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Spin-polarized quantum transport through an Aharonov–Bohm quantum-dot-ring*

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In this paper the quantum transport through an Aharonov–Bohm (AB) quantum-dot-ring with two dot-array arms described by a single-band tight-binding Hamiltonian is investigated in the presence of additional magnetic fields applied to the dot-array arms to produce spin flip of electrons. A far richer interference pattern than that in the charge transport alone is found. Besides the usual AB oscillation the tunable spin polarization of the current by the magnetic flux is a new observation and is seen to be particularly useful in technical applications. The spectrum of transmission probability is modulated by the quantum dot numbers on the up-arc and down-arc of the ring, which, however, does not affect the period of the AB oscillation.

Keywords: spin-polarized, Aharonov–Bohm ring, interference
PACC: 7210B, 7225, 7215C

1. Introduction

Traditional electronics is based on the charge transport, and the spin degree of electrons does not play a role. The idea of electronic devices in which exploited are both the charge and spin degrees of electrons has led to a new field known as spintronics. Spin polarized transports through quantum-dot devices have attracted considerable attention recently since these devices not only exhibit new fundamental physics but also are of promising applications in the emerging technologies of spintronics and quantum information.[1−3] Recent experimental advances have stimulated the study of spin-polarized transport, and hybrid mesoscopic structures have become one of the major focuses of the rapidly developing spintronics from the viewpoint of both fundamental physics and potential applications. In complex electronic devices consisting of quantum dots (QDs), quantum wires and mesoscopic rings,[4−7] where the size of structures is small with respect to the coherence length, the transmission probability, from which the conductance of the devices can be obtained with the help of the Laudauer–Büticker formula, plays an essential role in the study of the quantum transport through the systems. Various methods have been developed of numerically calculating the transmission probability, such as mode-matching methods,[8−11] recursive Green's function technique[12−19] and the scattering matrix.[20] The quantum transport through a mesoscopic AB ring coupled with reservoirs acting as a source of electrons and inelastic scatterers was first studied by Büttiker[21] and was subsequently extended to the case with a serial of rings.[22] The AB oscillations of the conductance and persistent current[23−26] in a mesoscopic ring have been also investigated both theoretically and experimentally.[27] All these studies are based on ballistic wave functions of the single-electron Schrödinger equation or the tight-binding model for a lattice ring.[28−33] However, the spin-polarized quantum transport combined with the AB phase interference has not yet been investigated, which may lead to new phenomena due to the mutual interference between the spin-up and spin-down currents.

It is the main goal of the paper to study the AB interference in spin-polarized transport. To this end we consider a mesoscopic device consisting of an AB lattice ring attached by two QD-array arms where two magnetic fields are applied separately. The first field applied to the left-arm gives rise to the energy level splitting, while the second field to the right-arm, making an angle of θ with the first one, results in the spin-flip and thus the transmission probability as well as the current depends on the spin-polarization. The AB

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oscillation of the spin-current is demonstrated. Our new observation is that the spin-polarization of the current can be tuned by the magnetic flux.

2. Model and formulation

The system under consideration is explained schematically in Fig.1, where the left arm of QD-array labelled by the lattice sites $-N_{\rm L},\ldots,-1$, is coupled with the dot-L of the ring, and the right-arm labelled by the lattice sites $0,1,\ldots,N_{\rm R}$, is coupled with the dot-R. Two electrodes (ideal wires) in connection with the dot-array arms are described by discrete lattice sites labelled as $-\infty,\ldots,-(N_{\rm L}+2),-(N_{\rm L}+1)$ and $N_{\rm R}+1,N_{\rm R}+2,\ldots,\infty$, respectively.

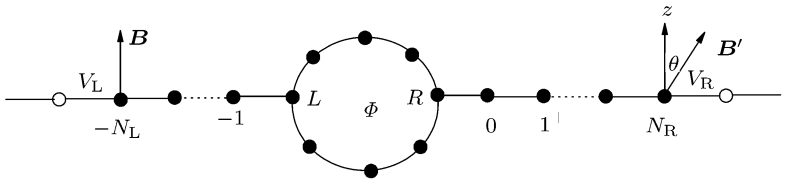


Fig.1. AB-ring with two QD-array arms.

