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Feasible schemes for preparing all five-photon graph states

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We propose feasible experimental schemes for preparing all five-photon graph states. Our schemes require only linear optical elements, photon detectors and post-selection, which are available in current experiment so that these schemes are within the reach of the current technology.

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Graph states as used in this paper are special pure multiparticle entangled states that correspond to mathematical graphs, where the vertices of the graph take the role of the physical systems and the edges represent Ising interactions.[1−4] They serve as essential resources in quantum information, such as quantum error correction,[5] quantum communication[6,7] and quantum computation.[8] Special instances of graph states are multiparticle Greenberger–Horne–Zeilinger (GHZ) states with applications to quantum communication and cluster states which are of significant importance in one-way quantum computation. It is thus important to prepare multiparticle graph states not only for the fundamental study but also for practical applications in quantum information.

On the other hand, the study of graph states has not been done until recently. Hein *et al*[1,4] have shown some properties of graph states. From their papers all connected graphs with four-vertices are equivalent to the star-graph or the linear graph by local unitary (LU) transformations and graph isomorphisms, while for five-vertices it is shown that there exist four different classes of connected graphs and each of them cannot be converted to any other one under LU transformations and graph isomorphisms. Some schemes[9−12] for the preparation of graph states have been proposed. Kiesel *et al*[13] have reported the experimental detection of a four-photon cluster state. Zhao *et al*[14] have experimentally shown the entanglement of a five-photon GHZ state. But how to prepare other classes of multiparticle graph states experimentally is still a challenge.

In this paper we propose experimentally feasible schemes for preparing other three types of five-photon graph states. The rest of this paper is organized as follows. We start with introducing the four different connected graphs with five vertices and proceed with showing graph states corresponding to these graphs. Then we discuss the preparation of the five-photon linear graph state in detail and describe simple changes of the setup for the preparation of the other two types of five-photon graph states. The schemes comprise linear optical elements, photon detectors, and post-selection which are all available in current technologies, so our schemes are feasible experimentally.

From Ref.[1], as Fig.1 shows, there are four families of graphs and any connected graph with five vertices is locally equivalent to one of them under LU transformations.

The states corresponding to these subgraphs in Fig.1 can be respectively written as follows:

$$\frac{1}{\sqrt{2}}(|H\rangle_1|+\rangle_2|+\rangle_3|+\rangle_4|+\rangle_5+|V\rangle_1|-\rangle_2|-\rangle_3|-\rangle_4|-\rangle_5), \tag{1}$$

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$$\frac{1}{2}(|+\rangle_1|H\rangle_2|+\rangle_3|H\rangle_4|+\rangle_5+|+\rangle_1|H\rangle_2|-\rangle_3|V\rangle_4|-\rangle_5 +|-\rangle_1|V\rangle_2|-\rangle_3|H\rangle_4|+\rangle_5+|-\rangle_1|V\rangle_2|+\rangle_3|V\rangle_4|-\rangle_5), \quad (2)$$

$$\frac{1}{2}(|+\rangle_1|+\rangle_2|H\rangle_3|H\rangle_4|+\rangle_5+|+\rangle_1|+\rangle_2|H\rangle_3|V\rangle_4|-\rangle_5 +|-\rangle_1|-\rangle_2|V\rangle_3|H\rangle_4|+\rangle_5-|-\rangle_1|-\rangle_2|V\rangle_3|V\rangle_4|-\rangle_5), \quad (3)$$

$$\frac{1}{2\sqrt{2}}(|+\rangle_1|H\rangle_2|+\rangle_3|H\rangle_4|H\rangle_5+|-\rangle_1|H\rangle_2|+\rangle_3|H\rangle_4|V\rangle_5 +|+\rangle_1|H\rangle_2|-\rangle_3|V\rangle_4|H\rangle_5-|-\rangle_1|H\rangle_2|-\rangle_3|V\rangle_4|V\rangle_5 +|-\rangle_1|V\rangle_2|-\rangle_3|H\rangle_4|H\rangle_5+|+\rangle_1|V\rangle_2|-\rangle_3|H\rangle_4|V\rangle_5 +|-\rangle_1|V\rangle_2|+\rangle_3|V\rangle_4|H\rangle_5-|+\rangle_1|V\rangle_2|+\rangle_3|V\rangle_4|V\rangle_5), \quad (4)$$

