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Continuous feedback control of KdV Burgers system*

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The chaos in the KdV Burgers equation describing a ferroelectric system has been successfully controlled by using a continuous feedback control. This system has two stationary points. In order to know whether the chaos is controlled or not, the instability of control equation has been analysed numerically. The numerical analysis shows that the chaos can be converted to one point by using one control signal, however, it can converted to the other point by using three control signals. The chaotic motion is converted to two desired stationary points and periodic orbits in numerical experiment separately.

Keywords: KdV Burgers equation, chaos attractor, stationary points, periodic orbit
PACC: 0545

1. Introduction

Dynamic chaos is a very interesting nonlinear effect which widely exists in physical systems. Chaotic systems are characterized by their extreme sensitivity to small perturbations in their initial conditions, which reduces the performance of systems. Therefore, the elimination of the chaos is very important. Furthermore, the key observation is that a chaotic attractor typically has embedded at some unstable fixed points or in an infinite number of unstable periodic orbits. In resent years, there has been aroused the tremendous interest in studying the control of chaos in nonlinear dynamical systems.[1−10] The control of chaos aims at suppressing the chaoticity of a system, e.g. controlling the unstable periodic orbits of the chaotic trajectory, and stabilizing the unstable fixed points into a steady state. Since Ott, Grebogi, and Yorke[10] suggested an efficient method to make small time-dependent perturbations in an accessible system parameter and thereby achieve a desired periodic output, many methods of controlling chaos have been presented. Grantham and Athalye[11,12] have presented a simple and effective method to suppress chaotic motions in the Duffing oscillator by adding a feedback control device consisting of a linear notch filter. Besides, they have presented a harmonic balance method and a numerical optimal parameter selection scheme to choose the notch filter parameters for achieving the desired k-period motion. The stabilization of unstable periodic orbits of a chaotic system has been achieved by a continuous feedback control.[13] In addition, the time-delay feedback control,[14] adaptive control,[15−17] and linear feedback control[18] have successfully eliminated the chaos of systems.

In plasmas and other systems, the KdV Burgers equation describes a large number of systems which contain dispersion and dissipation. Besides, the chaos appearing in plasmas has been extensively demonstrated and is very common. Zayko and Negedov[19] have described a new class of solutions of the KdV Burgers equation which demonstrates chaotic behaviours. In this paper, the chaos in the KdV Burgers equation describing a ferroelectric system has been successfully controlled by using a continuous feedback control. This system has two stationary points. In order to know whether the chaos is controlled or not, the instability of control equation has been analysed numerically. The numerical analysis shows that the chaos can be converted to one point by using one control signal, however, it can be converted to the other point by using three control signals. The chaotic motion is converted to two desired stationary points and some unstable periodic orbits by using the continuous feedback control in numerical experiment. The numerical analyse is consistent with the numerical experiment.

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2. Model

The KdV Burgers equation that arises from the theory of a ferroelectric system, for which the dynamical methods have been well developed,[19,20] is expressed as

$$P_\tau^{(1)} + AP^{(1)}P_\xi^{(1)} + DP_{\xi\xi}^{(1)} + BP_{\xi\xi\xi}^{(1)} = 0, \quad (1)$$

