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New expressions for the surface roughness length and displacement height in the atmospheric boundary layer*

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An alternative model for the prediction of surface roughness length is developed. In the model a new factor is introduced to compensate for the effects of wake diffusion and interactions between the wake and roughness obstacles. The experiments are carried out by the use of the hot wire anemometry in the simulated atmospheric boundary layer in a wind tunnel. Based on the experimental data, a new expression for the zero-plane displacement height is proposed for the square arrays of roughness elements, which highlights the influence of free-stream speed on the roughness length. It appears that the displacement height increases with the wind speed while the surface roughness length decreases with Reynolds number increasing. It is shown that the calculation results based on the new expressions are in reasonable agreement with the experimental data.

Keywords: roughness length, zero-plane displacement height, atmospheric boundary layer, free stream speed, packing density

PACC: 4725F, 9260F

1. Introduction

Boundary layer flows are very common and important.[1−3] The turbulence structure of air flow over a rough surface differs from that in a smooth-wall boundary layer in that the boundary layer depending on the surface roughness is characterized by a surface layer that can be subdivided into two parts: an inertial sublayer and a roughness sublayer.[4] It is now well received that, in the inertial sublayer, the vertical variation of the wind speed under neutral stability obeys the semi-logarithmic law [5−12]

$$\frac{U}{u_*} = \frac{1}{k}\ln\left(\frac{z-d}{z_0}\right), \tag{1}$$

where U is the average speed in the stream-wise direction, u_* is the friction speed, k is the von Karman constant, z is the vertical position coordinate, d is the zero-plane displacement height, and z_0 is the surface roughness length.

Consequently, the difficulties arise in determining the roughness length and the displacement height due to the complexities and uncertainties caused by roughness elements. A number of methods have been developed to estimate surface roughness,[10,13−15] most of which hinge on the preliminarily assumed value of the displacement height d, defined as a mean value of momentum absorption.[16]

However, as Raupach *et al*[4] have pointed out, the zero-plane displacement height might be well underestimated owing to the wake diffusion. This gives a clue that the wind speed might play an important role in estimating the surface roughness and displacement height. Iyengar and Farell[17] have noted the dependence of surface roughness and displacement height on the outer flow. Some other researchers have proposed that the arrays of roughness obstacles have a direct impact on the developing of the whole boundary layer.[18] Therefore, there maybe exist some inherent correlations among the aerodynamical parameters, the roughness geometry and the free-stream speed of air flow. In the present study, we further derive the new expressions for the surface roughness length and displacement height based on the experimental data, with the influence exerted by wind speed in addition to the packing densities of the roughness obstacles taken into consideration.

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2. Theoretical formulation

Generally, we assume that the roughness obstacles are three-dimensional with height h (in the vertical direction), breadth b (in the traverse direction) and length l (in the streamwise direction).The turbulent flow over the roughness surface covered uniformly by these obstacles is travelling at a constant speed of $U_{\rm e}$. To simulate the neutral atmospheric boundary layer, we also postulate that the equilibrium of the boundary layer is maintained and the total thickness of the boundary varies only slightly in the streamwise direction.[1]

Using the expressions from Ref.[15, 19], the frontal area density $\lambda_{\rm f}$ and plan area density $\lambda_{\rm p}$ can be respectively written as

$$\lambda_{\rm f} = \frac{A_{\rm f}}{A_{\rm d}}, \tag{2}$$

$$\lambda_{\rm p} = \frac{A_{\rm p}}{A_{\rm d}}, \tag{3}$$

where $A_{\rm f}$ is the total frontal area of the obstacles, $A_{\rm d}$ is the total area covered by obstacles, and $A_{\rm p}$ is the total plan area of roughness –obstacle.

