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Application of three-mode Einstein-Podolsky -Rosen entangled state with continuous variables to teleportation*

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We consider how to teleport two- and three-mode Einstein–Podolsky–Rosen entangled states ($|\eta\rangle$ and $|p_t, \chi_2, \chi_3\rangle$) via a $|p_t, \chi_2, \chi_3\rangle$ quantum channel for continuous variables. Using the complete and orthogonal representation of the entangled states, we can not only find the a complete basis set for the joint measurement but also propose the specific scheme of teleportation. Our calculation can be greatly simplified by using their Schmidt decompositions.

Keywords: entangled state representation, Einstein–Podolsky–Rosen (EPR) entangled state, quantum teleportation

PACC: 0365, 0367

1. Introduction

Quantum entanglement is an important resource for quantum information. In particular, quantum teleportation (QT)[1−6] has received much attention both theoretically and experimentally due to its potential applications. In QT, an unknown state is transmitted from a sender (Alice) to a receiver (Bob) via a quantum channel with the aid of some classical information. The process of teleportation may be regarded as sending and extracting quantum information via the quantum channel. Several kinds of schemes have been studied for the teleportation of quantum state with discrete variables,[7−10] which was firstly realized experimentally by the Innsbruck group.[11] Two schemes for teleporting an arbitrary three-particle state have been proposed.[12] In the first scheme, a two-particle entangled state and a three-particle entangled state (both non-maximally entangled states) are used as quantum channels, while in the second scheme, three non-maximally entangled particle pairs are employed as quantum channels. Then a scheme for probabilistic and controlled teleportation of an unknown arbitrary three-particle state is discussed by constructing a peculiar non-maximally entangled state as a controlled quantum channel.[13]

At present, more and more attention has been paid to teleporting quantum states with continuous variables (CVs). The theoretical analysis of the teleportation of continuous quantum states was first made by Vaidman,[14] and then followed by subsequent studies.[15,16] The QT of a two-mode squeezed entangled state is presented by Fan and Fan [17] and Cai,[18] separately, via two different quantum channels. The possibility of generating a multipartite entanglement for CVs, i.e. an entanglement shared by more than two parties[19], has recently opened up the possibility of constructing quantum teleportation networks. The specific form of a three-mode Einstein–Podolsky–Rosen (EPR) entangled state $|p_t, \chi_2, \chi_3\rangle$ has been proposed[20,21] in Fock space, which makes a new quantum mechanical representation due to the property of both completeness and orthogonality. Recently, $|p_t, \chi_2, \chi_3\rangle$ has been considered by Yuan and Qi[22] for the teleportation of single-mode state with CV. This scheme involves two-receiver (Bob and Charlie), and only one of them, not both, can recover the unknown state, the feature of which is that the teleportation between two sides (Alice and Bob) depends on the agreement with the third side (Charlie), who may participate in the process of QT as a supervisor.

In the present paper, we consider how to teleport two- and three-mode EPR entangled states ($|\eta\rangle$ and $|p_t, \chi_2, \chi_3\rangle$) via a $|p_t, \chi_2, \chi_3\rangle$ quantum channel for

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CVs. Using the complete and orthogonal representation ($|\eta\rangle$ and $|p_t, \chi_2, \chi_3\rangle$), we can not only find a complete basis set of the wavefunction for the joint measurement but also propose a specific scheme of teleportation. Our calculation also can be greatly simplified by using the Schmidt decompositions of both $|p_t, \chi_2, \chi_3\rangle$ and $|\eta\rangle$. The rest of this paper is arranged as follows. In Section 2 we briefly review the properties of $|p_t, \chi_2, \chi_3\rangle$. In Section 3, the relationship between two- and three-mode EPR entangled states is presented by analysing the quadrature measurement of both $|p_t, \chi_2, \chi_3\rangle$ and its conjugate state $|\chi, p_2, p_3\rangle$, which implies that it is a way of keeping the remaining particles in the entanglement to perform quadrature-momentum (coordinate) measurement for $|p_t, \chi_2, \chi_3\rangle$ ($|\chi, p_2, p_3\rangle$). In Section 4, the entanglement swapping of $|\eta\rangle$ via $|p_t, \chi_2, \chi_3\rangle$ is discussed. Section 5 is devoted to the scheme for the teleportation of $|\eta\rangle$ in which single $|p_t, \chi_2, \chi_3\rangle$ state and two $|p_t, \chi_2, \chi_3\rangle$ states are used as quantum channels, respectively. It is found that a continuous three-particle basis has been used for joint measurement for the former; while two bipartite EPR pairs are employed for the latter. The teleportation of $|p_t, \chi_2, \chi_3\rangle$ via 'mixed' quantum channels (composed of one $|p_t, \chi_2, \chi_3\rangle$ and one $|\eta\rangle$) is also considered in Section 6, in which a continuous five-particle basis is necessary for joint measurement. Finally, the conclusions drawn from the present study are given in Section 7.

2. Tripartite entangled state

We begin with briefly reviewing the properties of a three-mode entangled state $|p_t, \chi_2, \chi_3\rangle$. The state $|p_t, \chi_2, \chi_3\rangle$ is a common eigenvector of permutable operators: tripartite total momentum $\hat{P}_1 + \hat{P}_2 + \hat{P}_3$ and two relative coordinates $\hat{X}_1 - \hat{X}_2$ and $\hat{X}_1 - \hat{X}_3$. In the three-mode Fock space it reads[20,21]

$$|p_t, \chi_2, \chi_3\rangle = \frac{1}{\sqrt{3}\pi^{3/4}} \exp\left[-\frac{p_t^2}{6} - \frac{1}{3}\left(\chi_2^2 + \chi_3^2 - \chi_2\chi_3\right) + \frac{\mathrm{i}\sqrt{2}p_t}{3}\sum_{i=1}^{3}\hat{a}_i^\dagger + \frac{\sqrt{2}\chi_2}{3}(\hat{a}_1^\dagger - 2\hat{a}_2^\dagger + \hat{a}_3^\dagger) + \frac{\sqrt{2}\chi_3}{3}(\hat{a}_1^\dagger + \hat{a}_2^\dagger - 2\hat{a}_3^\dagger) + \frac{2}{3}\sum_{i<j}^{3}\hat{a}_i^\dagger\hat{a}_j^\dagger - \frac{1}{6}\sum_{i=1}^{3}\hat{a}_i^{\dagger 2}\right]|000\rangle, \quad (1)$$

which obeys the following equations:

$$\left(\hat{X}_1 - \hat{X}_2\right)|p_t, \chi_2, \chi_3\rangle = \chi_2\,|p_t, \chi_2, \chi_3\rangle, \quad (2)$$

$$\left(\hat{X}_1 - \hat{X}_3\right)|p_t, \chi_2, \chi_3\rangle = \chi_3\,|p_t, \chi_2, \chi_3\rangle, \quad (3)$$

$$\sum_{i=1}^{3}\hat{P}_i\,|p_t, \chi_2, \chi_3\rangle = p_t\,|p_t, \chi_2, \chi_3\rangle, \quad (4)$$

where p_t, χ_2 and χ_3 are real numbers. The state $|p_t, \chi_2, \chi_3\rangle$ is able to construct a new quantum mechanical representation and possesses the orthogonal property, i.e.

$$\int_{-\infty}^{\infty}\mathrm{d}p_t\mathrm{d}\chi_2\mathrm{d}\chi_3\,|p_t, \chi_2, \chi_3\rangle\langle p_t, \chi_2, \chi_3| = 1, \quad (5)$$

$$\langle p_t, \chi_2, \chi_3|\, p_t', \chi_2', \chi_3'\rangle = \delta(p_t - p_t')\delta(\chi_2 - \chi_2')\delta(\chi_3 - \chi_3'). \quad (6)$$