where $A = \dfrac{2\alpha}{K'(u)}$, $D = \dfrac{u}{K'(u)}$, $B = -\dfrac{u^2}{K'(u)}$, $K'(u) = \dfrac{\mathrm{d}K(u)}{\mathrm{d}u}$, and $K(u) = \dfrac{\omega_P^2 u^2}{c^2 - u^2} - \omega_0^2 - 2\alpha P_0$. $P^{(1)}$ is the first term of the series expansion of the polarization P with respect to a small attenuation coefficient σ; $K(u) = 0$ is the dispersive equation for wave speed u in the long-wave limit; $P_0 \simeq -\omega_0^2/\alpha$ is the equilibrium value of P; ω_p, and ω_0 are the characteristic frequencies corresponding to the optical and the acoustic branches of the spectrum. We consider the behaviour of this system beyond the point of phase transition ($T > T_\mathrm{c}$, T_c is the Curie temperature). Close to the Curie point, $u \ll c$ and $K(u) \simeq \omega_0^2 + \omega_p^2 u^2/c^2$. The equation, $K(u) = 0$, has pure imaginary solutions $u = \pm \mathrm{i}c\omega_0/\omega_p = \pm \mathrm{i}u_i$. The quantity $K'(u)$ becomes imaginary too i.e. $K'(u) = \pm 2\mathrm{i}\omega_0\omega_p/c$. Therefore, the coordinate $\xi = \sigma(z - ut)$ contains an imaginary contribution when $T > T_\mathrm{c}$. To eliminate this imaginary part, the new coordinates $\tau' = \tau$ and $\xi' = \xi + \mathrm{i}\sigma u_i t = \xi + \mathrm{i}u_i\tau/\sigma^2$ are introduced. Now Eq.(1) takes the form $P_\eta + \mathrm{i}PP_\eta - RP_{\eta\eta\eta} - \mathrm{i}\mu P_{\eta\eta} = 0$, where $\eta = (\xi' - V\tau)/c$, $P = -\dfrac{\sigma^2\omega_p}{c\omega_0}\left[V + \mathrm{i}\dfrac{c\alpha}{\omega_p\omega_0}P^{(1)}\right]$, $R = \dfrac{\sigma^2}{2\omega_p^2}$, and $\mu = \dfrac{\sigma^2}{2\omega_0\omega_p}$. By integrating it we can obtain $RP_{\eta\eta} + \mathrm{i}\mu P_\eta + \mathrm{i}\dfrac{P^2}{2} - P = C$, where $C = C_1 + \mathrm{i}C_2$ is a constant. Substituting $P = u + \mathrm{i}w$ into the equation yields a set of two second-order ordinal differential equations as follows:

$$\begin{aligned} &Ru_{\eta\eta} - u - uw - \mu w_\eta = C_1, \\ &Rw_{\eta\eta} - w - \frac{w^2}{2} + \frac{u^2}{2} + \mu u_\eta = C_2. \end{aligned} \quad (2)$$

Equation (2) can be written as a system of four first-order equations (for $C_1 = C_2 = 0$):

$$\begin{aligned} u_\eta &= u', \\ w_\eta &= w', \\ u'_\eta &= \frac{u}{R} + \frac{uw}{R} + \frac{\mu}{R}w', \\ w'_\eta &= \frac{w}{R} + \frac{w^2}{2R} - \frac{u^2}{2R} - \frac{\mu}{R}u', \end{aligned} \quad (3)$$

where μ and R are two important control parameters.

3. The stability of fixed points

Equation (3) has two stationary points $(u_1, w_1, u_1', w_1') = (0,0,0,0)$ and $(u_2, w_2, u_2', w_2') = (0,-2,0,0)$. For the linearized equation (3) near the first fixed point $(u_1, w_1, u_1', w_1') = (0,0,0,0)$, eigenvalues of the stationary linearization matrix are $\pm\dfrac{\sqrt{4R - \mu^2}}{2R} \pm \mathrm{i}\dfrac{\mu}{2R}$. In addition, the eigenvalues of stationary linearization matrix of the second fixed point $(u_2, w_2, u_2', w_2') = (0,-2,0,0)$ are $\pm\dfrac{\sqrt{-4R - \mu^2}}{2R} \pm \mathrm{i}\dfrac{\mu}{2R}$. It is easy to know that the first one (u_1, w_1, u_1', w_1') is a saddle point or a centre depending on $\mu^2 < 4R$ or $\mu^2 > 4R$, while the second one (u_2, w_2, u_2', w_2') is a centre in any term. According to the investigation of Zayko and Negedov[19] we know that the KdV Burgers equation describing the ferroelectric system has a chaos attractor when $R = 0.5$, and $\mu = 0.9$. Therefore, We convert the chaotic motion of KdV equation to two desired stationary points and periodic orbits by a continuous feedback control separately.

4. Controlling chaos by a continuous feedback control

4.1. Controlling chaos to a fixed point $(0,0,-2,0)$

In theory, by choosing a relevance scalar variable the chaotic motion can be controlled using a feedback. Usually, for chaos control a few negative direct-feedbacks are adopted in experiment. In order to convert chaos to a fixed point $(0,-2,0,0)$, we apply $-g_1(u-0)$, $-g_2(w-0)$, $-g_3(u'-2)$, and $-g_4(w'-0)$ respectively to the first, the second, the third, and the fourth equation of Eq.(3). The feedback is applied to the first equation of Eq.(3) as follows:

$$\begin{aligned} u_\eta &= u' - g_1 u, \\ w_\eta &= w', \\ u'_\eta &= \frac{u}{R} + \frac{uw}{R} + \frac{\mu}{R}w', \\ w'_\eta &= \frac{w}{R} + \frac{w^2}{2R} - \frac{u^2}{2R} - \frac{\mu}{R}u'. \end{aligned} \quad (4)$$

