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Controlling a time-delay system using multiple delay feedback control*

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In this paper multiple delay feedback control (MDFC) with different and independent delay times is shown to be an efficient method for stabilizing fixed points in finite-dimensional dynamical systems. Whether MDFC can be applied to infinite-dimensional systems has been an open question. In this paper we find that for infinite-dimensional systems modelled by delay differential equations, MDFC works well for stabilizing (unstable) steady states in long-, moderate- and short-time delay regions, in particular for the hyperchaotic case.

Keywords: MDFC, infinite-dimensional systems, hyperchaos
PACC: 0545

1. Introduction

In recent years, much attention has been focused on stabilizing unstable periodic orbits of a nonlinear dynamical system in terms of chaos control.[1−12] Among the most prominent approaches is the Ott–Grebogi–Yorke (OGY) method.[13] This method requires quite a lot of knowledge about the orbit, including its position and stability properties, that is not always straightforward to find in a real-life situation. A famous approach that is lacking the above shortcoming but achieves the same goal was proposed by Pyragas, also known as time delay autosynchronization (TDAS).[14,15] Then as the generalization of TDAS, extended time delay autosynchronization (ETDAS)[16,17] and N time delay autosynchronization (NTDAS)[18] were also introduced. For many dynamical systems it showed that Pyragas's TDAS method (and its extensions) is very successful for stabilizing unstable periodic orbits (UPOs) but less efficient to control unstable steady states. So Alexander Ahlborn suggested multiple delay feedback control (MDFC)[19] that is based on two or more different delay times which enter independent control terms. In contrast to (E) TDAS or NTDAS, the delay times are not integer multiples of each other. Therefore, the full control term only vanishes in the case of a steady state. By numerical simulations for the Rössler system, Lorentz system, Chua's circuit, Colpitts oscillator and even Baier-sahle,[20] the MDFC method has shown that it is superior for fixed point stabilization in terms of stability and flexibility, in particular for large delay times, and the efficiency of MDFC increases when including more independent delay times in the feedback loop.

So far, the research for MDFC is limited in finite-dimensional dynamical systems. Whether MDFC can apply to infinite-dimensional systems has been an open question. In this paper, we study the dynamics of a time-delay system with MDFC and verify the efficiency of the method in infinite-dimensional dynamical systems.

This paper is organized as follows. In Section 2 a simple model of a first-order delayed feedback system used in this paper is introduced. In Section 3 we discuss the dynamics of a time-delay system with MDFC in long-, moderate- and short-time delay regions, respectively. Finally we end this paper by a short conclusion in Section 4.

2. Model

Delay systems can be met in many research fields including physical, engineering, biological, economical systems. Problems of linear delay systems have been investigated for many years; in the recent twenty years attention has been drawn to the research of chaos in nonlinear dynamical systems.[21] In this paper, we consider a type of delayed feedback systems,[22] which can

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be modelled by a first-order delay differential equation

$$\tau'\dot{x} = -x(t) + f(x(t - t_{\mathrm{R}}), \mu). \tag{1}$$

As a result of the existence of the delay time t_{R}, the dynamical system is an infinite-dimensional system.[23,24] $x(t)$ is the dimensionless output of the system at time t, τ' is the response time of the nonlinear medium and μ is the bifurcation parameter. We study Eq.(1) with the special feedback function

$$f(x, \mu) = 1 - \mu x^2. \tag{2}$$

The special feedback function can be considered as the first nonlinear term of the Taylor expansion of a general nonlinear function $f(x, \mu)$ in the vicinity of a steady state, It should keep the general nonlinear properties of the delay feedback system, as pointed out by Li and Hao.[25] Then we use the above model as a chaotic system to which we apply MDFC. The equation is followed:

$$\tau'\dot{x} = -x(t) + f(x(t - t_{\mathrm{R}}), \mu) + u(t), \tag{3}$$

with a MDFC term

$$u(t) = k_1[x(t - \tau_1) - x(t)] + k_2[x(t - \tau_2) - x(t)]. \tag{4}$$

In Eq.(4) k_i are used to vary the strengths and influence (gain) of the feedback terms. Measuring the time in units of t_{R}, we find that Eq.(4) becomes

$$\tau\dot{x}(t) = -x(t) + f(x(t - 1), \mu) + u', \tag{5}$$

where $\tau = \tau'/t_{\mathrm{R}}$, which characterizes the effect of the delay when τ' is fixed. $u(t) = u'/t_{\mathrm{R}}$. On the other hand, in the limiting case of $\tau = 0$, Eq.(5) is reduced to

$$x(t) = f(x(t - 1), \mu). \tag{6}$$

Equation (6) is just the well-known discrete Logistic map model. As we know, it plays a crucial role in understanding the common behaviour of low-dimensional dissipative nonlinear systems and furthermore feedback system with delay is a system with infinite dimension, so we expect that Eq.(5) with the feedback function Eq.(2) may provide us with a fundamental scenario for the dynamics in infinite-dimensional systems with MDFC.

