Vol 16 No 8, August 2007
1009-1963/2007/16(08)/2211-04

Chinese Physics

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A simple scheme for generating multi-atom GHZ state via cavity QED*

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(Received 11 November 2006; revised manuscript received 27 November 2006)

This paper proposes a simple scheme for generating a three-atom GHZ state via cavity quantum electrodynamics (QED). The task can be achieved through the interaction between two EPR states, which can be prepared easily with current technology. In this scheme, the cavity field is only virtually excited during the interaction process, and no quantum information transfer between the atoms and the cavity is required. Thus it greatly prolongs the efficient decoherent time. Moreover, this scheme is also applicable for generating an N-atom GHZ state.

Keywords: multi-atom entanglement, GHZ state, cavity QED
PACC: 0365, 0367, 4250

1. Introduction

The state of two coupled quantum systems is entangled[1] if it cannot be expressed as product states of the individual systems. Entanglement has been recognized as a valuable resource for quantum information processing, and the EPR entangled state[1] has been used universally in many quantum applications such as quantum teleportation,[2] quantum computation[3] and quantum cryptography.[4] Recently, much attention has been devoted to the research on multi-particle entanglement, and showing that the multi-particle systems have many advantages over two-particle systems. For example, the three-particle GHZ state[5,6] is maximally stable against (white) noise, it maximally violates Bell-type inequalities,[7] the mutual information of measurement outcomes is maximal and one can locally obtain an EPR state shared between any two of the three particles from a GHZ state with unit probability. Moreover, the three-particle GHZ state can be utilized in the generation of an important four-particle state,[8] which can be used for realizing the teleportation-based control-not (C-Not) gate.[9−11] Therefore, the preparation of multi-particle entangled states[12−16] has become a critical technique in quantum information processing.

Recently, cavity QED has been proved to be a promising system for quantum information processing and quantum computation. Some schemes for generating the entanglement via resonant cavity QED have been proposed.[15,16] These schemes demand cavities with high quality factor. In 2000, Zheng and Guo[17] proposed a scheme for generating two-atom entanglement, which has been implemented experimentally by Osnahgi *et al.*[18] In this scheme, two atoms crossing a nonresonant cavity are entangled by coherent energy exchange. During the interaction process, the cavity field is only virtually excited and no quantum information transfer between the atomic system and the cavity is required. Therefore, the scheme is insensitive to cavity decay. Recently, much attention has been devoted to the research on the nonresonant cavity QED[19,20] and this system has been used universally in quantum information.[21−23]

In this paper, we propose a simple scheme for generating a three-atom GHZ state from two EPR states. In our scheme, two pairs of EPR states are prepared first. Then two atoms are chosen from each pair, and sent into a single-mode cavity simultaneously. After interacting for some time, we detect one of the two atoms. Finally, with some suitable single-atom rotation operations, we can obtain the desired GHZ state. In our scheme, the cavity field is only virtually excited during the interaction, and there is no energy exchange between the atomic system and the cavity. Thus the requirement for the quality factor of the cavity is greatly reduced and then the implementation is

*Project supported by the National Natural Science Foundation of China (Grant No 10574001) and the Program of the Education Department of Anhui Province (Grant No 2004kj029).
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expected. Moreover our scheme is also applicable for generating an N-atom GHZ state.

2. Scheme for generating multi-atom GHZ state via cavity QED

In this section, we describe our scheme in detail. Consider that two identical two-level Rydberg atoms interact simultaneously within a single-mode cavity field. The interaction Hamiltonian of this system in the interaction picture is

$$H_{\mathrm{I}} = g\sum_{j=1}^{2}(\mathrm{e}^{-\mathrm{i}\delta t}a^{\dagger}S_j^{-} + \mathrm{e}^{-\mathrm{i}\delta t}aS_j^{+}), \tag{1}$$

where $S_j^{+} = |e\rangle_j\langle g|$,$S_j^{-} = |g\rangle_j\langle e|$, and $|e\rangle_j$ ($|g\rangle_j$) is the excited (ground) state of atom j, $a^{\dagger}(a)$ is the creation (annihilation) operator for the cavity mode, g is the atom–cavity coupling strength, δ is the detuning between the atomic transition frequency and cavity frequency. In the case of $\delta \gg g$, there is no energy exchange between the atomic system and the cavity field. Assume that the cavity field is initially prepared in the vacuum state, then one can obtain the effective interaction Hamiltonian[17]

$$H_{\mathrm{e}} = \lambda\left[\sum_{j=1}^{2}|e\rangle_j\langle e| + \left(s_1^{+}s_2^{-} + s_1^{-}s_2^{+}\right)\right], \tag{2}$$

where $\lambda = g^2/\delta$. If the two atoms are initially in the state $|eg\rangle_{i,j}$, the state evolution of this system is

$$|eg\rangle_{i,j} \to \mathrm{e}^{-\mathrm{i}\lambda.\,t}\left[\cos(\lambda t)\,|eg\rangle_{i,j} - \mathrm{i}\sin(\lambda t)\,|ge\rangle_{i,j}\right]. \tag{3}$$