The linearized equation can be obtained by linearizing Eq.(4) near the fixed point $(0,-2,0,0)$ and this linearized equation is $\lambda^4 + g_1\lambda^3 + (4 + 4\mu^2)\lambda^2 + (2g_1 +$

$4g_1\mu^2)\lambda + 4 = 0$. Because this equation is difficult to solve analytically, we give numerical solutions for $R = 0.5$ and $\mu = 0.9$. Figure 1 shows how the latent roots vary with g_1. We know from Fig.1 that the latent roots are all less than zero when a feedback is applied to the first equation of Eq.(3), i.e. chaotic motion of the KdV Burgers system is successfully controlled to this fixed point. When the feedback is applied to the second, the third, or the fourth equation of Eq.(3) respectively, the same results are obtained as presented in Fig.1. Therefore, the chaos can be converted to the fixed point $(0, -2, 0, 0)$ as long as the control signal is applied to any equation of Eq.(3).

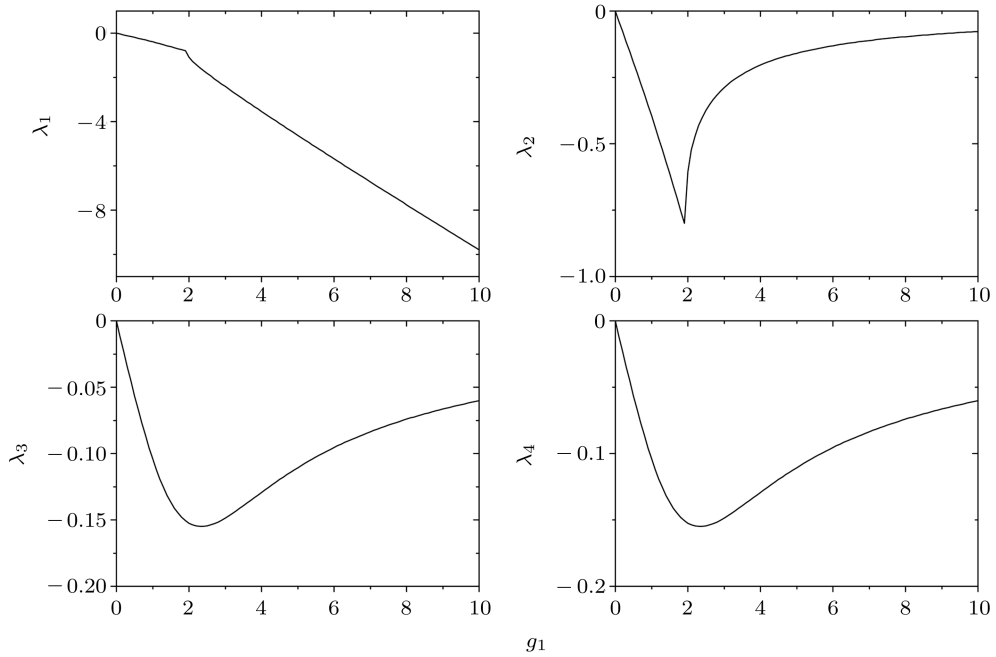


Fig.1. Eigenvalues of the Jabio matrix when applying a feedback to the first equation of Eq.(3) in order to control chaos to a fixed point (0, 0, −2, 0).