We start from the mean speed $U_{\rm h}$ at the top of the surface obstacles of height h:

$$U_{\rm h} = \frac{u_*}{k} \ln\left(\frac{h-d}{z_0}\right), \tag{4}$$

which is similar to Eq.(1). It seems that expression (4) needs some adjustments because the semi-logarithmic law is usually valid above $2h$ from the floor of the wind tunnel.[4,20] In addition, the friction speed at the top of an obstacle may be different from that at a vertical height of $2h$ according to the classical boundary layer theory.[21−23] As observed by some researchers,[4,17] the fluid Reynolds stress usually reaches the peak just above the roughness obstacles and then decreases and changes only slightly in a constant stress region. Hence, a new factor α is introduced here to compensate for the difference mentioned above, and expression (4) becomes

$$\frac{U_{\rm h}}{\alpha u_*} = \frac{1}{k} \ln\left(\frac{h-d}{z_0}\right). \tag{5}$$

With reference to the expressions from Ref.[24], the drag of the obstacle, $F_{\rm D}$, has the form of

$$F_{\rm D} = 0.5\rho U_{\rm h}^2 C_{\rm D} A_{\rm f}, \tag{6}$$

where ρ is the density of fluid and $C_{\rm D}$ is the drag coefficient.

Assume that the overall surface shear is dominated by the drag on the surface obstacles and the contribution of the ground shear stress between the obstacles is reasonably neglected,[15,16] then the relationship between the surface shear stress τ_0 and the friction speed will be established as

$$\tau_0 = \frac{F_{\rm D}}{A_{\rm d}} = \rho u_*^2. \tag{7}$$

Combining expressions (6) and (7) yields

$$\frac{u_*}{U_{\rm h}} = \left(0.5 C_{\rm D} \frac{A_{\rm f}}{A_{\rm d}}\right)^{\frac{1}{2}}. \tag{8}$$

Substituting $U_{\rm h}/u_*$ into expression (5), the final form for non-dimensional roughness length can be obtained as

$$\frac{z_0}{h} = \left(1 - \frac{d}{h}\right) \exp\left[-\left(0.5\frac{\alpha^2}{\kappa^2} C_{\rm D} \lambda_{\rm f}\right)^{-0.5}\right]. \tag{9}$$

Now the problems arise in determining the non-dimensional displacement height d/h and the wake factor α.

3. Experimental setup and procedure

In order to determine the displacement height and wake factor, the experiments were conducted in a closed-loop wind tunnel. A 0.6 m wide and 0.9 m long flat board was located in the central part of the working section with a diameter of 0.7 m and a length of 1 m as shown in Fig.1. The gap between the edge of the board and the tunnel surface was filled with a kind of polymer and sealed carefully with the tape in order to prevent the wind from leaking. The rotation speed of the fan was controlled by a speed controller so that the speed of the airflow in the wind tunnel could change in a range from 0 to approximately 15 m/s. We started from the equation of momentum conservation and deduced the equations for the design of the spires and the boundary layers suitable for our wind tunnel. As shown in Fig.1, triangular spires of elevation $H =$ 100 mm were used as turbulence generators positioned symmetrically at the entrance to the working section for artificially generating thick boundary layer. Combined with the spires, the three-dimensional obstacles with the frontal area $h \times b$ of 12 mm by 12 mm and the thickness l of 2 mm were glued uniformly to the whole floor of the flat board in order to simulate the planetary boundary layer.