If the two atoms are initially in the states $|gg\rangle_{i,j}$ and $|ee\rangle_{i,j}$, they will not be affected. But a phase factor $\mathrm{e}^{-\mathrm{i}(2\lambda t)}$ will be induced to the state $|ee\rangle_{i,j}$. With the choice of $\lambda t = \pi/2$, we can obtain all the possible results as follows:

$$\begin{aligned} &|ge\rangle_{i,j} \to -|eg\rangle_{i,j}, \quad |eg\rangle_{i,j} \to -|ge\rangle_{i,j} \\ &|gg\rangle_{i,j} \to |gg\rangle_{i,j}, \quad |ee\rangle_{i,j} \to -|ee\rangle_{i,j} \end{aligned}. \tag{4}$$

Next, we describe how to generate a three-atom GHZ state in detail. Consider two atoms prepared in the state of $|e\rangle_1 \otimes |g\rangle_2$. Let them interact simultaneously within a nonresonant cavity. The Hamiltonian of this system is given by Eq.(2). By setting $\lambda t = \pi/4$, then Eq.(3) becomes

$$|\Psi\rangle_{1,2}^{(1)} = 1\Big/\sqrt{2}\,(|eg\rangle - \mathrm{i}\,|ge\rangle)_{1,2}, \tag{5}$$

where the common phase factor $\pi/4$ is neglected in Eq.(5), and we can transform the entangled state into the following form by applying some single-atom rotation operations:

$$|\Psi\rangle_{1,2}^{(2)} = 1\Big/\sqrt{2}\,(|gg\rangle + |ee\rangle)_{1,2}. \tag{6}$$

Using the same method, we can also obtain the entanglement on another two atoms, A and 3. Up to here, the initial state for the generation of the three-atom GHZ state has been prepared already. Thus the initial state is given by the following equation:

$$|\Psi\rangle_{1,2,A,3}^{(0)} = 1/2\,(|gg\rangle + |ee\rangle)_{1,2} \otimes (|gg\rangle + |ee\rangle)_{A,3}. \tag{7}$$

Here, we introduce a Hadamard operation, which is defined as follows:

$$|g\rangle \to 1\Big/\sqrt{2}\,(|g\rangle + |e\rangle), \quad |e\rangle \to 1\Big/\sqrt{2}\,(|g\rangle - |e\rangle). \tag{8}$$

Now, we perform a Hadamard operation on atom 2, thus the evolution is

$$\begin{aligned} |\Psi\rangle_{1,2,A,3}^{(1)} = {} & \sqrt{2}\Big/4\,(|gg\rangle + |ge\rangle + |eg\rangle - |ee\rangle)_{1,2} \\ & \otimes\ (|gg\rangle + |ee\rangle)_{A,3}. \end{aligned} \tag{9}$$

Then let atoms 2 and A interact simultaneously within a nonresonant cavity. The Hamiltonian of this system is also given by Eq.(2). With the choice of $\lambda t = \pi/2$, the evolution is given by Eq.(4) and the state of the whole system is given as follows:

$$\begin{aligned} |\Psi\rangle_{1,2,A,3}^{(2)} = {} & \sqrt{2}/4(|gggg\rangle - |ggeg\rangle + |eggg\rangle \\ & + |egeg\rangle - |geee\rangle - |gege\rangle \\ & - |eege\rangle + |eeee\rangle)_{1,2,A,3}. \end{aligned} \tag{10}$$

Then perform the Hadamard operation on atom A, and Eq.(10) becomes

$$\begin{aligned} |\Psi\rangle_{1,2,A,3}^{(3)} &= 1/\sqrt{2}\,(|ggeg\rangle + |eggg\rangle - |gege\rangle - |eeee\rangle)_{1,2,A,3} \\ &= 1/\sqrt{2}\left[|g\rangle_A\,(|egg\rangle - |gee\rangle)_{1,2,3} + |e\rangle_A\,(|ggg\rangle - |eee\rangle)_{1,2,3}\right]. \end{aligned} \tag{11}$$

Next we detect the atom A; if it is in the state of $|e\rangle_A$, then the state of the other three atoms is

$$|\Psi\rangle^{(1)}_{1,2,3} = 1/\sqrt{2}\,(|ggg\rangle - |eee\rangle)_{1,2,3}\,. \qquad (12)$$

This is exactly the desired GHZ state. Therefore, our task has been achieved without doing anything else. If the atom A is in the state $|g\rangle_A$, then the state of the atoms 1, 2 and 3 is

$$|\Psi\rangle^{(2)}_{1,2,3} = 1/\sqrt{2}\,(|gee\rangle - |egg\rangle)_{1,2,3}\,. \qquad (13)$$

In this case, we can obtain the desired state by applying a classical field on the atom 1 to undergo the transition $|g\rangle \leftrightarrow |e\rangle$. Up to here, we have finished the generation of three-atom GHZ state with success probability of 100%.