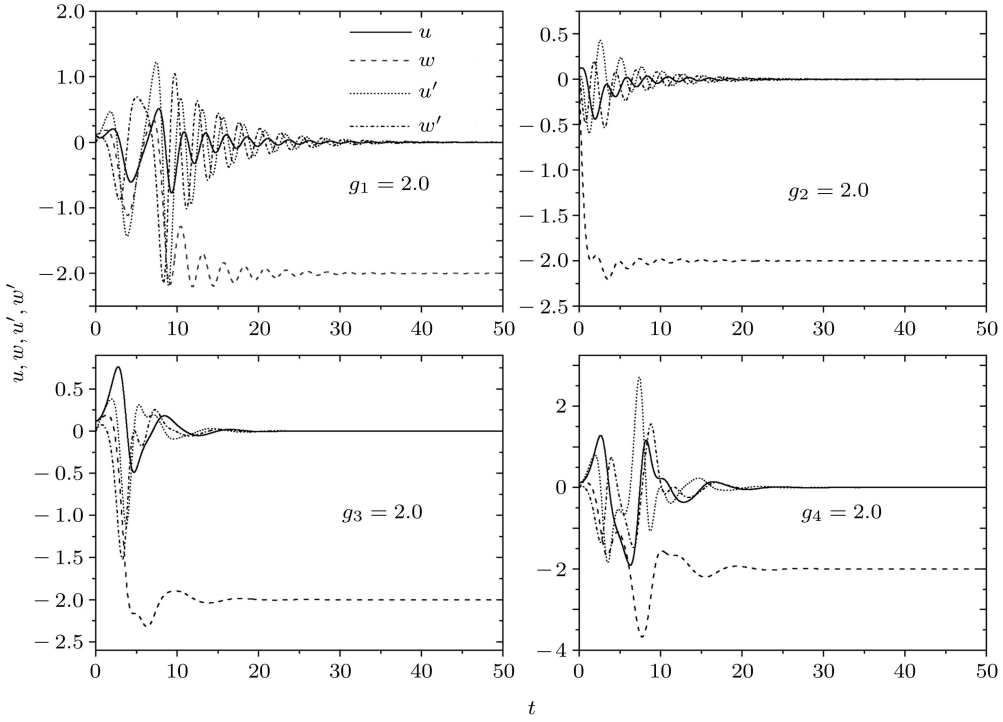


Fig.2. Controlling chaos to a fixed point (0, 0, −2, 0).

It is clear from Fig.2 that the chaotic attractor of the KdV Burgers system is successfully controlled to the fixed point $(0, -2, 0, 0)$ in numerical experiments. It is also seen that when the feedback is applied to the third equation the controlling effect is best, while the feedback is applied to the first equation the controlling effect is worst.

4.2. Controlling chaos to a fixed point $(0, 0, 0, 0)$

Using the same method, we investigate whether the chaos is controlled to a fixed point $(0, 0, 0, 0)$ or not. In order to convert chaos to a fixed point $(0, 0, 0, 0)$, we apply $-g_1(u-0)$, $-g_2(w-0)$, $-g_3(u'-0)$, and $-g_4(w'-0)$ respectively to the first, the second, the third, and the fourth equation of Eq.(3). When a feedback is applied to the first equation of Eq.(3), the linearized equation obtained by linearizing Eq.(4) near the fixed point $(0, 0, 0, 0)$ is $\lambda^4 + g_1\lambda^3 + (-4 + 4\mu^2)\lambda^2 + (-2g_1 + 4g_1\mu^2)\lambda + 4 = 0$. Figure 3 numerically illustrates how the latent roots of this linearized equation vary with g_1. It can be seen from Fig.3 that not all latent roots are less than zero, that is, the chaos cannot be converted to the fixed point $(0, 0, 0, 0)$ when a feedback is applied to the first equation of Eq.(3). When the feedback is applied to the second, the third, or the fourth equations of Eq.(3) respectively, the same results are obtained as presented in Fig.3. In other words, the chaotic attractor of the KdV Burgers equation cannot be controlled to the fixed point $(0, 0, 0, 0)$ by using a control signal.