Because $|p_t, \chi_2, \chi_3\rangle$ is the extension of the bipartite EPR entangled state $|\eta\rangle$,[20,21,23] we also call it three-mode EPR entangled state. The Schmidt decomposition of $|p_t, \chi_2, \chi_3\rangle$ can be expressed, in the coordinate basis and in the momentum basis, as

$$|p_t, \chi_2, \chi_3\rangle = \frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\mathrm{i}p_t(\chi_2+\chi_3)/3}\int_{-\infty}^{\infty}\mathrm{d}q\,|q\rangle_1 \otimes |q - \chi_2\rangle_2 \otimes |q - \chi_3\rangle_3\,\mathrm{e}^{\mathrm{i}qp_t}, \quad (7)$$

$$|p_t, \chi_2, \chi_3\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}u\mathrm{d}v\left|\frac{p_t}{3} - u - v\right\rangle_1 \otimes \left|\frac{p_t}{3} + u\right\rangle_2 \otimes \left|\frac{p_t}{3} + v\right\rangle_3 \mathrm{e}^{\mathrm{i}(u\chi_2 + v\chi_3)}, \quad (8)$$

where $|q\rangle_i$ and $|p\rangle_i$ are the eigenvectors of the coordinate and the momentum, respectively,

$$|q\rangle_i = \pi^{-1/4}\exp\left[-\frac{1}{2}q^2+\sqrt{2}q\hat{a}_i^\dagger-\frac{1}{2}\hat{a}_i^{\dagger 2}\right]|0\rangle_i, \quad (9)$$

$$|p\rangle_i = \pi^{-1/4}\exp\left[-\frac{1}{2}p^2+\mathrm{i}\sqrt{2}p\hat{a}_i^\dagger+\frac{1}{2}\hat{a}_i^{\dagger 2}\right]|0\rangle_i. \quad (10)$$

From expressions (7) and (8), one can see that $|p_t,\chi_2,\chi_3\rangle$ is a tripartite maximally entangled state, which can be used in QT for CVs. In particular, if $(p_t,\chi_2,\chi_3)=0$ one can find a maximally entangled state (denoted as $|\mathrm{GHZ}\rangle_q$)

$$\begin{aligned}|\mathrm{GHZ}\rangle_q &= \frac{1}{\sqrt{3}\pi^{3/4}}\exp\left[\frac{2}{3}\sum_{i<j}^{3}\hat{a}_i^\dagger\hat{a}_j^\dagger-\frac{1}{6}\sum_{i=1}^{3}\hat{a}_i^{\dagger 2}\right]|000\rangle=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}p\mathrm{d}p'\,|-p-p'\rangle_1\otimes|p\rangle_2\otimes|p'\rangle_3\\ &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathrm{d}q\,|q\rangle_1\otimes|q\rangle_2\otimes|q\rangle_3\,, \end{aligned} \quad (11)$$

which is just the three-mode Greenberger–Horne–Zeilinger (GHZ) entangled state with CVs.[24] Furthermore, the canonical conjugate state $|\chi,p_2,p_3\rangle$ of $|p_t,\chi_2,\chi_3\rangle$, i.e. the common eigenvector of $\hat{P}_1-\hat{P}_2$, $\hat{P}_1-\hat{P}_3$ and $\left(\hat{X}_1+\hat{X}_2+\hat{X}_3\right)$, is

$$\begin{aligned}|\chi,p_2,p_3\rangle = \frac{1}{\sqrt{3}\pi^{3/4}}\exp\Bigg[&-\frac{\chi^2}{6}-\frac{1}{3}\left(p_2^2+p_3^2-p_2p_3\right)+\frac{\sqrt{2}\chi}{3}\sum_{i=1}^{3}\hat{a}_i^\dagger+\frac{\mathrm{i}\sqrt{2}p_2}{3}(\hat{a}_1^\dagger-2\hat{a}_2^\dagger+\hat{a}_3^\dagger)\\ &+\frac{\mathrm{i}\sqrt{2}p_3}{3}(\hat{a}_1^\dagger+\hat{a}_2^\dagger-2\hat{a}_3^\dagger)-\frac{2}{3}\sum_{i<j}^{3}\hat{a}_i^\dagger\hat{a}_j^\dagger+\frac{1}{6}\sum_{i=1}^{3}\hat{a}_i^{\dagger 2}\Bigg]|000\rangle\,. \end{aligned} \quad (12)$$

The Schmidt decomposition of $|\chi,p_2,p_3\rangle$ is

$$|\chi,p_2,p_3\rangle=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}u\mathrm{d}v\left|\frac{\chi}{3}-u-v\right\rangle_1\otimes\left|\frac{\chi}{3}+u\right\rangle_2\otimes\left|\frac{\chi}{3}+v\right\rangle_3\mathrm{e}^{-\mathrm{i}(up_2+vp_3)}, \quad (13)$$

$$|\chi,p_2,p_3\rangle=\frac{1}{\sqrt{2\pi}}\mathrm{e}^{\mathrm{i}\chi(p_2+p_3)/3}\int_{-\infty}^{\infty}\mathrm{d}p\,|p\rangle_1\otimes|p-p_2\rangle_2\otimes|p-p_3\rangle_3\,\mathrm{e}^{-\mathrm{i}p\chi}. \quad (14)$$

In particular, letting $(\chi,p_2,p_3)=0$, one can find another maximally GHZ entangled state

$$\begin{aligned}|\mathrm{GHZ}\rangle_p &= \frac{1}{\sqrt{3}\pi^{3/4}}\exp\left[-\frac{2}{3}\sum_{i<j}^{3}\hat{a}_i^\dagger\hat{a}_j^\dagger+\frac{1}{6}\sum_{i=1}^{3}\hat{a}_i^{\dagger 2}\right]|000\rangle=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}q\mathrm{d}q'\,|-q-q'\rangle_1\otimes|q\rangle_2\otimes|q'\rangle_3\\ &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathrm{d}p\,|p\rangle_1\otimes|p\rangle_2\otimes|p\rangle_3\,. \end{aligned} \quad (15)$$

3. Relation between two- and three-mode EPR entangled states

From expressions (7) and (14), it is easy to see that once particle 1 is measured by making the quadrature-coordinate (momentum) measurement $|q'\rangle_{11}\langle q'|$ ($|p'\rangle_{11}\langle p'|$), then particles 2 and 3 simultaneously collapse to the coordinate (momentum) eigenvector $|q'-\chi_2\rangle_2\otimes|q'-\chi_3\rangle_3$ ($|p'-p_2\rangle_2\otimes|p'-p_3\rangle_3$) no matter how far the distance between the three particles is. In contrast, a question arises naturally: what are the results of measurement if we perform a quadrature-momentum (coordinate) measurement on $|p_t,\chi_2,\chi_3\rangle$ ($|\chi,p_2,p_3\rangle$) by $|p\rangle_{ii}\langle p|$ ($|q\rangle_{ii}\langle q|$), ($i=1,2,3$)? Does the remaining two-particle still keep the entangled property? Which type of entangled state they belong to? In order to see the results clearly, we use the Schmidt decomposition of $|p_t,\chi_2,\chi_3\rangle$ in expression (7) or (8) to directly make the calculation as follows:

$$
\begin{aligned}
{}_1\langle p\,|p_t,\chi_2,\chi_3\rangle &= \frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\mathrm{i}p_t(\chi_2+\chi_3)/3}\int_{-\infty}^{\infty}\mathrm{d}q_1\,\langle p\,|q\rangle_1\otimes|q-\chi_2\rangle_2\otimes|q-\chi_3\rangle_3\,\mathrm{e}^{\mathrm{i}qp_t}\\
&= \frac{1}{2\pi}\mathrm{e}^{-\mathrm{i}p_t(\chi_2+\chi_3)/3}\int_{-\infty}^{\infty}\mathrm{d}q\mathrm{e}^{-\mathrm{i}pq}\,|q-\chi_2\rangle_2\otimes|q-\chi_3\rangle_3\,\mathrm{e}^{\mathrm{i}qp_t}\\
&= \frac{1}{2\pi}\mathrm{e}^{\mathrm{i}\chi_3(p_t-p)/2-\mathrm{i}p_t(\chi_2+\chi_3)/3}U_p\mathrm{e}^{-\mathrm{i}\chi_3(p_t-p)/2}\int_{-\infty}^{\infty}\mathrm{d}q\,|q\rangle_2\otimes|q-\chi_3\rangle_3\,\mathrm{e}^{\mathrm{i}(p_t-p)q}\\
&= M_pU_p\,|\eta_1=\chi_3,\eta_2=p_t-p\rangle_{23}\,, \qquad (16)
\end{aligned}
$$

where

$$U_p=\exp\left(\mathrm{i}\hat{P}_2\chi_2\right), \qquad (17)$$

$$M_p=\frac{1}{2\pi}\exp\left[\mathrm{i}\left(\chi_3/2-\chi_2\right)p_t/3-\mathrm{i}\chi_3p/2\right], \qquad (18)$$

and we have used the formulas

$$\sqrt{2\pi}\,\langle q\,|p\rangle=\exp\left(\mathrm{i}pq\right), \qquad (19)$$

$$|q-q_0\rangle_{\mathrm{i}}=\mathrm{e}^{\mathrm{i}q_0\hat{P}_i}\,|q\rangle_{\mathrm{i}}\,; \qquad (20)$$

we have compared the integration result with the standard form of $\left|\eta=(\eta_1+\mathrm{i}\eta_2)/\sqrt{2}\right\rangle$, which is a bipartite EPR entangled state as given by in the two-mode Fock space,[24]

$$|\eta\rangle_{12}=\exp\left[-\frac{1}{2}|\eta|^2+\eta\hat{a}_1^{\dagger}-\eta^*\hat{a}_2^{\dagger}+\hat{a}_1^{\dagger}\hat{a}_2^{\dagger}\right]|00\rangle_{12}, \qquad (21)$$

and its Schmidt decomposition of $|\eta\rangle_{12}$ is[25]

$$|\eta\rangle_{12}=\mathrm{e}^{-\mathrm{i}\eta_1\eta_2/2}\int_{-\infty}^{\infty}\mathrm{d}q\;\;|q\rangle_1\otimes|q-\eta_1\rangle_2\,\mathrm{e}^{\mathrm{i}\eta_2q}, \qquad (22)$$

$$|\eta\rangle_{12}=\mathrm{e}^{-\mathrm{i}\eta_1\eta_2/2}\int_{-\infty}^{\infty}\mathrm{d}p\;\;|p+\eta_2\rangle_1\otimes|-p\rangle_2\,\mathrm{e}^{-\mathrm{i}p\eta_1}. \qquad (23)$$

In particular, if $\chi_3=0$, and $p_t=p$, expression (16) reduces further to a maximally two-mode entangled state $|\eta=0\rangle_{23}$. Similarly, we can obtain

$${}_2\langle p'\,|p_t,\chi_2,\chi_3\rangle=M'\,|\eta_1=\chi_3,\eta_2=p_t-p'\rangle_{13}\,, \qquad (24)$$

where

$$M_p'=\frac{1}{2\pi}\exp\left[\mathrm{i}\left(\chi_3/2-\chi_2\right)p_t/3+\mathrm{i}p'\left(\chi_2-\chi_3/2\right)\right];$$

$${}_3\langle p''\,|p_t,\chi_2,\chi_3\rangle=M_p''\,|\eta_1=\chi_2,\eta_2=p_t-p''\rangle_{12}\,, \qquad (25)$$

where

$$M_p''=\frac{1}{2\pi}\exp\left[\mathrm{i}\left(\chi_2/2-\chi_3\right)p_t/3+\mathrm{i}p''\left(\chi_3-\chi_2/2\right)\right].$$

In a similar way, the single-mode quadrature-amplitude measurement is performed for $|\chi,p_2,p_3\rangle$ and the results are as follows:

$$
\begin{aligned}
{}_1\langle x\,|\chi,p_2,p_3\rangle &= M_xU_x\,|\zeta_1=\chi,\zeta_2=p_3\rangle_{23}\,,\\
{}_2\langle x'\,|\chi,p_2,p_3\rangle &= M_x'U_x'\,|\zeta_1=\chi,\zeta_2=p_3\rangle_{13}\,,\\
{}_3\langle x''\,|\chi,p_2,p_3\rangle &= M_x''U_x''\,|\zeta_1=\chi,\zeta_2=p_2\rangle_{12}\,,
\end{aligned} \qquad (26)
$$

where $\left|\zeta=\left(\zeta_1+\mathrm{i}\zeta_2\right)/\sqrt{2}\right\rangle$ is the conjugate state of $|\eta\rangle$ and

$$
\begin{aligned}
M_x &= \frac{1}{2\pi}\exp\left[\mathrm{i}\chi\left(p_2-p_3/2\right)/3\right], U_x=\exp\left[\mathrm{i}\left(\hat{P}_2x-\hat{X}_2p_2\right)\right],\\
M_x' &= \frac{1}{2\pi}\exp\left\{\mathrm{i}\left[\chi\left(p_2-p_3/2\right)/3+p_2x'\right]\right\}, U_x'=\exp\left[\mathrm{i}\hat{P}_1x'\right],\\
M_x'' &= \frac{1}{2\pi}\exp\left\{\mathrm{i}\left[\chi\left(p_3-p_2/2\right)/3+p_3x''\right]\right\}, U_x'=\exp\left[\mathrm{i}\hat{P}_1x''\right].
\end{aligned} \qquad (27)
$$

From expressions (16), (24), and (25), one can see that, after single-mode quadrature-momentum (coordinate) measurement by $|p\rangle_{ii}\langle p|$ ($|x\rangle_{ii}\langle x|$), the remaining two-mode fields are in the form of $|\eta\rangle$ ($|\zeta\rangle$), i.e. the remaining two-particle is kept entangled. This implies that it is a way of keeping the remaining particles entangled to perform quadrature-momentum (coordinate) measurement.

