text{left}} = D_{\text{below}} = \dfrac{2\pi}{e}$(with $e = e_1 = e_2$ taken),

2. When $e_1 \neq e_2$, then $D_{\text{left}} = \dfrac{2\pi}{e_1}$, and $D_{\text{above}} = D_{\text{below}} = \dfrac{2\pi}{e_2}$ as shown in Fig.6.

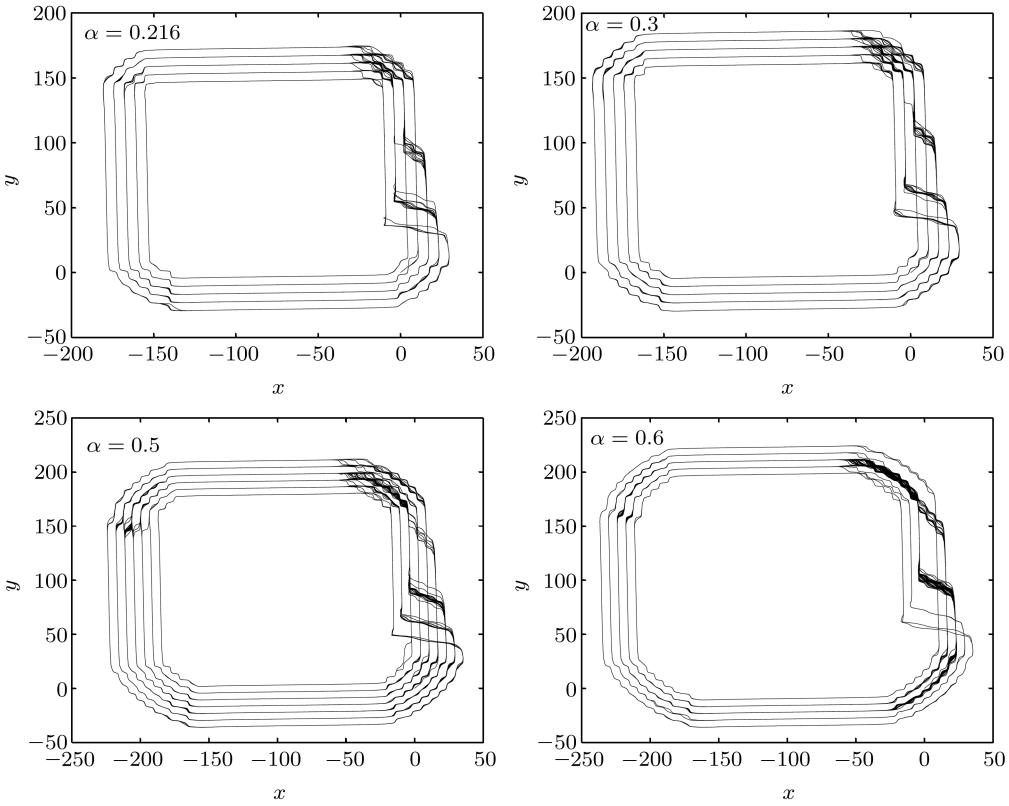


Fig.4. x-y projections with different values of α: 0.216, 0.3, 0.5 and 0.6 respectively.