where $|H\rangle_i$ and $|V\rangle_i$ represent a photon with horizontal and vertical polarization in spatial mode i respectively, $|+\rangle_i = 1/\sqrt{2}(|H\rangle_i + |V\rangle_i)$, and $|-\rangle_i = 1/\sqrt{2}(|H\rangle_i - |V\rangle_i)$. The polarization state (1) is exactly a five-photon GHZ state under local unitary transformations. Zhao *et al*[14] have reported a proof-of-principle demonstration of this state. So here we consider only the generation of the other three states.

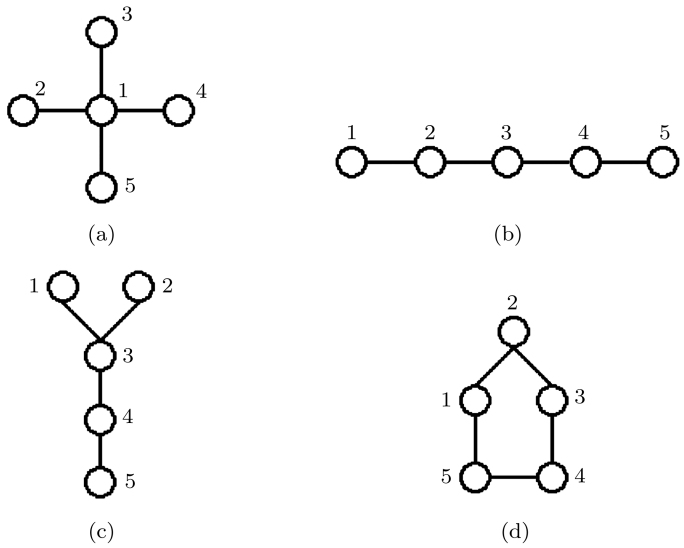


Fig.1. List of connected graphs with five vertices that are not equivalent to each other under LU transformations and graph isomorphism. (a) The five-photon star-graph state. (b) The five-photon linear cluster state. (c) The five-photon Y-shaped graph state. (d) The five-photon circle-shaped graph state.

Now, we describe the details of our schemes and the experimental setup, which is curtly shown in Fig.2. First, we assume that five photons are initially in the following state:

$$\frac{1}{\sqrt{2}}(|H\rangle_a|H\rangle_b+|V\rangle_a|V\rangle_b)\otimes\frac{1}{\sqrt{2}}(|H\rangle_c+|V\rangle_c) \otimes\frac{1}{\sqrt{2}}(|H\rangle_d+|V\rangle_d)\otimes\frac{1}{\sqrt{2}}(|H\rangle_e+|V\rangle_e), \quad (5)$$

The polarization beam splitters (PBSs) transmit the horizontal and reflect vertical polarizations (In the actual experiment, additional birefringent phase shifters are needed to make the relative phase can be set as desired, so by the post-selection the effect of PBS can be described by the operator $|HH\rangle\langle HH| + |VV\rangle\langle VV|$.[15]), so the state of photons in modes a, b and c is transformed into a tripartite GHZ state, after passing through PBS1 on the condition that there is only one photon in each of the out modes 3, s, t.