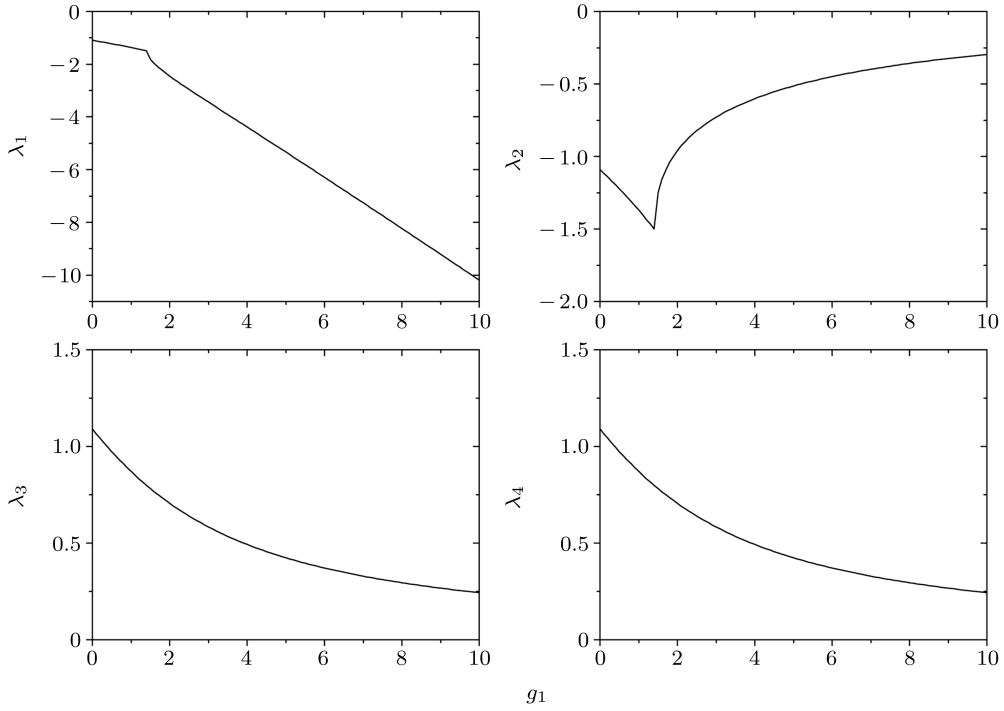


Fig.3. Eigenvalues of the Jabio matrix when applying a feedback to the first equation of Eq.(3) in order to control chaos to a fixed point (0,0,0,0).

Two feedbacks are applied to the first and the second equations of Eq.(3) as follows:

$$\begin{aligned} u_\eta &= u' - g_1 u, \\ w_\eta &= w' - g_2 w, \\ u'_\eta &= \frac{u}{R} + \frac{uw}{R} + \frac{\mu}{R} w', \\ w'_\eta &= \frac{w}{R} + \frac{w^2}{2R} - \frac{u^2}{2R} - \frac{\mu}{R} u'. \end{aligned} \tag{5}$$

The linearized equation obtained by linearizing Eq.(5) near the fixed point $(0, 0, 0, 0)$ is $\lambda^4 + (g_1 + g_2)\lambda^3 + (-4 + g_1 g_2 + 4\mu^2)\lambda^2 + (-2g_1 - 2g_2 + 4g_1\mu^2 + 4g_2\mu^2)\lambda + 4 = 0$. Figure 4 indicates that not all latent roots of the linearized equation are less than zero, that is, the chaos of the KdV Burgers equation cannot be controlled to the fixed point $(0, 0, 0, 0)$ by applying two feedbacks to first two equations of Eq.(3).

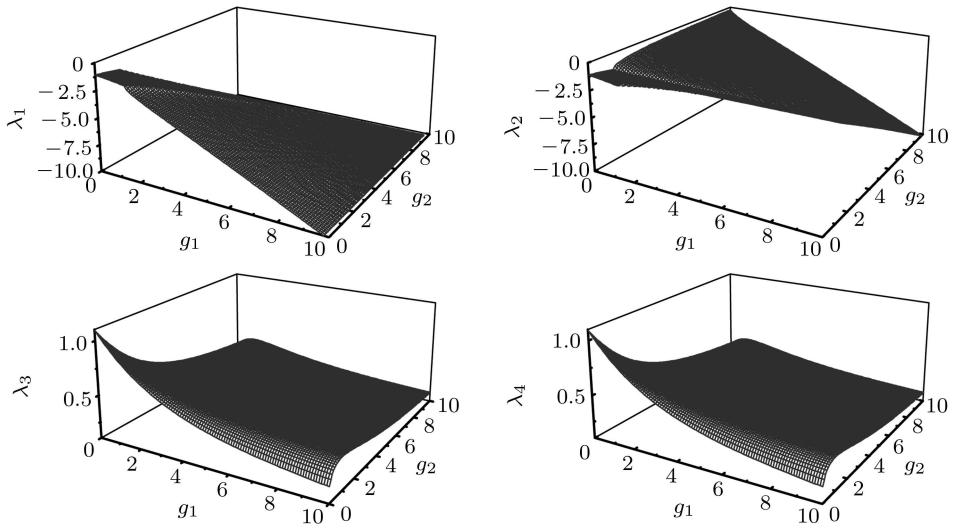


Fig.4. Eigenvalues of the Jabio matrix when applying feedbacks to the first and the second equations of Eq.(3) in order to control chaos to a fixed point (0,0,0,0).