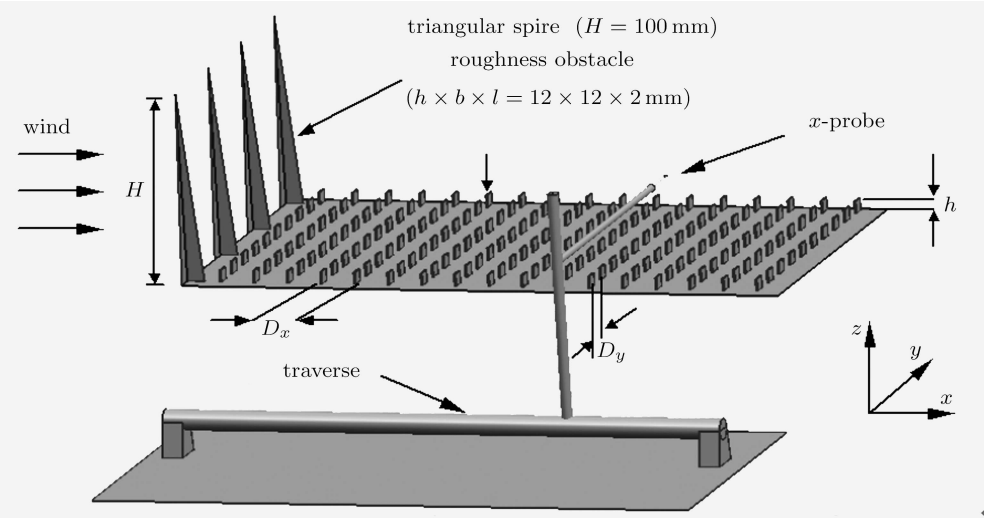


Fig.1. Schematic diagram of the simulation system.

The regular square arrays were adopted with three different frontal packing densities of 0.0156, 0.0625, and 0.25, denoted as RG1, RG2 and RG3, respectively. The corresponding plan packing densities are 0.0026, 0.0104 and 0.0417, respectively.

The speed measurements were carried out by the hot-wire anemometry (HWA) with a x-probe installed at the support rod of a traversing system that could smoothly move in three directions: laterally, vertically and longitudinally. The lowest position that the x-probe reached was half the height of the roughness elements, and the measurements were performed at $1h$ behind the rows of the roughness elements, where h is the height of obstacles. The HWA system was optimized by using some techniques such as Automatic Overheat Adjustment and Square Wave Test. Speed calibration of the probe was performed within a range from 0.5 to 18 m/s, generating a maximum curve-fit error of 0.74%. Directional calibration was accomplished with the x-probe placed in a plane flow field of constant speed and rotated by a holder through a set of yaw angles from –30° to 30°.

We evaluated the performances of the wind tunnel through a series of test measurements. Then the whole simulation system was positioned in the wind tunnel. The spires-roughness technique could produce the required boundary layer at a distance $6\,h$ or even $4.5\,h$ downstream from the spires. Therefore, the probe was mounted at the location of 520 mm from the spire arrays.

The vertical profiles were achieved with the seventeen point measurements at heights of $z/h = 0.5$, 0.75, 1, 1.25, 1.5, 1.75, 2.0, 2.25, 2.5, 2.75, 3.0, 3.5, 4, 5, 6, 7, 8. At each point of measurement the speed was recorded at a sampling rate of 20 Hz, with the sampling time of 1 min selected to maintain the stability of the mean values for the speed and other turbulence profiles. The data obtained from the measurements were processed in Excel, and loaded into the Matlab for calculations. To remove the lateral variability, the measurements were iterated at three different positions along the y-direction: the central line of the obstacle columns, the edge-line of the obstacle columns and the central line of the space between the two columns of the obstacles.

The measurements were first performed on the roughness arrays with $D_x = 4\,h$ and $D_y = 3\,h$ that yield the frontal packing density of 0.0625. Then the roughness concentration was doubled and the procedures were repeated. Finally, the experimental data were obtained from the roughness geometry of $D_x = 8\,h$ with $D_y = 7\,h$. For each roughness pattern, different measurements were achieved at different free stream speeds of wind.

4. Results and discussion

Since it is now widely received that the value of d cannot be obtained with a high accuracy by using the experimental methods,[16] the experimentally determined values of the displacement height d are spread in a range and cannot be expressed as a single definitive value. All these values are within an acceptable error. Based on the experimental data, the expression

for the displacement height d/h is established from our experimental dataset as follows:

$$\frac{d}{h} = \left(\frac{U_{\rm e}}{30}\right)^{(1-\lambda_{\rm p})} \lambda_{\rm p}^{0.05}, \tag{10}$$

where $U_{\rm e}$ is the free stream speed of wind.

The experimentally determined values of non-dimensional displacement height d/h are shown in Fig.2 with two dashed lines representing the up-limits and low-limits of wind tunnel data, respectively. The calculation results from expression (10) are also drawn in Fig.2. It can be seen that the predictions from the model for the displacement height are in reasonable agreement with the wind tunnel data and the displacement height increases significantly with the wind speed.