The half wave plates (HWPs) make the polarizations of the two photons in modes s and t rotate by 45°, i.e. $|H\rangle \rightarrow 1/\sqrt{2}(|H\rangle + |V\rangle)$, and $|V\rangle \rightarrow 1/\sqrt{2}(|H\rangle - |V\rangle)$, resulting in

$$
\begin{aligned}
& \frac{1}{\sqrt{2}}(|H\rangle_a|H\rangle_b+|V\rangle_a|V\rangle_b)\otimes\frac{1}{\sqrt{2}}(|H\rangle_c+|V\rangle_c)\otimes\frac{1}{\sqrt{2}}(|H\rangle_d+|V\rangle_d)\otimes\frac{1}{\sqrt{2}}(|H\rangle_e+|V\rangle_e)\\
\xrightarrow{\text{afterPBS1}} \quad & \frac{1}{2\sqrt{2}}(|H\rangle_3|H\rangle_t|H\rangle_s+|V\rangle_3|V\rangle_t|V\rangle_s)\otimes(|H\rangle_d+|V\rangle_d)\otimes(|H\rangle_e+|V\rangle_e)\\
\xrightarrow{\text{afterHWP1,2}} \quad & \frac{1}{2\sqrt{2}}(|H\rangle_3|+\rangle_t|+\rangle_s+|V\rangle_3|-\rangle_t|-\rangle_s)\otimes(|H\rangle_d+|V\rangle_d)\otimes(|H\rangle_e+|V\rangle_e). \qquad (6)
\end{aligned}
$$

After photons in modes d and s have passed through PBS2 and photons in mode e and t have passed through PBS3, by using post-selection to keep only one photon in each of modes 1, 2, 3, 4 and 5, we obtain

$$
\begin{aligned}
& \frac{1}{2\sqrt{2}}(|H\rangle_3|+\rangle_t|+\rangle_s+|V\rangle_3|-\rangle_t|-\rangle_s)\otimes(|H\rangle_d+|V\rangle_d)\otimes(|H\rangle_e+|V\rangle_e)\\
= \quad & \frac{1}{4}(|+\rangle_3|H\rangle_t|H\rangle_s+|-\rangle_3|H\rangle_t|V\rangle_s+|+\rangle_3|V\rangle_t|V\rangle_s+|-\rangle_3|V\rangle_t|H\rangle_s)\otimes(|H\rangle_d+|V\rangle_d)\otimes(|H\rangle_e+|V\rangle_e)\\
\xrightarrow{\text{afterPBS2}} \quad & \frac{1}{2\sqrt{2}}(|H\rangle_t|+\rangle_3|H\rangle_4|H\rangle_5+|H\rangle_t|-\rangle_3|V\rangle_4|V\rangle_5+|V\rangle_t|+\rangle_3|V\rangle_4|V\rangle_5\\
& +|V\rangle_t|-\rangle_3|H\rangle_4|H\rangle_5)\otimes(|H\rangle_e+|V\rangle_e)\\
\xrightarrow{\text{afterPBS3}} \quad & \frac{1}{2}(|H\rangle_1|H\rangle_2|+\rangle_3|H\rangle_4|H\rangle_5+|H\rangle_1|H\rangle_2|-\rangle_3|V\rangle_4|V\rangle_5+|V\rangle_1|V\rangle_2|+\rangle_3|V\rangle_4|V\rangle_5\\
& +|V\rangle_1|V\rangle_2|-\rangle_3|H\rangle_4|H\rangle_5), \qquad (7)
\end{aligned}
$$

which is equivalent to Eq.(2) under LU transformations.

If we remove the HWP1 as shown in Fig.3, the evolution of the whole state is as follows (similarly we consider only the case in which there is only one photon in each of the output modes):