3. Numerical results

In this paper, we study the dynamics of the Eq.(5) in wide range of delay. The parameter regions are classified as three different regions (long-, moderate- and short-time delay regions) according to the delay time.

3.1. Hyperchaos in the long-delay time region

For Eq.(5) in the long-time delay region (i.e. τ is very small), we investigated the chosen parameters $\mu = 1.76803$, $\tau = 0.05$ for numerical studies in controlling system. For these parameter values the unperturbed system has multiple positive Lyapunov exponents, as shown in Figs.1 and 2. We draw the evolution curve of the variable $x(t)$ via the time t and the Lyapunov exponents spectra where μ is varied from 1.0 to 2.0 at $\tau = 0.05$. If the control term u is activated to the system at $t = 515$, in Figs.3 and 4 we can clearly see that the value of $x(t)$ is quickly stabilized at the fixed point $x = 0.52068$. Here $k_1 = k_2 = 0.7$. Figure 5 demonstrates the corresponding control perturbations u versus the control time t. When the system is locked to the (original unstable) state, the perturbations become very small.

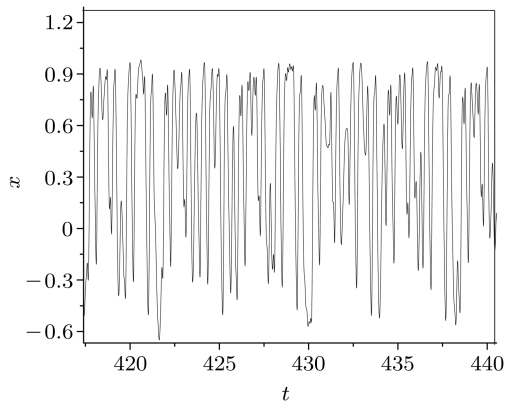


Fig.1. Time series of x variable at $\tau = 0.05$ where $\mu = 1.76803$.

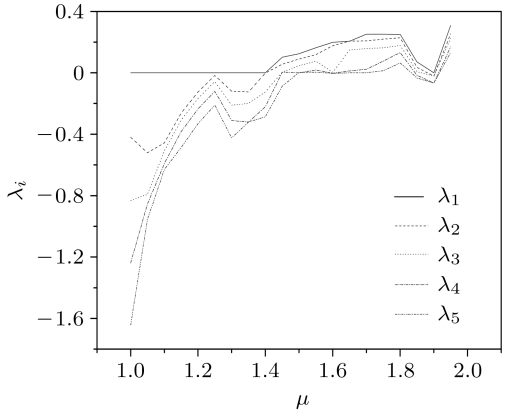


Fig.2. Lyapunov exponents spectra where $\mu \in [1.0, 2.0]$ at $\tau = 0.05$

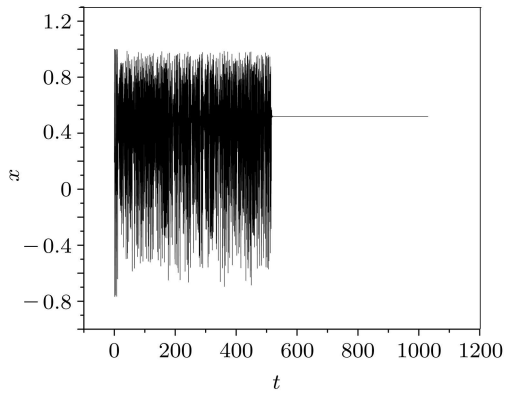


Fig.3. x variable with control being activated at $t = 515$.

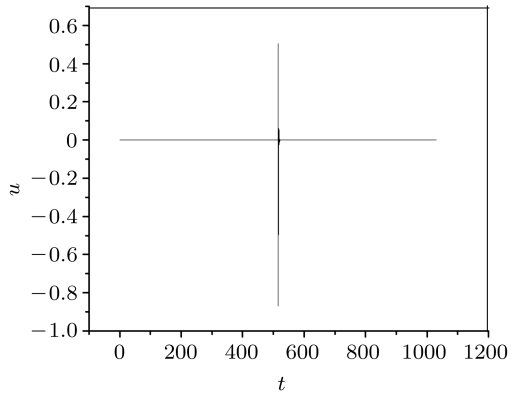


Fig.4. Feedback signal u of the system with control being activated at $t = 515$.