4. Entanglement swapping of $|\eta\rangle$ via $|p_t, \chi_2, \chi_3\rangle$

Now we consider how to realize the entanglement swapping of $|\eta\rangle$ by using two $|p_t, \chi_2, \chi_3\rangle$ states as quantum channels, where $|\eta\rangle\langle\eta|$ is viewed as a continuous Bell basis. The two quantum channels are denoted by $|p_t, \chi_2, \chi_3\rangle_{123}$ and $|p'_t, \chi'_2, \chi'_3\rangle_{456}$. Particles 1 and 4, 2 and 5, 3 and 6 belong to Alice, Charlie and Bob, respectively. Alice performs a joint Bell measurement (quadrature phase measurement) of $\hat{X}_1 - \hat{X}_4$ and $\hat{P}_1 + \hat{P}_4$ on particles 1 and 4 (expressed by $|\eta\rangle_{1414}\langle\eta|$). Thus the projected state of the whole system is given by

$$|\psi\rangle \equiv {}_{14}\langle\eta|\, p_t, \chi_2, \chi_3\rangle_{123} \otimes |p'_t, \chi'_2, \chi'_3\rangle_{456}\,. \tag{28}$$

Using the Schmidt decompositions of expressions (7) and (22), we can rewrite expression (28) as

$$\begin{aligned}
|\psi\rangle = & \frac{\mathrm{e}^{\mathrm{i}\eta_1\eta_2/2}}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}q\ {}_1\langle q| \otimes {}_4\langle q-\eta_1|\ \mathrm{e}^{-\mathrm{i}\eta_2 q} \\
& \times \mathrm{e}^{-\mathrm{i}p_t(\chi_2+\chi_3)/3}\int_{-\infty}^{\infty}\mathrm{d}q'\,|q'\rangle_1 \otimes |q'-\chi_2\rangle_2 \otimes |q'-\chi_3\rangle_3\,\mathrm{e}^{\mathrm{i}q'p_t} \\
& \times \mathrm{e}^{-\mathrm{i}p'_t(\chi'_2+\chi'_3)/3}\int_{-\infty}^{\infty}\mathrm{d}q''\,|q''\rangle_4 \otimes |q''-\chi'_2\rangle_5 \otimes |q''-\chi'_3\rangle_6\,\mathrm{e}^{\mathrm{i}q''p'_t} \\
= & \ \Omega\int_{-\infty}^{\infty}\mathrm{d}q\ \mathrm{e}^{-\mathrm{i}\left(\eta_2-p_t-p'_t\right)q}\,|q-\chi_2\rangle_2 \otimes |q-\chi_3\rangle_3 \\
& \otimes |q-\eta_1-\chi'_2\rangle_5 \otimes |q-\eta_1-\chi'_3\rangle_6 \\
= & \ \Omega U_c\int_{-\infty}^{\infty}\mathrm{d}q\ \mathrm{e}^{-\mathrm{i}\left(\eta_2-p_t-p'_t\right)q}\,|q\rangle_2 \otimes |q-\chi_3\rangle_3 \\
& \otimes |q-\eta_1\rangle_5 \otimes |q-\eta_1-\chi'_3\rangle_6\,,
\end{aligned} \tag{29}$$

where

$$\Omega \equiv \frac{1}{2\pi}\mathrm{e}^{\mathrm{i}\eta_1\eta_2/2}\mathrm{e}^{-\mathrm{i}p_t(\chi_2+\chi_3)/3}\mathrm{e}^{-\mathrm{i}p'_t(\chi'_2+\chi'_3)/3}\mathrm{e}^{-\mathrm{i}\eta_1 p'_t}, \tag{30}$$

$$U_c \equiv \exp\left[\mathrm{i}\left(\hat{P}_2\chi_2 + \hat{P}_5\chi'_2\right)\right]. \tag{31}$$

Then one can see that expression (29) implies the quadrature measurement $|\eta\rangle_{1414}\langle\eta|$ for two tripartite entangled states ($|p_t, \chi_2, \chi_3\rangle$ and $|p'_t, \chi'_2, \chi'_3\rangle$) can produce a new four-mode entangled state, which can be viewed as an eigenvector of operator $\left(\hat{X}_2 - \hat{X}_3\right) + \left(\hat{X}_5 - \hat{X}_6\right)$. In this sense, it is possible to construct $2n$-mode quantum teleportation networks by performing EPR joint measurements. In order to realize the entanglement swapping from particles 1 and 4 to particles 3 and 6, Charlie must do some measurements on particles 2 and 5; on the other hand, after Charlie's measurement, particles 2 and 3 should still reside in an entangled state for realizing their goal. Based on these, after Alice has transferred the results of measurement (η_1, η_2) to Charlie and Bob, Charlie performs a further unitary transformation $U_c^\dagger$ and then makes a quadrature-momentum measurement $|p'\rangle_{55}\langle p'|$ and $|p\rangle_{22}\langle p|$ on particles 2 and 5 separately, i.e.

$$
\begin{aligned}
{}_5\langle p'|\otimes_2\langle p|\,U_c^{\dagger}\,|\psi\rangle &= \Omega\int_{-\infty}^{\infty}\mathrm{d}q\;\mathrm{e}^{-\mathrm{i}\left(\eta_2-p_t-p_t'\right)q}\,\langle p\,|q\rangle\,\langle p'\,|q-\eta_1\rangle\,|q-\chi_3\rangle_3\otimes|q-\eta_1-\chi_3'\rangle_6\\
&= \frac{\Omega}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}q\;\mathrm{e}^{-\mathrm{i}\left(\eta_2-p_t-p_t'\right)q}\mathrm{e}^{-\mathrm{i}pq-\mathrm{i}p'(q-\eta_1)}\,|q-\chi_3\rangle_3\otimes|q-\eta_1-\chi_3'\rangle_6\\
&= \frac{\Omega}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}q\;\mathrm{e}^{-\mathrm{i}\left(\eta_2-p_t-p_t'\right)q}\mathrm{e}^{-\mathrm{i}pq-\mathrm{i}p'(q-\eta_1)}\mathrm{e}^{\mathrm{i}\hat{P}_3\chi_3}\,|q\rangle_3\otimes\mathrm{e}^{\mathrm{i}\hat{P}_6\chi_3'}\,|q-\eta_1\rangle_6\\
&= \frac{\Omega}{2\pi}\mathrm{e}^{\mathrm{i}\left(\hat{P}_3\chi_3+\hat{P}_6\chi_3'\right)}\mathrm{e}^{-\mathrm{i}\left(p\hat{X}_3+p'\hat{X}_6\right)}\int_{-\infty}^{\infty}\mathrm{d}q\;\mathrm{e}^{-\mathrm{i}\left(\eta_2-p_t-p_t'\right)q}\,|q\rangle_3\otimes|q-\eta_1\rangle_6\,. \qquad (32)
\end{aligned}
$$

A comparison between expressions (22) and (32) yields

$$
\mathrm{Eq.}\,(32)=\frac{\Omega\mathrm{e}^{\mathrm{i}\eta_1'\eta_2'/2}}{2\pi}U_b\,|\eta_1'=\eta_1,\eta_2'=\eta_2-p_t-p_t'\rangle_{36}\,, \qquad (33)
$$

where

$$
\begin{aligned}
U_b \equiv{}& \exp\left[\mathrm{i}\left(\hat{P}_3\chi_3+\hat{P}_6\chi_3'\right)\right]\\
&\times\exp\left[-\mathrm{i}\left(p\hat{X}_3+p'\hat{X}_6\right)\right], \qquad (34)
\end{aligned}
$$

and we have used formulas (19) and (20). After receiving the information (p,p') through a classical channel from Charlie, Bob performs a unitary transformation $U_b^{\dagger}$ (up to a trivial phase factor Ω). Finally, the particles 3 and 6 are of the type of $|\eta_1',\eta_2'=\eta_2-p_t-p_t'\rangle_{36}$. Thus the entanglement swapping for $|\eta\rangle$ is realized successfully by using a continuous four-particle basis for joint measurement of two-separate-momentum of single particle and total momentum and relative position of two other particles, i.e. $|p'\rangle_{55}\langle p'|\otimes|p\rangle_{22}\langle p|\otimes|\eta\rangle_{1414}\langle\eta|$. From the above description, the feature of scheme is that swapping between two sides (Alice and Bob) depends on the agreement with the third side (Charlie), who may participate in the process of QT as a supervisor. That is to say, it is impossible for Bob to obtain the expectant state by himself without the help of Charlie, which is evidently different from the process of QT by using two $|\eta\rangle$ states as quantum channels.