In the same way, we know that the same results as presented in Fig.4 are gained by applying two feedbacks to two arbitrarily equations of (3). Therefore, using one or two control signals the chaotic motion of KdV Burgers system cannot be controlled to the fixed point $(0, 0, 0, 0)$. When applying three feedbacks to the first, the third and the fourth equations or the second, the third and the fourth equations of the Eq.(3), we obtain two linearized equations, respectively, $\lambda^4 + (5 + g_3 + g_4)\lambda^3 + (-4 + 5g_3 + 5g_4 + g_3g_4 + 4\mu^2)\lambda^2 + (-10 - 2g_3 - 2g_4 + 5g_3g_4 + 4\mu^2)\lambda + 4 - 10g_3 = 0$ for $g_1 = 5$ and $\lambda^4 + (5 + g_3 + g_4)\lambda^3 + (-4 + 5g_3 + 5g_4 + g_3g_4 + 4\mu^2)\lambda^2 + (-10 - 2g_3 - 2g_4 + 5g_3g_4 + 4\mu^2)\lambda + 4 - 10g_4 = 0$ for $g_2 = 5$. It is found by numerically solving the above two equations that the first and the third talent roots are less than zero but the second and the fourth talent roots are not always less than zero. In order to conveniently analyse the second and the fourth latent roots, Fig.5 indicates how the second and the fourth latent roots are distributed in (g_3, g_4) plane for $g_1 = 5$ and $g_2 = 5$, respectively.

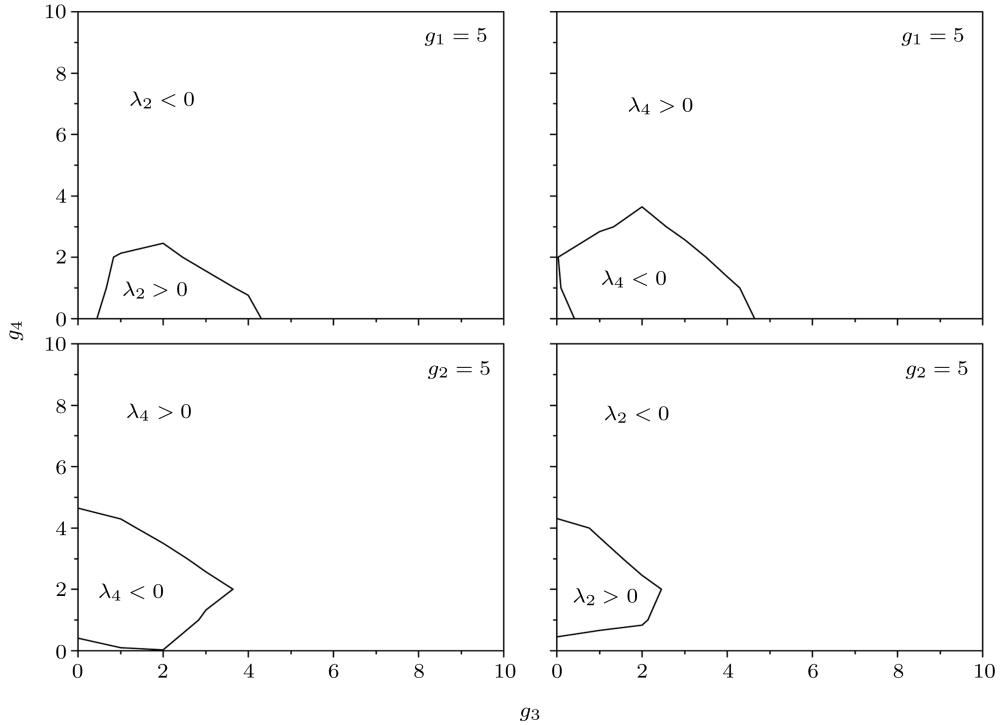


Fig.5. The distributions of eigenvalues in (g_3, g_4) plane for $g_1 = 5$ and $g_2 = 5$ in order to control chaos to a fixed point (0,0,0,0).

It is also clear that when the second latent root is less than zero, the fourth latent root is larger than zero, and when the second latent root is larger than zero, the fourth latent root is less than zero. Namely, the second and the fourth latent roots are not less than zero at the same time. Therefore, in the above two instances the chaos cannot be converted to the fixed point $(0,0,0,0)$. When applying three feedbacks to the first, the second and the third equations or the first, the second and the fourth equations of Eq.(3) respectively, we obtain two linearized equations, respectively, $\lambda^4+(5+g_2+g_3)\lambda^3+(-4+5g_2+5g_3+g_2g_3+4\mu^2)\lambda^2+(-10-2g_2-2g_3+5g_2g_3+20\mu^2+4g_2\mu^2)\lambda+4-10g_3+20g_2\mu^2=0$ for $g_1=5$ and $\lambda^4+(5+g_2+g_4)\lambda^3+(-4+5g_2+5g_4+g_2g_4+4\mu^2)\lambda^2+(-10-2g_2-2g_4+5g_2g_4+20\mu^2+4g_2\mu^2)\lambda+4-2g_2g_4+20g_2\mu^2=0$ for $g_1=5$. Their numerical results show obtain that the first, the second, and the third latent roots are less than zero but the fourth latent root is not always less than zero. In Fig.6, we can see the distribution of the fourth latent root in (g_2,g_3) plane and (g_2,g_4) plane for $(g_1=5)$ respectively, and the results of controlling. It is also seen from Fig.6 that the fourth latent root may be less than zero. Therefore, setting random values to (g_2,g_3) and (g_2,g_4) in region where the fourth latent root is less than zero, the chaotic motion can be converted to the fixed point $(0,0,0,0)$. Furthermore, in Fig.6 the chaos of the KdV Burgers system is successfully controlled to the fixed point $(0,0,0,0)$ in numerical experiment.