The first magnetic field is applied to the dot- $-N_{\rm L}$ in the z-direction, and the other to the dot- $N_{\rm R}$ forms an angle θ with the z-axis. In the single-band tight-binding approximation the Hamiltonian of the system can be written as

$$H = H_0 + H_{\rm R} + H_{\rm I} \tag{1}$$

with

$$\begin{aligned}
H_0 =& \sum_{n=-N_{\rm L}+1,\sigma}^{N_{\rm R}-1} \varepsilon_n a^+_{n,\sigma} a_{n,\sigma} - \sum_{n=-N_{\rm L},\sigma(n\neq -1)}^{N_{\rm R}-1} (t_{n,n+1}(\sigma) a^+_{n+1,\sigma} a_{n,\sigma} + {\rm h.c}) + (\varepsilon_{-N_{\rm L}} + \sigma g\mu_B B) a^+_{-N_{\rm L},\sigma} a_{-N_{\rm L},\sigma} \\
&+ (\varepsilon_{N_{\rm R}} + \sigma g\mu_{B'} B'\cos\theta) a^+_{N_{\rm R},\sigma} a_{N_{\rm R},\sigma} + g\mu_{B'} B' \sin\theta (a^+_{N_{\rm R},\sigma} a_{N_{\rm R},-\sigma} + {\rm h.c}) + \sum_{n\le -(N_{\rm L}+1),\sigma} \varepsilon_{\rm L} a^+_{n,\sigma} a_{n,\sigma} \\
&- \sum_{n\le -(N_{\rm L}+2),\sigma} (t_{n,n+1}(\sigma) a^+_{n+1,\sigma} a_{n,\sigma} + {\rm h.c}) + \sum_{n\ge N_{\rm R}+1,\sigma} \varepsilon_{\rm R} a^+_{n,\sigma} a_{n,\sigma} - \sum_{n\ge N_{\rm R}+2,\sigma} (t_{n,n+1}(\sigma) a^+_{n+1,\sigma} a_{n,\sigma} + {\rm h.c}) \\
&- V_{\rm L}(\sigma)(a^+_{-N_{\rm L}-1} a_{-N_{\rm L}} + {\rm h.c}) - V_{\rm R}(\sigma)(a^+_{N_{\rm R}+1} a_{N_{\rm R}} + {\rm h.c}), \\
H_{\rm R} =& \sum_{j=1}^{N} \varepsilon_j c^+_{j,\sigma} c_{j,\sigma} - \sum_{j=1}^{N-1} (t_c(\sigma) c^+_{j+1,\sigma} c_{j,\sigma} {\rm e}^{{\rm i}\gamma} + {\rm h.c}) - t_c(\sigma)({\rm e}^{{\rm i}\gamma} c^+_{1,\sigma} c_{N,\sigma} + {\rm h.c}), \\
H_{\rm I} =& - V_0(\sigma)(c^+_{{\rm L},\sigma} a_{-1,\sigma} + {\rm h.c}) - V_0(\sigma)(c^+_{{\rm R},\sigma} a_{0,\sigma} + {\rm h.c}),
\end{aligned} \tag{2}$$

where $H_{\rm R}$ describes the AB lattice-ring, H_0 is the Hamiltonian of the two QD-array arms along with electrodes , and $H_{\rm I}$ is the interaction Hamiltonian between the QD-array arm and the AB-ring. ε_n with $-N_{\rm L} \le n \le N_{\rm R}$, denotes the energy eigenvalue of the nth QD in the QD array, $\varepsilon_{\rm R}$ and $\varepsilon_{\rm L}$ are the on-site energies of the left and right electrodes respectively. $a^+_{n,\sigma}$ (with $\sigma = \uparrow,\downarrow$) is the creation operator of the electrons with spin index σ at the nth site, where $-\sigma$ denotes the opposite spin polarization with respect to σ. $t_{n,n+1}(\sigma) = \langle n+1|H|n\rangle = t$ is the hopping integral between the nearest QDs, and it is independent of spin polarization, except for $n = N_{\rm R}, N_{\rm R}-1$ and $n = -N_{\rm L}-1$, $-N_{\rm L}$ where the Zeeman energy-level splitting induced by the magnetic fields leads to the imbalance of couplings between electrons with opposite spin-polarization that $t_{N_{\rm R},N_{\rm R}+1}(\sigma) = V_{\rm R}(\sigma)$, $t_{-N_{\rm L}-1,-N_{\rm L}}(\sigma) = V_{\rm L}(\sigma)$ and