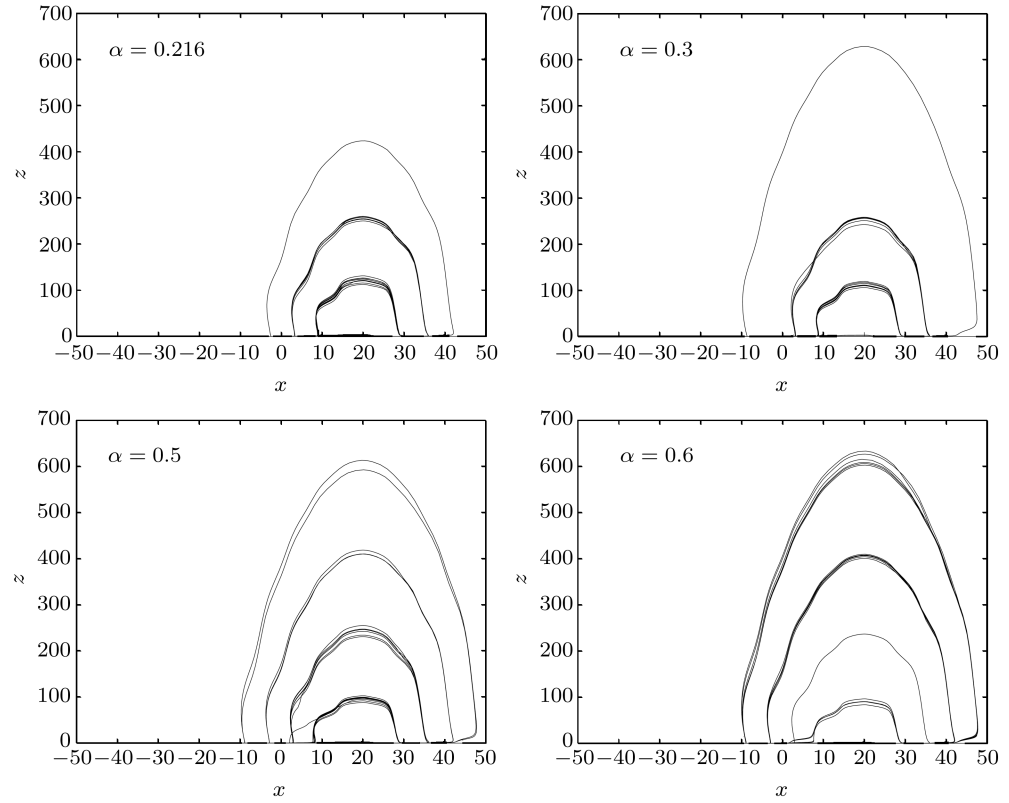


Fig.5. x-z projections of the system orbits with different values of α: 0.216, 0.3, 0.5 and 0.6 respectively.

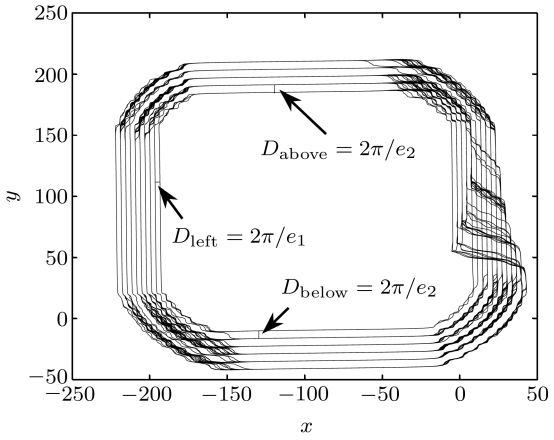


Fig.6. x-y projection with $e_1 = 2$ and $e_2 = 1$.

This may show the reason why f needs selecting as a periodic function.

The number of stripes is usually 7; though for different noise levels of α, the number may change a little (randomly with the initial condition selection). Tables 1–5 show some numerical results, with the values of α taken as 0.216, 0.3, 0.4, 0.5, 0.55 and 0.6.

Table 1. Number of stripes n vs parameter α, with the initial condition taken as $x_0 = 0,\ y_0 = 0$, and $z_0 = 10^{-10}$.

α	0.216	0.3	0.4	0.45	0.5	0.55	0.6
n	6	7	7	7	8	7	7

Table 2. Number of stripes n vs parameter α, with the initial condition taken as $x_0 = 0,\ y_0 = 0$, and $z_0 = 10^{-1}$.

α	0.216	0.3	0.4	0.45	0.5	0.55	0.6
n	6	7	7	7	7	7	8

Table 3. Number of stripes n vs parameter α, with the initial condition taken as $x_0 = 10^{-10},\ y_0 = 10^{-10}$, and $z_0 = 10^{-10}$.

α	0.216	0.3	0.4	0.45	0.5	0.55	0.6
n	6	8	7	7	7	7	7

Table 4. Number of stripes n vs parameter α, with the initial condition taken as $x_0 = 10^{-1},\ y_0 = 10^{-1}$, and $z_0 = 10^{-1}$.

α	0.216	0.3	0.4	0.45	0.5	0.55	0.6
n	7	7	9	8	7	8	7

Table 5. Number of stripes n vs parameter α, with the initial condition taken as $x_0 = 0.5,\ y_0 = 0.5$, and $z_0 = 0.5$.

α	0.216	0.3	0.4	0.45	0.5	0.55	0.6
n	7	7	7	9	7	7	7

4. Conclusions

In this paper, the steady-state trajectories in striped rectangular shape have been generated by Rossler systems with an additive noise. The shape of generated system orbits can be adjusted by changing the value of parameter α. Moreover, some discussion on the parameter selection has been made through the numerical analysis. The noise-induced phenomena show the capability of noise in generating complex phenomena. However, nonlinear systems with noises are hard to analyse in a strict way, more attention should be paid to the nonlinear dynamics in the future.

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