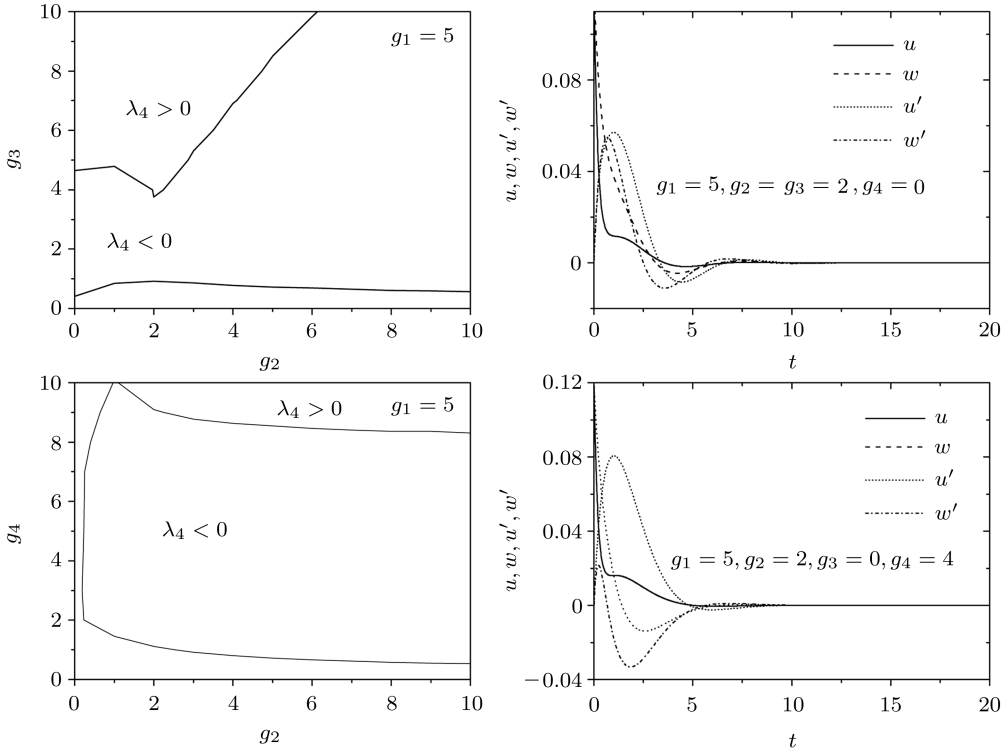


Fig.6. Left: The distributions of eigenvalues in (g_2,g_3) and (g_2,g_4) planes right: controlling chaos to a fixed point (0,0,0,0).

4.3. Controlling chaos to some periodic orbits

The key observation is that a chaotic attractor typically can be embedded not only to many fixed points but also within an infinite number of unstable periodic orbits. We eliminate the chaos in KdV Burgers system by controlling chaos to some unstable periodic orbits using the continuous feedback control. The control is achieved by stabilizing already existing unstable periodic orbits rather than creating some new periodic orbits without knowing the dynamics of the system for other parametric choices. If appropriate control signals are chosen, the chaos can be converted to some existing unstable periodic orbits. In order to convert chaos to unstable periodic orbits, we apply $-g_1u$, $-g_2w$, $-g_3u'$, and $-g_4w'$ to the first, the second, the third, and the fourth equations of Eq.(3). According to the numerical experiment, when the strong control signal is applied to the second equation of

Eq.(3) the chaos is converted into an unstable periodic orbit. Furthermore, when applying three feedbacks to the second, the third and the fourth equations of Eq.(3), the chaotic motion is eliminated, and with the intensity of the control signals varying, the unstable periodic orbits turn different. It is clear from Fig.7 that the chaotic motion is successfully converted into some unstable periodic orbits using the continuous feedback control in numerical experiment. The left sub-diagrams of Fig.7 indicate the phase portraits of trajectory in the space of variables u, w, and the right sub-diagrams of Fig.7 show the time series corresponding to the left sub-diagrams. Moreover, Fig.7 shows when the control signals are different, the chaos is converted into the unstable periodic orbits with different periods. It is well known that these periodic orbits are unstable by adding external control parameters. Therefore, these unstable periodic orbits cannot be realized in this KdV Burgers system.