$t_{-N_{\rm L},-N_{\rm L}+1}(\sigma) = t_{N_{\rm R}-1,N_{\rm R}}(\sigma) = t'_\sigma$, which depend on the spin-polarization. ε_j with $1 \le j \le N$, denotes the energy eigenvalues on jth QD in the AB ring with N being the total number of the QDs; $c^+_{j,\sigma}$ and $t_{\rm c}(\sigma)$ are the corresponding creation operator and hopping integral respectively. The magnetic flux is measured in terms of the quantum flux unit $\Phi_0 = hc/e$. $\gamma = 2\pi\alpha/N$ is the phase of electron wavefunction acquired by hopping between adjacent QDs on the AB ring with $\alpha = \Phi/\Phi_0$, where Φ is the magnetic flux. Here we have worked on the symmetric gauge. $V_0(\sigma)$ is the tunnel coupling between the AB-ring,[34] and the QD-array.[35] The eigenstates of the whole Hamiltonian can be written as

$$|\Psi\rangle = \sum_{n=-\infty,\sigma}^{\infty} C_{n,\sigma} a^+_{n,\sigma}|0\rangle + \sum_{j=1,\sigma}^{N} b_{j,\sigma} c^+_{j,\sigma}|0\rangle. \quad (3)$$

The expansion coefficients, $\{C_{n,\sigma}\}$, satisfy the recurring equation

$$(E-\varepsilon'_n)C_{n,\sigma} + t_{n-1,n}(\sigma)C_{n-1,\sigma} + t_{n,n+1}(\sigma)C_{n+1,\sigma}, \quad (4)$$

$$(E-\varepsilon'_{-1})C_{-1,\sigma} = -V_0(\sigma)b_{L,\sigma} - t_{-2,-1}(\sigma)C_{-2,\sigma},$$

$$(E-\varepsilon'_0)C_{0,\sigma} = -V_0(\sigma)b_{R,\sigma} - t_{0,1}(\sigma)C_{1,\sigma}, \quad (5)$$

while the coefficients, $\{b_{j,\sigma}\}$, satisfy the equation

$$(E-\varepsilon_j)b_{j,\sigma} + t_{\rm c}(\sigma)\mathrm{e}^{-\mathrm{i}\gamma}b_{j-1,\sigma} + t_{\rm c}(\sigma)\mathrm{e}^{\mathrm{i}\gamma}b_{j+1,\sigma} = 0, \quad j \ne L, R,$$

$$(E-\varepsilon_L)b_{L,\sigma} + t_{\rm c}(\sigma)\mathrm{e}^{-\mathrm{i}\gamma}b_{L-1,\sigma} + t_{\rm c}(\sigma)\mathrm{e}^{\mathrm{i}\gamma}b_{L+1,\sigma} + V_0(\sigma)C_{-1,\sigma} = 0,$$

$$(E-\varepsilon_R)b_{R,\sigma} + t_{\rm c}(\sigma)\mathrm{e}^{-\mathrm{i}\gamma}b_{R-1,\sigma} + t_{\rm c}(\sigma)\mathrm{e}^{\mathrm{i}\gamma}b_{R+1,\sigma} + V_0(\sigma)C_{0,\sigma} = 0. \quad (6)$$