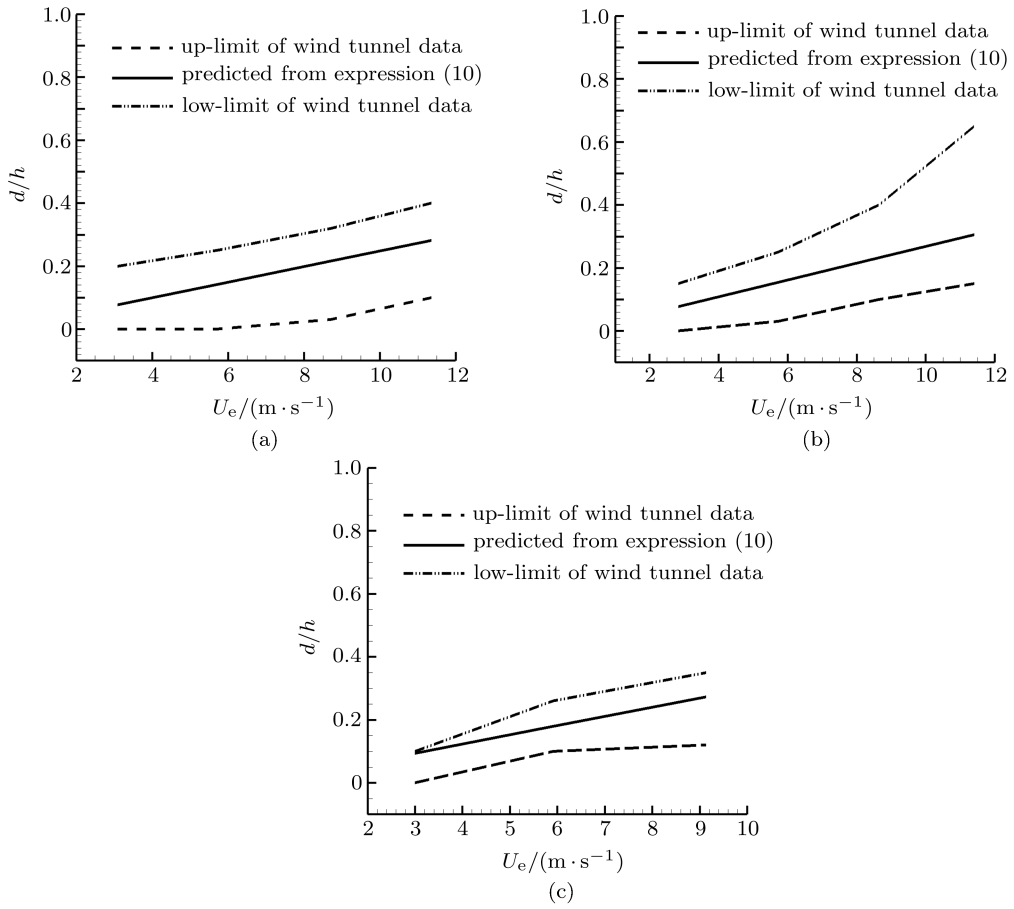


Fig.2. Relationships of displacement height and wind speed for the three roughness arrangements. (a) Plan packing density $\lambda = 0.0026$. (b) Plan packing density $\lambda = 0.01$. (c) Plan packing density $\lambda = 0.042$.

For a comparison, the experimentally determined displacement heights from Ref.[4], together with the calculations from expression (10), are drawn in Fig.3(a). In the experiment of Ref.[4], the wind speed of free stream was 17.5 m/s and the obstacle height was 6 mm. The roughness surfaces had two square arrangements with the values of the nearest-neighbour separation D being 40 mm and 20 mm, and consequently, the plan packing densities were 0.013 and 0.042, respectively. It is shown that the estimations from expression (10) are in reasonable agreement with the experimental data in Ref.[4]. It means that the model for the displacement height developed in this paper can be used in the other cases in which the air speed, packing density or roughness elements are different from the above.