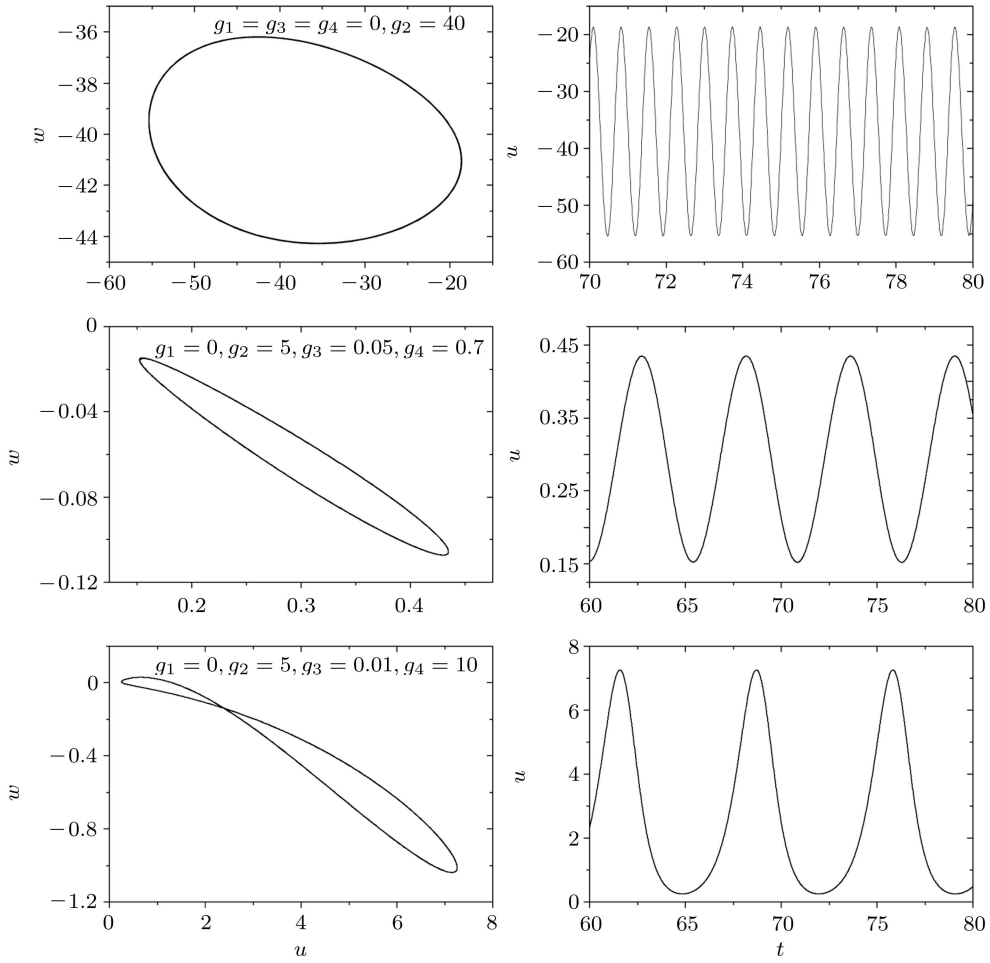


Fig.7. Results of controlling chaos to unstable periodic orbits.

5. Conclusion

In summary, we successfully suppress the chaotic motion in the KdV Burgers system by converting chaos to two desired fixed points ($(u_1, w_1, u_1', w_1') = (0, 0, 0, 0)$ and $(u_2, w_2, u_2', w_2') = (0, -2, 0, 0)$) and some unstable period orbits using continuous feedback control in numerical experiment. The KdV Burgers equation describes a large number of systems which contain dispersion and dissipation in plasmas and other systems. Besides, the chaos appearing in plasmas has been extensively demonstrated and is very common. Therefore, it is important for the further investigation of chaos control in a plasma.

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