We assume that there are n QDs $(1, 2, \ldots, n)$ on the up-arc and m QDs $(-1, -2, \ldots, -m)$ on the down-arc of the AB ring, so this AB ring consists of $n + m + 2$ QDs. Equation (6) can be written matrix-wise as follows:

$$\begin{bmatrix} b_{R,\uparrow} \\ b_{n,\uparrow} \\ b_{R,\downarrow} \\ b_{n,\downarrow} \end{bmatrix} = M(j,E)^n \begin{bmatrix} b_{1,\uparrow} \\ b_{L,\uparrow} \\ b_{1,\downarrow} \\ b_{L,\downarrow} \end{bmatrix}, \quad (7)$$

where $M(j,E)$ is a transfer matrix in the quantum ring defined as

$$M(j,E) = \begin{bmatrix} -\dfrac{E-\varepsilon_j}{t_{\rm c}(\sigma)}\mathrm{e}^{-\mathrm{i}\gamma} & -\mathrm{e}^{-2\mathrm{i}\gamma} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -\dfrac{E-\varepsilon_j}{t_{\rm c}(-\sigma)}\mathrm{e}^{-\mathrm{i}\gamma} & -\mathrm{e}^{-2\mathrm{i}\gamma} \\ 0 & 0 & 1 & 0 \end{bmatrix}. \quad (8)$$

Then we can obtain $b_{1,\uparrow}$ and $b_{1,\downarrow}$

$$b_{1,\uparrow} = \frac{1}{M^n_{11}}C_{R,\uparrow} - \frac{M^n_{12}}{M^n_{11}}C_{L,\uparrow}, \qquad b_{1,\downarrow} = \frac{1}{M^n_{33}}C_{R,\downarrow} - \frac{M^n_{34}}{M^n_{33}}C_{L,\downarrow}, \quad (9)$$

similarly

$$b_{-1,\uparrow} = \frac{1}{(M^m_{11})^*}C_{R,\uparrow} - \frac{(M^m_{12})^*}{(M^m_{11})^*}C_{L,\uparrow}, \qquad b_{-1,\downarrow} = \frac{1}{(M^m_{33})^*}C_{R,\downarrow} - \frac{(M^n_{34})^*}{(M^m_{33})^*}C_{L,\downarrow}. \quad (10)$$

From Eqs.(6),(9) and (10), we obtain

$$\begin{bmatrix} C_{0,\uparrow} \\ b_{R,\uparrow} \\ C_{0,\downarrow} \\ b_{R,\downarrow} \end{bmatrix} = M(R,E)M(L,E) \begin{bmatrix} b_{L,\uparrow} \\ C_{-1,\uparrow} \\ b_{L,\downarrow} \\ C_{-1,\downarrow} \end{bmatrix} = P \begin{bmatrix} b_{L,\uparrow} \\ C_{-1,\uparrow} \\ b_{L,\downarrow} \\ C_{-1,\downarrow} \end{bmatrix}, \quad (11)$$

where the transfer matrices of the site L and site R, i.e. $M(R,E)$ and $M(L,E)$, have the following form:

$$M(R,E)=\begin{bmatrix} -\dfrac{E-\varepsilon'_R}{V_0(\uparrow)} & -\dfrac{t_R}{V_0(\uparrow)} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -\dfrac{E-\varepsilon'_R}{V_0(\uparrow)} & -\dfrac{t_R}{V_0(\downarrow)} \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

and

$$M(L,E)=\begin{bmatrix} -\dfrac{E-\varepsilon'_L}{t_L} & -\dfrac{V_0(\uparrow)}{t_L} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -\dfrac{E-\varepsilon'_L}{t_L} & -\dfrac{V_0(\downarrow)}{t_L} \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{12}$$

where ε'_L and ε'_R are effective on-site energies; t_L, and t_R are the effective hopping integrals

$$\varepsilon'_L=\varepsilon'_R=\varepsilon-t_{\rm c}(\sigma){\rm e}^{{\rm i}\gamma}\frac{M^n_{12}}{M^n_{11}}-t_{\rm c}(\sigma){\rm e}^{-{\rm i}\gamma}\frac{(M^m_{12})^*}{(M^m_{11})^*},$$

$$t_L=\frac{t_{\rm c}(\sigma){\rm e}^{{\rm i}\gamma}}{M^n_{11}}+t_{\rm c}(\sigma){\rm e}^{-{\rm i}\gamma}\frac{1}{(M^m_{11})^*},\qquad t_R=(t_L)^*. \tag{13}$$