$$
\begin{aligned}
& \frac{1}{\sqrt{2}}(|H\rangle_a|H\rangle_b+|V\rangle_a|V\rangle_b)\otimes\frac{1}{\sqrt{2}}(|H\rangle_c+|V\rangle_c)\otimes\frac{1}{\sqrt{2}}(|H\rangle_d+|V\rangle_d)\otimes\frac{1}{\sqrt{2}}(|H\rangle_e+|V\rangle_e)\\
\xrightarrow{\text{afterPBS1,HWP2}} \quad & \frac{1}{2\sqrt{2}}(|H\rangle_3|H\rangle_t|+\rangle_s+|V\rangle_3|V\rangle_t|-\rangle_s)\otimes(|H\rangle_d+|V\rangle_d)\otimes(|H\rangle_e+|V\rangle_e)\\
\xrightarrow{\text{afterPBS2}} \quad & \frac{1}{2\sqrt{2}}(|H\rangle_t|H\rangle_3|H\rangle_4|H\rangle_5+|H\rangle_t|H\rangle_3|V\rangle_4|V\rangle_5+|V\rangle_t|V\rangle_3|H\rangle_4|H\rangle_5\\
& -|V\rangle_t|V\rangle_3|V\rangle_4|V\rangle_5)\otimes(|H\rangle_e+|V\rangle_e)\\
\xrightarrow{\text{afterPBS3}} \quad & \frac{1}{2}(|H\rangle_1|H\rangle_2|H\rangle_3|H\rangle_4|H\rangle_5+|H\rangle_1|H\rangle_2|H\rangle_3|V\rangle_4|V\rangle_5+|V\rangle_1|V\rangle_2|V\rangle_3|H\rangle_4|H\rangle_s\\
& -|V\rangle_1|V\rangle_2|V\rangle_3|V\rangle_4|V\rangle_s), \qquad (8)
\end{aligned}
$$

which can be converted to Eq.(3) by using LU transformations.

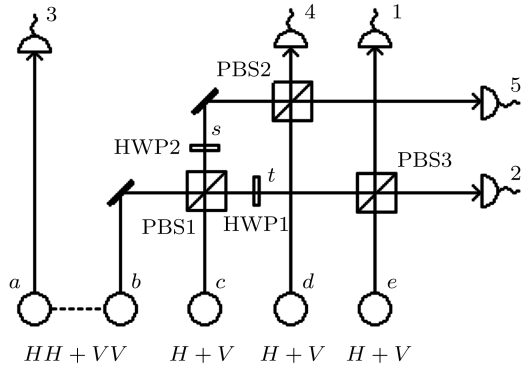


Fig.2. Experimental setup for the preparation of a five-photon linear graph state.

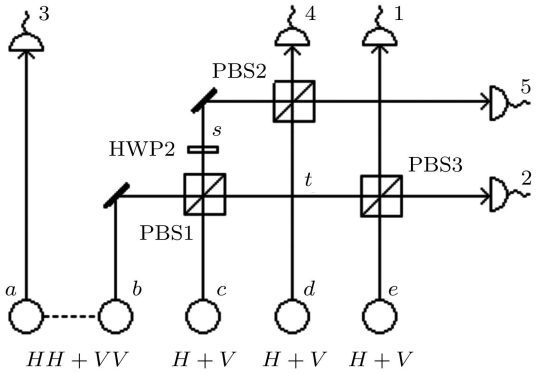


Fig.3. Experimental arrangement for generating a five-photon Y-shaped graph state.

To prepare a circle-shaped graph state, we need a controlled phase (C-phase) gate applied to qubits 3 and 4 and qubits 1 and 5. As shown in Ref.[13], such

an optical C-phase gate is available using the other three polarization-dependent beam splitters (PDBSs) in the setup as shown in Fig.4.

Two input photons are overlapped on a PDBS1 (PDBS4), with transformation for horizontal polarization $T_{\rm h}=1$, and for vertical polarization $T_{\rm v}=\frac{1}{3}$. The other two beam splitters PDBS2 (PDBS5) and PDBS3 (PDBS6) with $T_{\rm h}=\frac{1}{3}$ and $T_{\rm v}=1$ are used to equalize the transmittances for all input polarizations.[13] And we assume the initial state to be

$$\frac{1}{\sqrt{2}}(|H\rangle_a|H\rangle_b+|V\rangle_a|V\rangle_b)\otimes\frac{1}{\sqrt{2}}(|H\rangle_c+|V\rangle_c)\otimes\frac{1}{\sqrt{2}}(|H\rangle_d|H\rangle_e+|V\rangle_d|V\rangle_e). \tag{9}$$