Figure 3(b) shows a comparison between our calculations from expression (10) and the wind tunnel data from Iyengar and Farell's experiment,[17] in which both the frontal packing density and the plan packing density were 0.0784. Iyengar and Farell applied two different methods to dealing with the same set of experimental data which were obtained under three wind speeds of 11.9, 20.2 and 30 m/s, separately. Hence, method 3 was applied in this paper to the cases of wind speeds of 11.9 and 20.2 m/s while method 2 was

applied to the case of wind speed of 30 m/s. One can see that the calculations from expression (10) agree well with the wind tunnel data given by Iyengar and Farell. Therefore, it is concluded that, to some extent, expression (10) is a satisfactory representation for describing the influence of wind speed and plan packing density on the displacement height.

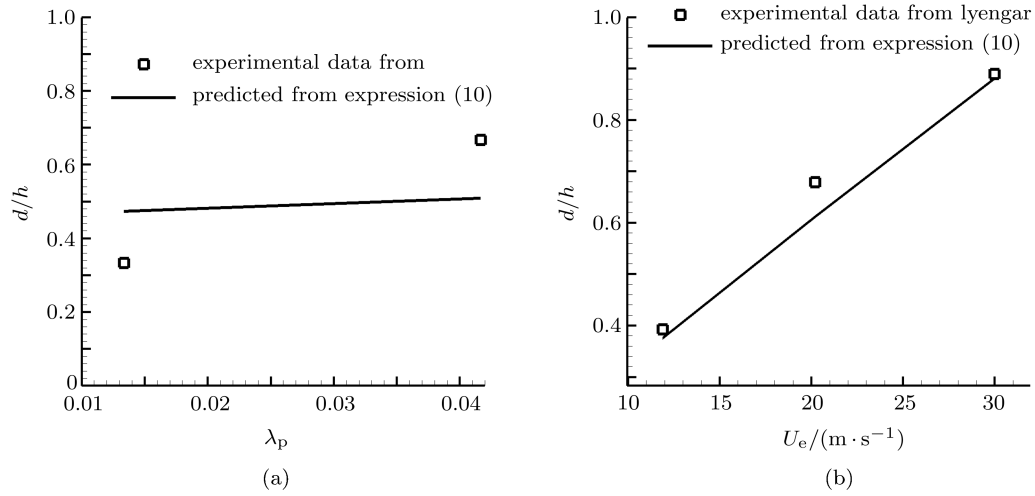


Fig.3. Comparisons between calculations and measurements. (a) Comparison between calculations from expression (10) and measurements from Ref.[4]. (b) Comparison between calculations from expression (10) and measurements from Ref.[17].

Since the flow field is very complicated owing to the existence of roughness obstacles, it is a difficult task to determine the newly introduced factor α. However, this problem can be simplified by noticing some dominant parameters. In general, the free stream speed and the packing density are mainly responsible for the wake diffusion and interaction between the wake and the roughness elements. By analysing our wind tunnel data, it is found that the wake factor α almost remains constant when wind speed goes up but changes with the packing density of roughness elements. The changing tendency of wake factor with the frontal packing density can be given as follows:

$$\alpha = \begin{cases} 1.02 + 5.12\lambda_{\mathrm{f}}, & \lambda_{\mathrm{f}} < 0.07, \\ 1.45 - 1.39\lambda_{\mathrm{f}}, & \lambda_{\mathrm{f}} \geq 0.07. \end{cases} \tag{11}$$

It is shown that the wake factor rises first and then decreases with the frontal packing density, which means that the influence of wake diffusion and interactions between the wake and the obstacles varies with packing density and probably reaches a peak at $\lambda_{\mathrm{f}} = 0.07$.

Figure 4 represents the calculations from expression (9) for the surface roughness length z_0/h. It is seen that the surface roughness lengths for all the three rough surfaces descend slightly with the wind speed. The dependence of roughness length on frontal packing density is also clearly shown in Fig.4, which is in accord with the results available from many other publications such as Refs.[4,15].