From the above equations, we can see that matrix P transfers the coefficients of the electron wave from the site- -1 on the left-arm of QD array to the site- 0 on the right-arm. Rewrite Eqs.(4) and (5) matrix-wise, then we will have

$$\begin{bmatrix} C_{n+1,\sigma} \\ C_{n,\sigma} \\ C_{n+1,-\sigma} \\ C_{n,-\sigma} \end{bmatrix}=M(n,E)\begin{bmatrix} C_{n,\sigma} \\ C_{n-1,\sigma} \\ C_{n,-\sigma} \\ C_{n-1,-\sigma} \end{bmatrix}, \tag{14}$$

where $M(n,E)$ is called the scattering matrix which links the expansion coefficient vector $(C_{n+1,\sigma},C_{n,\sigma},C_{n+1,-\sigma},C_{n,-\sigma})^{\rm T}$ to the vector $(C_{n,\sigma},C_{n-1,\sigma},C_{n,-\sigma},C_{n-1,-\sigma})^{\rm T}$. Thus it is also called the transfer matrix defined as

$$M(n,E)=\begin{bmatrix} -\dfrac{E-\varepsilon'_n}{t_{n,n+1}(\sigma)} & -\dfrac{t_{n-1,n}(\sigma)}{t_{n,n+1}(\sigma)} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -\dfrac{E-\varepsilon'_n}{t_{n,n+1}(-\sigma)} & -\dfrac{t_{n-1,n}(-\sigma)}{t_{n,n+1}(-\sigma)} \\ 0 & 0 & 1 & 0 \end{bmatrix},\quad n\neq N_{\rm R}, \tag{15}$$

$$M(N_{\rm R},E)=\begin{bmatrix} -\dfrac{E-\varepsilon'_{N_{\rm R}}(+)}{t_{N_{\rm R},N_{\rm R}+1}(\sigma)} & -\dfrac{t_{N_{\rm R}-1,N_{\rm R}}(\sigma)}{t_{N_{\rm R},N_{\rm R}+1}(\sigma)} & \dfrac{g\mu_{B'}B'\sin\theta}{t_{N_{\rm R},N_{\rm R}+1}(\sigma)} & 0 \\ 1 & 0 & 0 & 0 \\ \dfrac{g\mu_{B'}B'\sin\theta}{t_{N_{\rm R},N_{\rm R}+1}(-\sigma)} & 0 & -\dfrac{E-\varepsilon'_{N_{\rm R}}(-)}{t_{N_{\rm R},N_{\rm R}+1}(-\sigma)} & -\dfrac{t_{N_{\rm R}-1,N_{\rm R}}(-\sigma)}{t_{N_{\rm R},N_{\rm R}+1}(-\sigma)} \\ 0 & 0 & 1 & 0 \end{bmatrix},\qquad (n=N_{\rm R}), \tag{16}$$

where $\varepsilon'_{(n,0)}$ is the effective on-site energy given by

$$\varepsilon'_n=\varepsilon_n,\qquad n\neq -N_{\rm L},N_{\rm R}, \tag{17}$$

$$\varepsilon'_{-N_{\rm L}}=\varepsilon_{-N_{\rm L}}+\sigma g\mu_B B,\quad n=-N_{\rm L},\quad \varepsilon'_{N_{\rm R}}(\pm)=\varepsilon_{N_{\rm R}}\pm g\mu_{B'}B'\cos\theta,\quad n=-N_{\rm R}. \tag{18}$$

The wavefunction of the electron in the left-electrode is a superposition of incoming and reflecting plane waves and in the right-electrode there is only the transmissive wave with amplitude t_σ. Taking this into account we obtain

$$C_{n,\sigma}={\rm e}^{{\rm i}K^{\rm L}_\sigma na}+r_\sigma{\rm e}^{-{\rm i}K^{\rm L}_\sigma na},\qquad n\leq-(N_{\rm L}+1),\qquad C_{n,\sigma}=t_\sigma{\rm e}^{{\rm i}K^{\rm L}_\sigma na},\qquad n\geq N_{\rm R}+1. \tag{19}$$