Next we describe how to obtain the N-atom GHZ state from an (N-1)-atom GHZ state. Assume that the (N-1)-atom GHZ state has been prepared already. The state is

$$|\mathrm{GHZ}\rangle^{(0)}_{1,2,\ldots(N-1)} = 1/\sqrt{2}\left[\bigotimes_{i=1}^{N-1}|g\rangle_i + \bigotimes_{i=1}^{N-1}|e\rangle_i\right]. \qquad (14)$$

Moreover, an EPR state has been generated on the atoms N and A, and the state is

$$|\Psi\rangle_{A,N} = 1/\sqrt{2}\,(|gg\rangle + |ee\rangle)_{A,N}\,. \qquad (15)$$

Now we perform a Hadamard operation on the atom $(N-1)$ first, thus Eq.(14) becomes

$$|\mathrm{GHZ}\rangle^{(1)}_{1,2,\ldots,(N-1)} = \frac{1}{2}\left[\bigotimes_{i=1}^{N-2}|g\rangle_i\,(|g\rangle + |e\rangle)_{N-1} + \bigotimes_{i=1}^{N-2}|e\rangle_i\,(|g\rangle - |e\rangle)_{N-1}\right]. \qquad (16)$$

Then we let atoms $N-1$ and A interact simultaneously within a nonresonant cavity. By setting $\lambda t = \pi/2$, then the evolution is

$$\begin{aligned}|\Psi\rangle^{(1)}_{1,\ldots(N-1),A,N} = \sqrt{2}\Big/4\Bigg[&\bigotimes_{i=1}^{N-2}|g\rangle_i \otimes (|ggg\rangle - |geg\rangle - |ege\rangle - |eee\rangle)_{N-1,A,N} \\ &+ \bigotimes_{i=1}^{N-2}|e\rangle_i \otimes (|ggg\rangle + |geg\rangle - |ege\rangle + |eee\rangle)_{N-1,A,N}\Bigg]. \end{aligned} \qquad (17)$$

Next we perform a Hadamard operation on the atom A. Thus Eq.(17) becomes

$$\begin{aligned}|\Psi\rangle^{(2)}_{1,\ldots,(N-1),A,N} &= \frac{1}{2}\left[\bigotimes_{i=1}^{N-2}|g\rangle_i \otimes (|geg\rangle - |ege\rangle)_{(N-1),A,N} + \bigotimes_{i=1}^{N-2}|e\rangle_i \otimes (|ggg\rangle - |eee\rangle)_{(N-1),A,N}\right] \\ &= \frac{1}{2}\left[|e\rangle_A\,(|g\ldots ggg\rangle - |e\ldots eee\rangle)_{1,\ldots,N} + |g\rangle_A\,(|e\ldots egg\rangle - |g\ldots gee\rangle)_{1,\ldots,N}\right]. \end{aligned} \qquad (18)$$

Now we detect the atom A. If it is in the state $|e\rangle_A$, then the other N atoms are in the state

$$|\Psi\rangle^{(1)}_{1,\ldots,N} = 1\Big/\sqrt{2}\,(|g\ldots ggg\rangle - |e\ldots eee\rangle)_{1,\ldots,N}\,. \qquad (19)$$

This is exactly the desired state. Therefore, our task has been achieved without doing anything else. If the atom A is in the state $|g\rangle_A$, then the state of the other N atoms is

$$|\Psi\rangle^{(2)}_{1,\ldots,N} = 1\Big/\sqrt{2}\,(|g\ldots gee\rangle - |e\ldots egg\rangle)_{1,\ldots,N}\,. \qquad (20)$$

In this case, we can obtain the desired state by applying a classical field to undergo the transition $|g\rangle \leftrightarrow |e\rangle$ on the atoms $N-1$ and N.

Next we give a brief discussion on the experimental matters. For Rydberg atoms with principal quantum numbers 49, 50 and 51, the radiative time is about $T_r = 3\times10^{-2}$ s, and the coupling constant $g = 2\pi\times 24$ kHz.[24] Setting $\delta = 10\,g$, therefore the essential atom–cavity–field interaction time is approximately $\pi\delta/g^2 \approx 2\times10^{-4}$ s. Thus the time needed to complete the whole procedure is approximately 10^{-3} s, much shorter than T_r. Therefore, our scheme is experimentally feasible with current technology.

3. Conclusions

In summary, we have proposed a simple scheme for generating the three-atom GHZ state via cavity QED. In the scheme, the initial states are two EPR entangled pairs and they can be prepared easily with

current technology. Our task can be achieved through the interaction between two atoms within a nonresonant cavity. Moreover, we can generate the N-atom GHZ state step by step. The most distinct advantage of our scheme is that the cavity field is only virtually excited, and no quantum information transfer between the atomic system and the cavity is required. Thus the requirement for the quality factor of the cavity is greatly reduced and then the implementation is expected.

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