5. Teleportation of two-mode quantum states by $|p_t,\chi_2,\chi_3\rangle$

5.1. Teleportation scheme using single $|p_t,\chi_2,\chi_3\rangle$ state

In this section, we investigate the teleportation of $|\eta\rangle$ state by using single $|p_t,\chi_2,\chi_3\rangle$ state as quantum channel. Suppose that the quantum channel $|p_t,\chi_2,\chi_3\rangle_{123}$ is shared by Alice and Bob, where particle 3 and particle pair 1 and 2 belong to Alice and Bob, respectively. The $|\eta\rangle_{45}$ which will be teleported to Bob is shared by Alice (particle 4) and the supervisor "Charlie" (particle 5). In order to realize the teleportation, a Bell state measurement of $\hat{X}_1-\hat{X}_4$ and $\hat{P}_1+\hat{P}_4$, i.e. $|\eta'\rangle_{1414}\langle\eta'|$ on particles 1 and 4 is made by Alice, which will project particles 2, 3 and 5 onto the following state:

$$
{}_{14}\langle\eta'\,|p_t,\chi_2,\chi_3\rangle_{123}\otimes|\eta\rangle_{45}\,. \qquad (35)
$$

Using expressions (7) and (22), we can further rewrite expression (35) as

$$
\begin{aligned}
|\psi\rangle_{235} \equiv{}& {}_{14}\langle\eta'\,|p_t,\chi_2,\chi_3\rangle_{123}\otimes|\eta\rangle_{45}\\
={}& N\cdot\int_{-\infty}^{\infty}\mathrm{d}q\mathrm{e}^{-\mathrm{i}\eta_2'q}\int_{-\infty}^{\infty}\mathrm{d}q'\mathrm{e}^{\mathrm{i}q'p_t}\;{}_1\langle q|\;q'\rangle_1\,{}_4\langle q-\eta_1'|\;\eta\rangle_{45}\otimes|q'-\chi_2\rangle_2\otimes|q'-\chi_3\rangle_3\\
={}& N\cdot\mathrm{e}^{-\mathrm{i}\eta_1\eta_2/2}\int_{-\infty}^{\infty}\mathrm{d}q\mathrm{e}^{-\mathrm{i}\eta_2'q}\mathrm{e}^{\mathrm{i}xp_t}\mathrm{e}^{\mathrm{i}\eta_2 q}\,|q-\chi_2\rangle_2\otimes|q-\chi_3\rangle_3\otimes|q-\eta_1-\eta_1'\rangle_5\,, \qquad (36)
\end{aligned}
$$

where

$$N \equiv \frac{1}{\sqrt{2\pi}} \mathrm{e}^{\mathrm{i}\eta_1'\eta_2'/2 - \mathrm{i}p_t(\chi_2+\chi_3)/3}. \tag{37}$$

From expression (36) we see that $|\psi\rangle_{235}$ is actually an entangled state of particles 2, 3 and 5. Expression (36) means that Charlie must do a measurement on particle 5 in order to teleport the information taken by particle 4 and 5 to particles 2 and 3. Thus Charlie chooses the quadrature-momentum measurement basis $|p\rangle_{55}\langle p|$. So after the measurement, using formula (19), expression (36) becomes

$${}_5\langle p\,|\psi\rangle_{235} = \frac{N}{\sqrt{2\pi}} \mathrm{e}^{-\mathrm{i}\eta_1\eta_2/2} \int_{-\infty}^{\infty} \mathrm{d}q \mathrm{e}^{-\mathrm{i}\eta_2' q} \mathrm{e}^{\mathrm{i}qp_t} \mathrm{e}^{\mathrm{i}\eta_2 q} \mathrm{e}^{-\mathrm{i}p\left(q-\eta_1-\eta_1'\right)} |q-\chi_2\rangle_2 \otimes |q-\chi_3\rangle_3 . \tag{38}$$

In particular, if the parameter of shift position η_1' is assumed to be known (letting $\eta_1' = 0$), expression (38) becomes

$$\begin{aligned} {}_5\langle p\,|\psi\rangle_{235} = & \; N'U_N \int_{-\infty}^{\infty} \mathrm{d}q\,|q\rangle_2 \otimes |q\rangle_3\, \mathrm{e}^{\mathrm{i}\eta_2' q} \\ = & \; N'U_N\,|\eta_1'=0,\eta_2'\rangle_{23}\,, \end{aligned} \tag{39}$$

where

$$\begin{aligned} N' &= \frac{N}{\sqrt{2\pi}} \mathrm{e}^{\mathrm{i}p\eta_1'}, \\ U_N &= \mathrm{e}^{\mathrm{i}\left(\hat{P}_2\chi_2+\hat{P}_3\chi_3\right)} \mathrm{e}^{-\mathrm{i}p\hat{X}_2} \mathrm{e}^{\mathrm{i}\left(p_t-\eta_2'\right)\hat{X}_3}. \end{aligned} \tag{40}$$

From expression (39) we see that after Alice and Charlie have informed Bob of the results of measurement (η, p), Bob performs a unitary transformation $U_N^{\dagger}$ (up to a trivial phase factor N') and an unknown two-mode entangled state $|\eta_1'=0,\eta_2'\rangle_{45}$ can be reproduced on particles 2 and 3. Thus the teleportation is carried out successfully through single $|p_t,\chi_2,\chi_3\rangle$ state by using a continuous three-particle basis for joint measurement of one-momentum of single particle and total momentum and relative position of two other particles, i.e. $|p\rangle_{55}\langle p| \otimes |\eta\rangle_{1414}\langle\eta|$. It is natural for us to consider how to realize the teleportation of $|\eta_1', \eta_2'=0\rangle$ via single tripartite entangled state. To realize it, we resort to the $|\chi, p_2, p_3\rangle$ representation in expression (12). Similarly, one-coordinate of single particle and total momentum and relative position of two other particles, i.e. $|x\rangle_{55}\langle x| \otimes |\eta\rangle_{1414}\langle\eta|$ must be used as a continuous three-particle basis for joint measurement. Thus we obtain

$$\begin{aligned} & {}_5\langle x|_{14}\langle\eta\,|\chi,p_2,p_3\rangle_{123} \otimes |\eta'\rangle_{45} \\ = & \; N''U_N'' \otimes \Pi_{p2}\,|\eta_1',\eta_2'=0\rangle_{23}\,, \end{aligned} \tag{41}$$

where

$$N'' = \frac{1}{2\pi} \mathrm{e}^{\mathrm{i}\chi(p_2+p_3)/3} \mathrm{e}^{-\mathrm{i}\eta_2(\eta_1/2+\chi)}, \tag{42}$$

$$U_N'' = \mathrm{e}^{\mathrm{i}\left[\eta_2\left(\hat{X}_2+\hat{X}_3\right)-p_2\hat{X}_2-p_3\hat{X}_3\right]} \mathrm{e}^{\mathrm{i}\hat{P}_3(x-\eta_1-\chi)}, \tag{43}$$

$$\Pi_{p2} = \int_{-\infty}^{\infty} \mathrm{d}p\,|-p\rangle_{22}\langle p|\,. \tag{44}$$

So the teleportation of $|\eta_1', \eta_2'=0\rangle$ is also performed perfectly via single tripartite entangled state. Generally, for $|\eta_1' \neq 0,\ \eta_2' \neq 0\rangle$, Alice cannot teleport perfectly the state $|\eta\rangle$ by performing a three-particle joint measurement $|x\rangle_{55}\langle x| \otimes |\eta\rangle_{1414}\langle\eta|$ $(|p\rangle_{55}\langle p| \otimes |\eta\rangle_{1414}\langle\eta|)$, the basis for which is a set of the three-particle states involving two entangled particles only.