It can be shown that the transmissive amplitude t_σ is related to the incident coefficients by the equation

$$\begin{bmatrix} t_\sigma \\ 0 \\ t_{-\sigma} \\ 0 \end{bmatrix} = T(E) \begin{bmatrix} 1 \\ r_\sigma \\ 1 \\ r_{-\sigma} \end{bmatrix}, \tag{20}$$

where the transfer matrix $T(E)$ is given by

$$\begin{aligned} T(E) = & \begin{bmatrix} \mathrm{e}^{-\mathrm{i}K_\sigma^{\mathrm{R}}(N_{\mathrm{R}}+1)a} & 0 & 0 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}K_\sigma^{R}(N_{\mathrm{R}}+1)a} & 0 & 0 \\ 0 & 0 & \mathrm{e}^{-\mathrm{i}K_{-\sigma}^{\mathrm{R}}(N_{\mathrm{R}}+1)a} & 0 \\ 0 & 0 & 0 & \mathrm{e}^{\mathrm{i}K_{-\sigma}^{\mathrm{R}}(N_{\mathrm{R}}+1)a} \end{bmatrix} \\ & \times \begin{bmatrix} \mathrm{e}^{\mathrm{i}k_\sigma^{\mathrm{R}}a} & \mathrm{e}^{-\mathrm{i}k_\sigma^{\mathrm{R}}a} & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & \mathrm{e}^{\mathrm{i}k_{-\sigma}^{\mathrm{R}}a} & \mathrm{e}^{-\mathrm{i}k_{-\sigma}^{\mathrm{R}}a} \\ 0 & 0 & 1 & 1 \end{bmatrix}^{-1} \prod_{n=N_{\mathrm{R}}+1}^{0} M(n,E) M(R,E) M(L,E) \prod_{n=-1}^{-N_{\mathrm{L}}-1} M(n,E) \\ & \times \begin{bmatrix} \mathrm{e}^{\mathrm{i}k_\sigma^{\mathrm{L}}a} & \mathrm{e}^{-\mathrm{i}k_\sigma^{\mathrm{L}}a} & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & \mathrm{e}^{\mathrm{i}k_{-\sigma}^{\mathrm{L}}a} & \mathrm{e}^{-\mathrm{i}k_{-\sigma}^{\mathrm{L}}a} \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{e}^{-\mathrm{i}K_\sigma^{\mathrm{L}}(N_{\mathrm{L}}+1)a} & 0 & 0 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}K_\sigma^{\mathrm{L}}(N_{\mathrm{L}}+1)a} & 0 & 0 \\ 0 & 0 & \mathrm{e}^{-\mathrm{i}K_{-\sigma}^{\mathrm{L}}(N_{\mathrm{L}}+1)a} & 0 \\ 0 & 0 & 0 & \mathrm{e}^{\mathrm{i}K_{-\sigma}^{\mathrm{L}}(N_{\mathrm{L}}+1)a} \end{bmatrix}. \end{aligned} \tag{21}$$

So from Eq.(20) we can obtain the reflection amplitudes $r_\uparrow$ and $r_\downarrow$ as

$$\begin{aligned} r_\uparrow &= \frac{-\left(\dfrac{T_{21}}{T_{22}} + \dfrac{T_{23}}{T_{22}}\right) + \dfrac{T_{24}}{T_{22}}\left(\dfrac{T_{41}}{T_{44}} + \dfrac{T_{43}}{T_{44}}\right)}{1 - \dfrac{T_{24}}{T_{22}}\dfrac{T_{42}}{T_{44}}}, \\ r_\downarrow &= \frac{-\left(\dfrac{T_{41}}{T_{44}} + \dfrac{T_{43}}{T_{44}}\right) + \dfrac{T_{42}}{T_{44}}\left(\dfrac{T_{21}}{T_{22}} + \dfrac{T_{23}}{T_{22}}\right)}{1 - \dfrac{T_{24}}{T_{22}}\dfrac{T_{42}}{T_{44}}}, \end{aligned} \tag{22}$$

and the transmission amplitudes $t_\uparrow$ and $t_\downarrow$

$$\begin{aligned} t_\uparrow(E) &= T_{11} + T_{12}r_\uparrow + T_{13} + T_{14}r_\downarrow, \\ t_\downarrow(E) &= T_{31} + T_{32}r_\uparrow + T_{33} + T_{34}r_\downarrow, \end{aligned} \tag{23}$$

while the spin-polarization dependent reflection and transmission probabilities are

$$R_\sigma(E) = \frac{1}{2}|r_\sigma|^2, \quad T_\sigma(E) = \frac{1}{2}\frac{V_{\mathrm{R},\sigma}}{V_{\mathrm{L},\sigma}}|t_\sigma|^2. \tag{24}$$