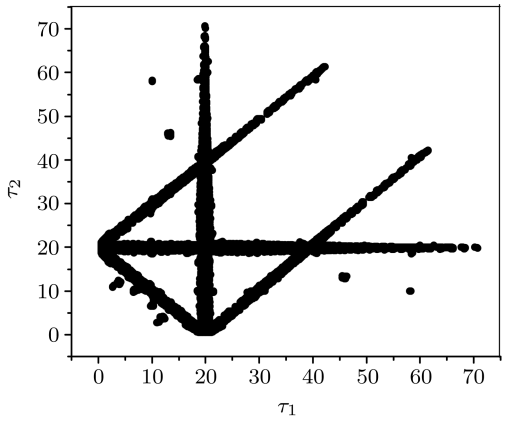


Fig.5. Stability diagram of the controlled system (Eq.(5)).

3.2. Chaos in the moderate-delay time case

At $\tau = 0.8$ and $\mu = 4.65$, the system is chaotic without feedback, see Figs.6 and 7. According to Eq.(5), we add the control to $x(t)$ with $k_1 = k_2 = 0.7$ at $t = 515$. Figures 8 and 9 depicts $x(t)$ and u versus t for the MDFC trial of Eq.(5). Here k_1, k_2 are set to 0.7. From the figure we can find that it also successfully stabilizes the chaos motion to the fixed point.

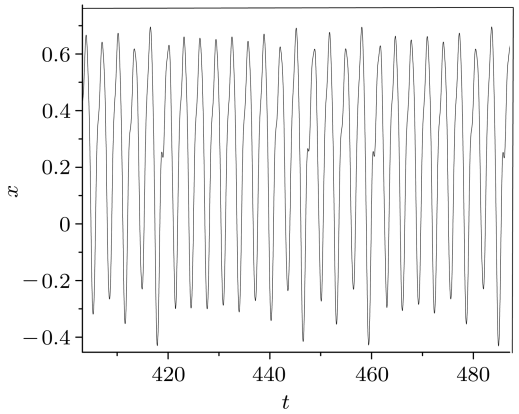


Fig.6. The evolution curves of x at $\tau = 0.8$ where $\mu = 4.65$.

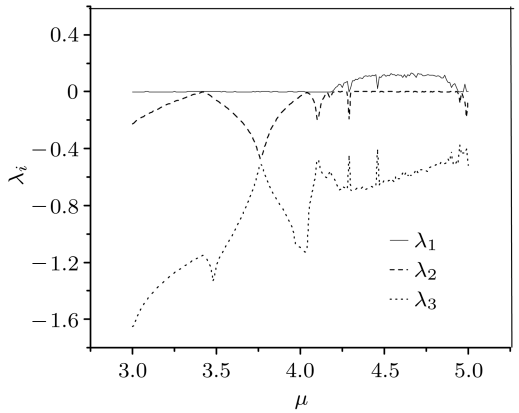


Fig.7. Lyapunov exponents of Eq.(5) versus μ at $\tau = 0.8$ where $\mu \in [3.0, 5.0]$.

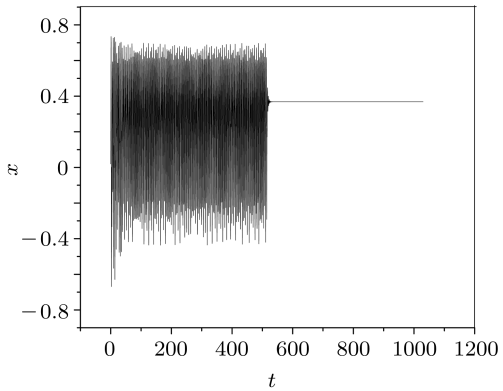


Fig.8. $x(t)$ of the system with control being activated at $t = 515$.

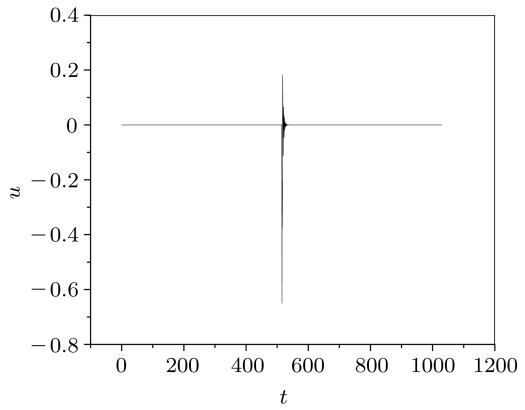


Fig.9. $u(t)$ of the system with control being activated at $t = 515$.