5.2. Teleportation scheme using two $|p_t, \chi_2, \chi_3\rangle$ states

Next, we consider how to teleport an arbitrary bipartite entangled state by using tripartite channels. Two $|p_t,\chi_2,\chi_3\rangle$ states are used as quantum channels to teleport the unknown entangled state. Suppose that the two quantum channels $|p_t,\chi_2,\chi_3\rangle_{135} \otimes |p_t',\chi_2',\chi_3'\rangle_{246}$ are shared by Alice, Bob and the supervisor Charlie, where particles 1 and 2, 5 and 6, 3 and 4 belong to Alice, Charlie and Bob, respectively. Alice receives an unknown entangled state $|\psi\rangle_{78}$ of particles 7 and 8 and wants to teleport $|\psi\rangle_{78}$ to Bob. Thus the total initial state of the whole system (eight particles) is expressed as

$$|p_t,\chi_2,\chi_3\rangle_{135} \otimes |p_t',\chi_2',\chi_3'\rangle_{246} \otimes |\psi\rangle_{78}\,. \tag{45}$$

The teleportation can be described as follows. Firstly, Alice must perform a joint Bell measurement of $\hat{X}_7 - \hat{X}_1$ and $\hat{P}_7 + \hat{P}_1$, i.e. $|\eta'\rangle_{7171}\langle\eta'|$ on particles 1 and 7, $\hat{X}_8 - \hat{X}_2$ and $\hat{P}_8 + \hat{P}_2$, i.e. $|\eta''\rangle_{8282}\langle\eta''|$ on particles 2 and 8, respectively. So the total projection state is $|\eta'\rangle_{7171}\langle\eta'| \otimes |\eta''\rangle_{8282}\langle\eta''|$. According to expressions (7) and (23), after the measurement the projected state for particles 3 and 4, 5 and 6 is

$$
\begin{aligned}
|\Psi\rangle \equiv\ & {}_{82}\langle\eta'| \otimes {}_{71}\langle\eta\ \ |p_t,\chi_2,\chi_3\rangle_{135} \otimes |p'_t,\chi'_2,\chi'_3\rangle_{246} \otimes |\psi\rangle_{78} \\
=\ & \frac{\Pi_1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}p' \mathrm{e}^{\mathrm{i}p'\eta'_1}\ {}_8\langle p'+\eta'_2| \otimes {}_2\langle -p'| \\
& \otimes \int_{-\infty}^{\infty} \mathrm{d}p \mathrm{e}^{\mathrm{i}p\eta_1}\ {}_7\langle p+\eta_2| \otimes {}_1\langle -p|\ \psi\rangle_{78} \\
& \otimes \int_{-\infty}^{\infty} \mathrm{d}q\, |q\rangle_1 \otimes |q-\chi_2\rangle_3 \otimes |q-\chi_3\rangle_5\, \mathrm{e}^{\mathrm{i}qp_t} \\
& \otimes \int_{-\infty}^{\infty} \mathrm{d}q'\, |q'\rangle_2 \otimes |q'-\chi'_2\rangle_4 \otimes |q'-\chi'_3\rangle_6\, \mathrm{e}^{\mathrm{i}q'p'_t},
\end{aligned} \tag{46}
$$

where Π_1 is a phase factor

$$
\Pi_1 \equiv \mathrm{e}^{\mathrm{i}\left\{(\eta''_1\eta''_2+\eta'_1\eta'_2)/2-\left[p_t(\chi_2+\chi_3)+p'_t(\chi'_2+\chi'_3)\right]/3\right\}}. \tag{47}
$$

Using formula (19), we can rewrite expression (46) as

$$
\begin{aligned}
|\Psi\rangle = & \frac{\Pi_1}{(2\pi)^2}\int_{-\infty}^{\infty} \mathrm{d}p' \mathrm{e}^{\mathrm{i}p'\eta'_1} \int_{-\infty}^{\infty} \mathrm{d}p \mathrm{e}^{\mathrm{i}p\eta_1} \int_{-\infty}^{\infty} \mathrm{d}q \int_{-\infty}^{\infty} \mathrm{d}q' \mathrm{e}^{\mathrm{i}\left(p'q'+pq\right)} \\
& \times {}_8\langle p'+\eta'_2| \otimes {}_7\langle p+\eta_2|\ \psi\rangle_{78}\, \mathrm{e}^{\mathrm{i}\left(qp_t+q'p'_t\right)} \\
& \times |q-\chi_2\rangle_3 \otimes |q'-\chi'_2\rangle_4 \otimes |q-\chi_3\rangle_5 \otimes |q'-\chi'_3\rangle_6 .
\end{aligned} \tag{48}
$$

Then one can see that the unknown state $|\psi\rangle_{78}$ projects onto ${}_8\langle p'+\eta'_2| \otimes {}_7\langle p+\eta_2|$ and simultaneously produces the state $|q-\chi_2\rangle_3 \otimes |q'-\chi'_2\rangle_4 \otimes |q-\chi_3\rangle_5 \otimes |q'-\chi'_3\rangle_6$, which is related to not only particles 5 and 6 (belonged to Bob) but also particles 3 and 4 (belonging to Charlie), i.e. after the measurement performed by Alice, the four particles are involved in correlation with each other. Thus it is necessary for Charlie to make a further measurement to realize the teleportation from Alice to Bob. Now we are concerned for the situation when $|\psi\rangle_{78}$ is a two-mode squeezed entangled vacuum state

$$
|\psi\rangle_{78} = \sec h\gamma \exp\left[\hat{a}_7^{\dagger}\hat{a}_8^{\dagger}\tanh\gamma\right]|00\rangle_{78}, \tau = \mathrm{e}^{\gamma}, \tag{49}
$$

which has its natural expression in the $|\eta\rangle_{78}$ representation as follows:[26]

$$
\begin{aligned}
|\psi\rangle_{78} = & \frac{1}{\tau}\int \frac{\mathrm{d}^2\eta}{\pi} |\eta/\tau\rangle_{7878}\langle\eta|\ 00\rangle_{78} \\
= & \frac{1}{\tau}\int \frac{\mathrm{d}^2\zeta}{\pi} \mathrm{e}^{-|\zeta|^2/2} |\zeta/\tau\rangle_{78} .
\end{aligned} \tag{50}
$$

By using the Schmidt decomposition of $|\zeta/\tau\rangle_{78}$ in Expression (23) we have

$$
\begin{aligned}
& {}_8\langle p'+\eta'_2| \otimes {}_7\langle p+\eta_2|\ \zeta/\tau\rangle_{78} \\
= & \ \mathrm{e}^{-\mathrm{i}\zeta_1\zeta_2/2\tau^2}\ {}_8\langle p'+\eta'_2| \otimes {}_7\langle p+\eta_2| \cdot \int_{-\infty}^{\infty} \mathrm{d}p''\, |p''+\zeta_2/\tau\rangle_7 \otimes |-p''\rangle_8\, \mathrm{e}^{-\mathrm{i}p''\zeta_1/\tau} \\
= & \ \mathrm{e}^{-\mathrm{i}\zeta_1\zeta_2/2\tau^2}\, \delta\left(p+p'+\eta_2+\eta'_2-\zeta_2/\tau\right) \mathrm{e}^{-\mathrm{i}(p+\eta_2-\zeta_2/\tau)\zeta_1/\tau}.
\end{aligned} \tag{51}
$$