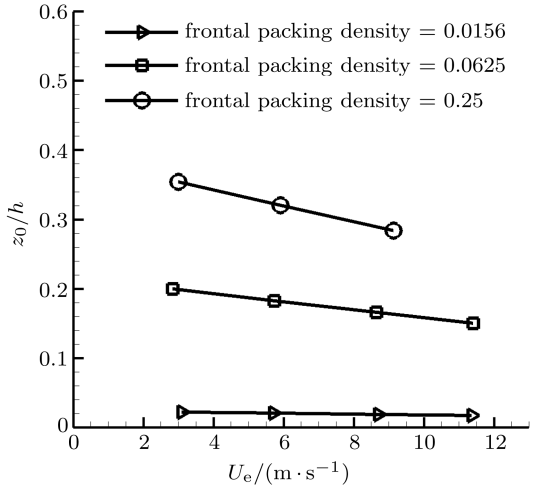


Fig.4. Dependence of surface roughness on wind speed and frontal packing density.

A comparison of predictions from expression (9) with the experimental data is shown in Fig.5. The maximum wind speed of 11.4 m/s in our experiment, and the wind tunnel data: 17.5 m/s taken from Ref.[4] and 11.9 m/s and 20.2 m/s cited from Ref.[17] are all drawn in Fig.5. The curve obtained from expression (9) is irregular, which is only because the variable

z_0/h has many independent variables such as packing density, displacement height, wind speed, etc. It is demonstrated that the model for z_0/h established in this paper yields almost the same values as the experimental data.

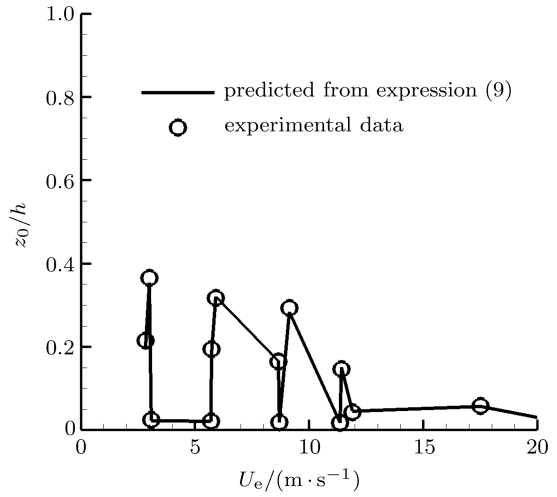


Fig.5. Comparison between calculations from expression (9) and measurements.

Based on the above results we make the following points.

(1) Models developed in this paper have some advantages in that they include the effects of wind speed in addition to the packing densities. In particular, an attempt has been made to separate the influences of different factors from each other, such as wind speed, frontal packing density, plan packing density, drag coefficient, wake diffusion on the displacement height and surface roughness length, though there may exist some correlations among these parameters.

(2) In the mathematical form of d/h, 30 m/s is adopted for non-dimensional purpose because the wind tunnel data from Ref.[17] indicate that the displacement height may not change when wind speed is high enough. Expression(10) satisfies the constraints $d/h = 0$ when $\lambda_p = 0$ and $d/h = 1$ when $\lambda_p = 1$ provided by Ref.[15].

(3) There are some differences between Macdonald *et al*'s models[15] and the models developed here. Firstly, model for roughness length of expression (9) adopts a wake factor to make up for the items neglected by Macdonald *et al.*[15] Secondly, expression (9) does not employ the concept of effective height proposed by Macdonald *et al.*[15] Finally, the characteristic of wake factor indicates that α is responsible for the appearance of a peak when z_0/h changes with the frontal packing density. However, the model of Macdonald *et al* implies that $(1 - d/h)$ mainly accounts for this phenomenon of peak in z_0/h.

5. Conclusions

The new expression for the surface roughness length is established in this paper, in which a new factor defined as a wake factor is introduced so that the impacts of the wake diffusion and interactions between the wake and the roughness elements can be analysed separately. On basis of our wind tunnel data, a starting effort to describe the qualitative characteristics of wake factor is made in this paper. In addition, a novel model for the zero-plane displacement height is presented here based on our experimental dataset, which has demonstrated that the wind speed has a significant influence on the displacement height. It is demonstrated that the surface roughness length for all the three rough surfaces declines slightly with the wind speed. The models presented in this paper are justified by the wind tunnel data from other researches, hence the wind speed applicable for these models can be enhanced up to 30 m/s.

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