By the post-selection, the final state is obtained as follows:

$$\begin{aligned}
&\frac{1}{\sqrt{2}}(|H\rangle_a|H\rangle_b+|V\rangle_a|V\rangle_b)\otimes\frac{1}{\sqrt{2}}(|H\rangle_c+|V\rangle_c)\otimes\frac{1}{\sqrt{2}}(|H\rangle_d|H\rangle_e+|V\rangle_d|V\rangle_e)\\
\xrightarrow{\text{afterPBSandHWP1,2,3}}\ &\frac{1}{2}(|H\rangle_2|+\rangle_s|+\rangle_t+|V\rangle_2|-\rangle_s|-\rangle_t)\otimes(|H\rangle_d|+\rangle_e+|V\rangle_d|-\rangle_e)\\
\xrightarrow{\text{afterPDBS1,2,3}}\ &\frac{1}{2}(|H\rangle_2|+\rangle_3|H\rangle_4|+\rangle_t|+\rangle_e+|V\rangle_2|-\rangle_3|H\rangle_4|-\rangle_t|+\rangle_e\\
&+|H\rangle_2|-\rangle_3|V\rangle_4|+\rangle_t|-\rangle_e+|V\rangle_2|+\rangle_3|V\rangle_4|-\rangle_t|-\rangle_e)\\
\xrightarrow{\text{afterPDBS4,5,6}}\ &\frac{1}{2\sqrt{2}}(|+\rangle_1|H\rangle_2|+\rangle_3|H\rangle_4|H\rangle_5+|-\rangle_1|H\rangle_2|+\rangle_3|H\rangle_4|V\rangle_5\\
&|-\rangle_1|V\rangle_2|-\rangle_3|H\rangle_4|H\rangle_5+|+\rangle_1|V\rangle_2|-\rangle_3|H\rangle_4|V\rangle_5\\
&+|+\rangle_1|H\rangle_2|-\rangle_3|V\rangle_4|H\rangle_5-|-\rangle_1|H\rangle_2|-\rangle_3|V\rangle_4|V\rangle_5\\
&+|-\rangle_1|V\rangle_2|+\rangle_3|V\rangle_4|H\rangle_5-|+\rangle_1|V\rangle_2|+\rangle_3|V\rangle_4|V\rangle_5\,,
\end{aligned} \tag{10}$$

which is exactly the state in Eq. (4). So we can prepare the five-photon linear cluster state, Y-shaped graph state and circle-shaped graph state using a similar setup.

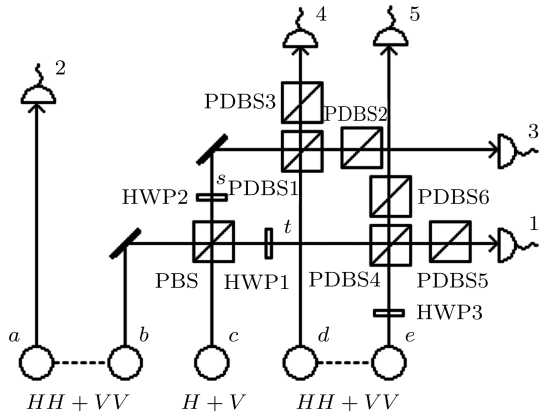


Fig.4. Preparation of a five-photon circle-shaped graph state.

In summary, we have proposed simple schemes to prepare several classes of five-photon graph states. Besides the possible applications of the graph states to multiparty quantum communication and quantum computation, the study of their features is important for a better understanding of entanglement of many parties. So our schemes have profound significance for the development of quantum information. In addition, we should point out that the linear optical elements, photon detectors and post-selection technology needed in the schemes are all often used in experiments. We can prepare the initial states easily with resources of five photons or three entangled photon pairs when one of the six photons serves as a trigger by parametric down conversion. In fact our experimental team has studied five photons experimentally[14] in 2004 and the experiment on six photons has been also reported recently.[15] In our proposals, the final states are selected by the coincidence detection so that the main errors from multiphoton emissions by parametric down converters are eliminated. Therefore, our schemes are feasible by using current technologies.

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