Substituting expression (51) into expression (50) and then into expression (48), we can rewrite expression (48) as

$$
\begin{aligned}
|\Psi\rangle = & \frac{\Pi_1}{(2\pi)^2}\int \frac{\mathrm{d}^2\zeta}{\tau\pi} \mathrm{e}^{-|\zeta|^2/2-\mathrm{i}\zeta_1\zeta_2/2\tau^2} \int_{-\infty}^{\infty} \mathrm{d}q \int_{-\infty}^{\infty} \mathrm{d}q' \mathrm{e}^{\mathrm{i}\left(qp_t+q'p'_t\right)} \int_{-\infty}^{\infty} \mathrm{d}p' \mathrm{e}^{\mathrm{i}p'\left(\eta'_1+q'\right)} \\
& \times \int_{-\infty}^{\infty} \mathrm{d}p \mathrm{e}^{\mathrm{i}p(\eta_1+q)}\, \delta\left(p+p'+\eta_2+\eta'_2-\zeta_2/\tau\right) \cdot \mathrm{e}^{-\mathrm{i}(p+\eta_2-\zeta_2/\tau)\zeta_1/\tau} \\
& \times |q-\chi_2\rangle_3 \otimes |q'-\chi'_2\rangle_4 \otimes |q-\chi_3\rangle_5 \otimes |q'-\chi'_3\rangle_6
\end{aligned}
$$

$$= \Pi_2 U_{b1} \otimes U_{c1} \int \frac{\mathrm{d}^2\zeta}{\tau\pi} \mathrm{e}^{-|\zeta|^2/2} \mathrm{e}^{-\mathrm{i}\zeta_1\zeta_2/2\tau^2} \times \int_{-\infty}^{\infty} \mathrm{d}q'' \mathrm{e}^{\mathrm{i}q''\zeta_2/\tau} \left|q''\right\rangle_3 \otimes \left|q'' - \zeta_1/\tau\right\rangle_4 \otimes \left|q''\right\rangle_5 \otimes \left|q'' - \zeta_1/\tau\right\rangle_6, \tag{52}$$

where we have used formula (20) and $\hat{X}_i \left|x\right\rangle_i = x \left|x\right\rangle_i$,

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}k \exp(\mathrm{i}kx), \tag{53}$$

$$U_{b1} \equiv \mathrm{e}^{\mathrm{i}\left(\hat{X}_5 p_t + \hat{X}_6 p_t'\right)} \mathrm{e}^{\mathrm{i}\hat{P}_5(\eta_1+\chi_3)} \mathrm{e}^{\mathrm{i}\hat{P}_6\left(\eta_1'+\chi_3'\right)}, \tag{54}$$

$$U_{c1} \equiv \mathrm{e}^{\mathrm{i}\hat{P}_4\left(\eta_1'+\chi_2'\right)} \mathrm{e}^{-\mathrm{i}\eta_2'\hat{X}_4} \mathrm{e}^{\mathrm{i}\hat{P}_3(\eta_1+\chi_2)} \mathrm{e}^{-\mathrm{i}\hat{X}_3\eta_2}, \tag{55}$$

$$\Pi_2 \equiv \frac{1}{2\pi} \Pi_1 \mathrm{e}^{\mathrm{i}\left(\chi_3 p_t + \chi_3' p_t'\right) - \mathrm{i}\left(\eta_2+\eta_2'\right)\eta_1} \mathrm{e}^{\mathrm{i}\eta_1\left(\eta_2+\eta_2'\right)}, \tag{56}$$

where ($U_{b1}$and U_{c1}) and (Π_1 and Π_2) are unitary operators and inessential phase factors, respectively. From expression (52) we see that two pairs of particles 3 and 4, 5 and 6 are involved in a four-mode entangled state. To assist Alice and Bob, it is necessary for Charlie to make a further unitary transformation $U_{c1}^{\dagger}$ and associated measurement on particles 3 and 4 on his side. Thus, after receiving the measurement results from Alice via a classical channel, Charlie performs the unitary transformation and makes the measurement by $\left|p\right\rangle_{33}\left\langle p\right|$ and $\left|p'\right\rangle_{44}\left\langle p'\right|$, thus the particles 5 and 6 will be the following state:

$$\begin{aligned} {}_3\left\langle p\right| \otimes_4 \left\langle p'\right| \left|\Psi\right\rangle = {} & \Pi_3 U_{b1} \int \frac{\mathrm{d}^2\zeta}{\tau\pi} \mathrm{e}^{-|\zeta|^2/2} \cdot \mathrm{e}^{-\mathrm{i}\zeta_1\zeta_2/2\tau^2} \int_{-\infty}^{\infty} \mathrm{d}q'' \mathrm{e}^{\mathrm{i}q''\zeta_2/\tau} \\ & \times {}_3\left\langle p\right| \left|q''\right\rangle_{34} \left\langle p'\right| \left|q'' - \zeta_1/\tau\right\rangle_4 \left|q''\right\rangle_5 \otimes \left|q'' - \zeta_1/\tau\right\rangle_6 \\ = {} & \frac{1}{2\pi} \Pi_3 U_{b3} \left|\psi\right\rangle_{56}, \end{aligned} \tag{57}$$

where

$$U_{b2} = U_{b1} \mathrm{e}^{-\mathrm{i}\left(\hat{X}_5 p + \hat{X}_6 p'\right)}. \tag{58}$$

Finally, as soon as Bob receives the classical information about the results of measurement from Alice (η, η') and Charlie (p, p'), Bob makes the unitary transformation $U_{b2}^{\dagger}$ to obtain $\left|\psi\right\rangle_{78}$. In this way the teleportation process is carried out successfully with the cooperation of Charlie. In this scheme, a continuous six-partite basis must be used for joint measurement of two-separate-momentum of single particle and two pairs of total momentum and relative position of four other particles, i.e. $\left|p\right\rangle_{33}\left\langle p\right| \otimes \left|p'\right\rangle_{44}\left\langle p'\right| \otimes \left|\eta'\right\rangle_{8282}\left\langle \eta'\right| \otimes \left|\eta\right\rangle_{7171}\left\langle \eta\right|$. Due to the completeness property of $\left|\eta\right\rangle$, any bipartite state $\left|\,\right\rangle_{78}$ can be expressed as

$$\left|\,\right\rangle_{78} = \int_{-\infty}^{\infty} \frac{\mathrm{d}^2\eta}{\pi} \pounds \left|\eta\right\rangle_{78}, \tag{59}$$

where $\pounds = {}_{78}\left\langle \eta\right|\left\rangle_{78}$, thus any bipartite state with a definite $\pounds$ can be teleported in this way.