The total transmissive and reflection probabilities are defined as

$$\begin{aligned} T_{\mathrm{tot}}(E) &= T_\uparrow(E) + T_\downarrow(E), \\ R_{\mathrm{tot}}(E) &= R_\uparrow(E) + R_\downarrow(E). \end{aligned} \tag{25}$$

And the spin current is related to the net spin-polarization transmission probability by

$$\Delta T(E) = T_\uparrow(E) - T_\downarrow(E). \tag{26}$$

The conductance of the device can be derived from the transmission probability with the help of the Laudauer–Büticker formula

$$G = \left(\frac{2e^2}{h}\right)T, \tag{27}$$

where T is the transmission probability.

3. Numerical result and discussion

In the numerical evaluation we assume the hopping integral values to be $t(\sigma) = t = V_{\mathrm{c}}(\sigma) = V_0(\sigma) = 1$ and $t'_\sigma = t' = 0.8t$; $V_{L,\sigma} = V_{R,\sigma} = 0.5t$, and eight QDs in the quantum ring ($N = 8$). Figure 2 shows the relative transmission probability between the spin-up and the spin-down components, i.e. the spin transition probability, as a function of energy E with ε_n $(-N_{\mathrm{L}} \le n \le N_{\mathrm{R}}) = \varepsilon_j$ $(1 \le j \le N) = \varepsilon_{\mathrm{L}} = \varepsilon_{\mathrm{R}} = 2t$, and $g\mu_B B = g\mu_{B'}B' = 1t$, $\theta = \pi/2$. The solid line is for the magnetic flux of $\alpha = 0.3$ and the dotted line is for $\alpha = 0.8$. It can be seen that the spin transition spectrum is modulated strongly by the magnetic flux. In some energy range (for example, the electron energy is between $1.24t$ and $1.41t$) the output spin current defined by $\Delta T(E) = T_\uparrow(E) - T_\downarrow(E)$ is positive (i.e. the spin current with spin-up polarization)

for the magnetic flux $\alpha = 0.8$; while $\Delta T(E)$ is negative (namely the spin-down current) for $\alpha = 0.3$. The phenomenon that the spin polarization can be turned over by the magnetic flux is due to the mutual interference between spin-up and down currents and is the most interesting observation in our study. Moreover, the spin current spectrum is influenced strongly by the positions of the two QD-array arms seen from Fig.3 where the solid line is the spectrum with only one QD in the up-arc ($n = 1$) of AB ring and five QDs in the down-arc ($m = 5$) for the fixed magnetic flux value $\alpha = 0.8$ and the dotted-line is the plot for $n = 2$ and $m = 4$. It can be seen that when the electron energy is between $1.53t$ and $1.88t$, the output spin current $\Delta T(E)$ is negative in the former while positive in the latter.

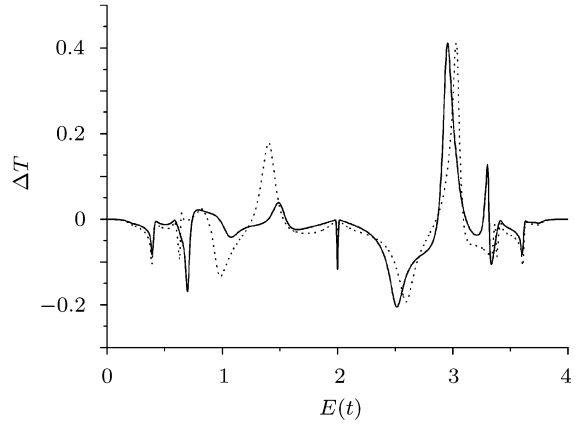


Fig.2. Spin transmission probability as a function of energy for $n = m = 3$; $\alpha = 0.3$ (solid line), $\alpha = 0.8$ (dotted line).