3.3. Chaos in the short-delay time region

In the case of short-time delay, Eq.(5) will approach a normal one-dimensional ordinary differential equation. When $\mu = 47.31$ and $\tau = 1/0.22$, chaotic motion occurs for the unperturbed system ($k_1 = k_2 = 0.0$), see Figs.10 and 11. The same control law with the same control coefficients (i.e. $u = k_1(x(t-\tau_1) - x(t)) + k_2(x(t-\tau_2) - x(t))$ when $k_1 = k_2 = 2.0$) is applied to stabilize the unstable state. Figures 12 and 13 demonstrate the numerical results which successfully stabilize the unstable states at the parameters. One can see that the stabilizing procedure works well.

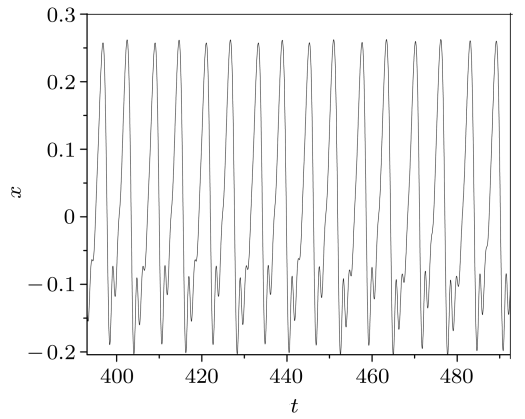


Fig.10. The evolution curves of x for t at $\tau = 1/0.22$ where $\mu = 47.31$.

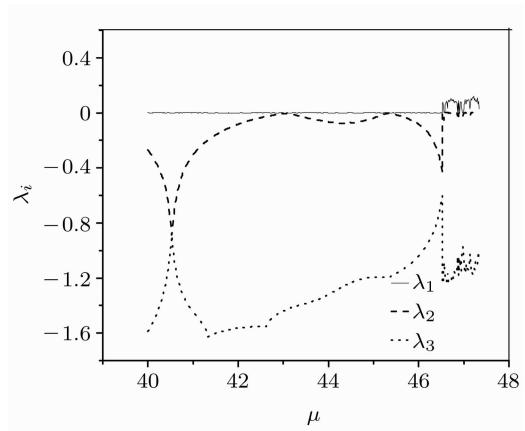


Fig.11. Lyapunov exponents of Eq.(5) versus μ at $\tau = 1/0.22$ where $\mu \in [40.0, 48.0]$.

To study the question further, in Fig.5, we take the parameter $\mu = 1.76803$, $\tau = 0.05$ as example and plot stability diagram of the steady states controlled with two delay times in the parameter plane (τ_1, τ_2) where for fixed values of the coupling strengths $k_1 = 0.7 = k_2$. Regions where a successful stabilization of the system's fixed points is possible are marked in black. Alexander Ahlborn and Ulrich Parlitz suggested that applications to high dimensional systems need more than two delay times, but our results show that without more than two delay times, MDFC works well in the infinite-dimensional system.

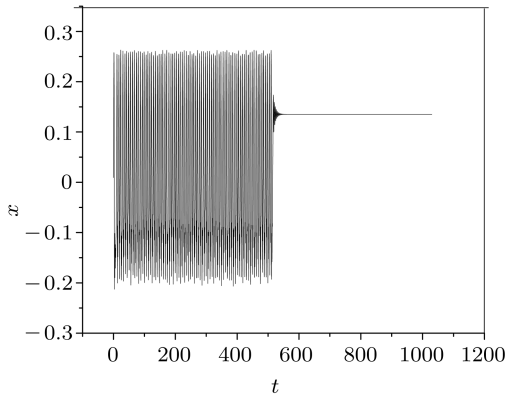


Fig.12. x variable with control being activated at $t = 515$.

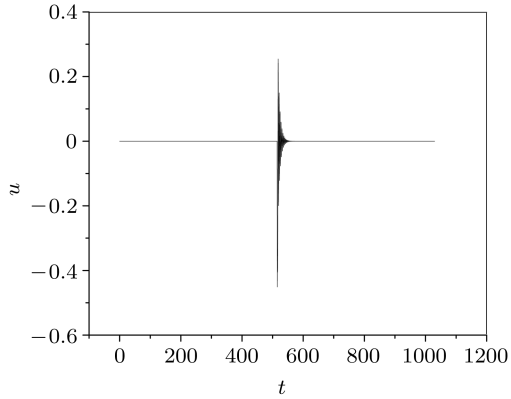


Fig.13. Feedback signal u of the system with control being activated at $t = 515$.

4. Conclusion

In conclusion, numerical simulation for a first-order time-delay system in long-delay time, moderate-

delay time, short-delay time regions indicates that MDFC is efficient for stabilizing fixed points in the system, in particular for the unstable states embedded in hyperchaotic attractor. The dynamical system described by delay differential equations differs from the system modelled by ordinary differential equations and is an infinite-dimensional system. So our work is mainly devoted to extending MDFC from finite-dimensional systems to infinite-dimensional systems.

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