6. Teleportation via 'mixed' quantum channels

In this section, a scheme for the teleportation of $\left|p_t, \chi_2, \chi_3 = 0\right\rangle_{135}$ is presented by using 'mixed' quantum channels (denoted as $\left|p_t, \chi_2, \chi_3\right\rangle_{724} \otimes \left|\eta\right\rangle_{86}$). Particles (1,3,7,8), (2,4,6) and 5 belong to Alice, Bob and Charlie, respectively. It is necessary to use a continuous five-partite basis for joint measurement of one-momentum of single particle and two pairs of total momentum and relative position of four other particles, i.e. $\left|p\right\rangle_{55}\left\langle p\right|$ and $\left|\eta''\right\rangle_{3838}\left\langle \eta''\right| \otimes \left|\eta'\right\rangle_{1717}\left\langle \eta'\right|$, which is done by Charlie and Alice, respectively. After the measurement, particles 2, 4 and 6 will project onto the following state:

$$\begin{aligned} \left|\Psi\right\rangle \equiv {} & {}_5\left\langle p\right| \otimes_{38} \left\langle \eta''\right| \otimes {}_{17}\left\langle \eta'\right| \left|p_t, \chi_2, \chi_3\right\rangle_{135} \otimes \left|p_t', \chi_2', \chi_3'\right\rangle_{724} \otimes \left|\eta\right\rangle_{86} \\ = {} & \frac{\Pi_4 \mathrm{e}^{-\mathrm{i}\left(\eta_1' p_t' + \eta_2 \eta_1''\right)}}{\sqrt{2\pi}} \mathrm{e}^{-\mathrm{i}p_t(\chi_2+\chi_3)/3} \int_{-\infty}^{\infty} \mathrm{d}q \mathrm{e}^{\mathrm{i}qp_t} \left\langle p\right| \left|q - \chi_3\right\rangle \end{aligned}$$

$$
\begin{aligned}
&\otimes \mathrm{e}^{\mathrm{i}q\left(p_t'-\eta_2'\right)}\left|q-\eta_1'-\chi_2'\right\rangle_2 \otimes \left|q-\eta_1'-\chi_3'\right\rangle_4 \\
&\otimes \mathrm{e}^{\mathrm{i}\eta_2(q-\chi_2)}\mathrm{e}^{-\mathrm{i}\eta_2''(q-\chi_2)}\left|q-\chi_2-\eta_1''-\eta_1\right\rangle_6 \\
= \;& \Pi_4 \mathrm{e}^{-\mathrm{i}\left(\eta_1' p_t'+\eta_2\eta_1''\right)} U_4 \otimes \frac{\mathrm{e}^{-\mathrm{i}p_t(\chi_2+\chi_3)/3}}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}q \mathrm{e}^{\mathrm{i}qp_t}\mathrm{e}^{\mathrm{i}p\chi_3} \\
&\otimes \left|q\right\rangle_2 \otimes \left|q\right\rangle_4 \otimes \left|q-\chi_2\right\rangle_6 , \qquad (60)
\end{aligned}
$$

where

$$
\Pi_4 \equiv \frac{1}{\sqrt{2\pi}}\mathrm{e}^{\mathrm{i}\left[\left(\eta_1'\eta_2'+\eta_1''\eta_2''-\eta_1\eta_2\right)/2-p_t'\left(\chi_2'+\chi_3'\right)/3\right]}, \qquad (61)
$$

$$
U_4 \equiv \mathrm{e}^{\mathrm{i}\hat{P}_2\left(\eta_1'+\chi_2'\right)}\mathrm{e}^{\mathrm{i}\hat{X}_2\left(p_t'-\eta_2'-p\right)}\mathrm{e}^{\mathrm{i}\hat{P}_4\left(\eta_1'+\chi_3'\right)}\mathrm{e}^{\mathrm{i}\hat{P}_6\left(\eta_1''+\eta_1\right)}\mathrm{e}^{\mathrm{i}\left(\eta_2-\eta_2''\right)\hat{X}_6}. \qquad (62)
$$

When $\chi_3 = 0$, expression (60) can be further written as

$$
\left|\Psi\right\rangle = \frac{\Pi_4 \mathrm{e}^{-\mathrm{i}\left(\eta_1' p_t'+\eta_2\eta_1''\right)}}{\sqrt{2\pi}} U_4 \left|p_t, \chi_2, \chi_3=0\right\rangle_{264}. \qquad (63)
$$

After receiving the result of measurement (η', η'', p) from Alice and Charlie via the classical channel, Bob makes a unitary transformation $U_4^{\dagger}$ and obtains the input state $\left|p_t, \chi_2, \chi_3=0\right\rangle$. The case is true for $\chi_2 = 0$.

There is no doubt that the above "mixed" quantum channels (denoted as $\left|p_t, \chi_2, \chi_3\right\rangle_{135} \otimes \left|\eta''\right\rangle_{24}$) can also be used to realize the teleportation of bipartite EPR entangled state ($\left|\eta_a\right\rangle_{78}$). Similarly, a five-particle basis $\left|p\right\rangle_{33}\left\langle p\right| \otimes \left|\eta'\right\rangle_{8282}\left\langle \eta'\right| \otimes \left|\eta\right\rangle_{7171}\left\langle \eta\right|$ must be used. After measurement, the projected state for particles 4 and 5 is

$$
\begin{aligned}
&{}_3\left\langle p\right|_{82}\left\langle \eta'\right| \otimes {}_{71}\left\langle \eta\right. \left|p_t, \chi_2, \chi_3\right\rangle_{135} \otimes \left|\eta''\right\rangle_{24} \otimes \left|\eta_a\right\rangle_{78} \\
= \;& \mathcal{D}' U_c' U_b' \left|\eta_a\right\rangle_{54}, \qquad (64)
\end{aligned}
$$

where

$$
U_c' = \mathrm{e}^{\mathrm{i}\hat{P}_3(\eta_1+\chi_2)}, \qquad (65)
$$

$$
U_b' = \mathrm{e}^{\mathrm{i}\hat{P}_5(\eta_1+\chi_3)}\mathrm{e}^{\mathrm{i}\hat{X}_5(p_t-\eta_2-p)}\mathrm{e}^{\mathrm{i}\hat{P}_4\left(\eta_1'+\eta_1''\right)}\mathrm{e}^{\mathrm{i}\hat{X}_4\left(\eta_2''-\eta_2'\right)}, \qquad (66)
$$

$$
\mathcal{D}' = \frac{1}{2\pi}\mathrm{e}^{-\mathrm{i}p_t(\chi_2+\chi_3)/3}\mathrm{e}^{\mathrm{i}\left(\eta_1\eta_2+\eta_1'\eta_2'-\eta_1''\eta_2''\right)/2}\mathrm{e}^{-\mathrm{i}\left(\eta_1 p_t+\eta_1'\eta_2''\right)}. \qquad (67)
$$

Thus after receiving the results of measurement from Alice via a classical channel, Charlie performs a unitary transformation $U_c'^{\dagger}$ on particle 3, then makes a quadrature-momentum measurement $\left|p\right\rangle_{33}\left\langle p\right|$ and informs Bob of his result (p). Then Bob can reproduce $\left|\eta_a\right\rangle$ on particles 4 and 5 by making the unitary transformation $U_b'^{\dagger}$ (up to a trivial phase factor $\mathcal{D}'$). It should be pointed out that not all 'mixed' quantum channels realize the teleportation of $\left|\eta_a\right\rangle$. For example, the 'mixed' $\left|p_t, \chi_2, \chi_3\right\rangle$ and $\left|\chi, p_2, p_3\right\rangle$ states are used as quantum channels, which can carry out successfully the teleportation of $\left|\eta\right\rangle$ with $\eta_1 = 0$.

7. Conclusions

Teleportations of two- and three-mode EPR entangled states ($\left|\eta\right\rangle$ and $\left|p_t, \chi_2, \chi_3\right\rangle$) are presented via $\left|p_t, \chi_2, \chi_3\right\rangle$ and 'mixed' quantum channels for CVs, respectively. We can not only find the a complete basis set of the wavefunction for the joint measurement but also propose the specific scheme of teleportation by using the complete and orthogonal representation of the entangled states. Our calculation can be greatly simplified with their Schmidt decompositions.

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