Figure 4(a) is the plot of spin transmission probability ΔT as a function of magnetic flux for the fixed energy $E = 0.5t$ for a symmetric AB ring i.e. $n = m$. Obviously, the spin transmission probability is a periodic function of magnetic flux, indicating that the AB oscillation is similar to the result of charge transport in Ref.[22]. For an unsymmetrical AB ring the period of AB oscillation is not affected by the position of two arms seen from Fig.4(b) where the solid line denotes the plot of the case with $n = 1$ and $m = 5$; the dotted line is for the plot with $n = 2$, $m = 4$. We have shown that the quantum phase interference effect is much richer in the quantum transport when the spin degree is awoke since both the spin-up and spin-down components are involved in the AB interference. In our model the spin-flip is induced by two unparalleled magnetic fields applied to two single QDs for the simplicity of analysis. As a matter of fact, the results are qualitatively the same when the magnetic fields are separately applied to the two QD-array arms, and the latter is easier to manipulate technically.

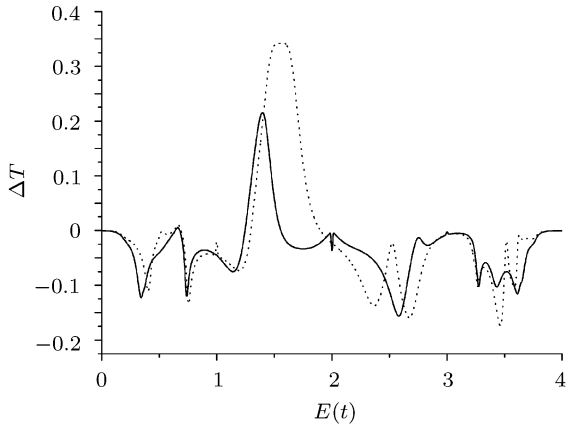


Fig.3. Spin transmission probability as a function of energy for $n = 1$, $m = 5$ (solid line), $n = 2$, $m = 4$ (dotted line).

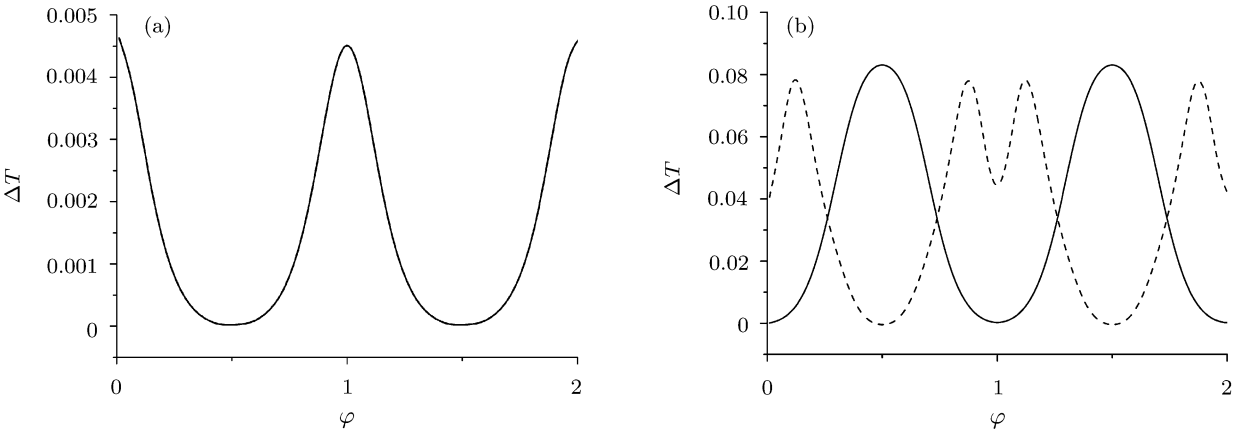


Fig.4. Spin transmission probability as a function of magnetic flux α for a symmetric AB-ring $n = m = 3$ (a); $n = 1$, $m = 5$ (solid line), and $n = 2$, $m = 4$ (dotted line) (b).

4. Summary and conclusion

The quantum transport combining the AB effect with the spin degree displays richer quantum interference phenomena. Although the AB oscillation is the same as in the case of charge transport, the tunable spin polarization of the current by the magnetic flux is a new observation and is particularly useful in the manipulation of spintronic devices.

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