Vol 16 No 7, July 2007
1009-1963/2007/16(07)/2023-05

Chinese Physics

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Effect of initial linear chirp on collision characteristics of two solitons in the birefringent fibre*

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(Received 9 July 2006; revised manuscript received 13 August 2006)

The collision characteristics of the orthogonally polarized solitons with initial linear frequency chirp in the linear birefringent fibre for $\beta_2 < 0$ are numerically studied. It is found that initial chirp changes the threshold value of solitons to form the bound-state in the birefringent fibre. The effect of initial positive chirp on the threshold value is more obvious than that of negative chirp. In the case of $\delta = 0.7$ and initial interval $2\tau_0 = 1.25$, the two solitons are mutually bound for $0.2 \leq C \leq 1$, and they do not form the bound-state for $-1 \leq C < 0.2$. Frequency shifts increase with the increase of chirp parameter C for $-1 \leq C < 0.2$, and have the oscillatory structure for $C \geq 0.2$. The effect of positive chirp on temporal FWHM is greater than that of negative chirp. The peak of temporal waveform oscillates with the propagation distance. The period and amplitude of the oscillation for the chirped case are greater than those for the unchirped case, and they vary with the increase of $|C|$. The peak of output temporal waveform can be controlled by changing the initial chirp.

Keywords: frequency chirp, solitons collision, linear birefringent fibre, soliton-bound state
PACC: 4281D, 4281F, 4265S

1. Introduction

Soliton collision in birefringent fibre has attracted attention for its importance in theory and practical applications of optical logic devices and polarization multiplexing systems.[1−14] The initial interval between two polarized solitons can be adjusted in practical applications. In an optical logic device, one soliton is the data signal, the other is the control signal. Whether two solitons are bound can be controlled by changing their initial interval and chirp in the transmitter. The time delay of signals for the soliton bound-state case is less than that for the unbound-state case; one can either receive the data signal or not by setting the corresponding delay in the receiver. In a polarization multiplexing system, two polarized soliton signals are independent and their initial interval and chirp can be set according to the practical applications. The soliton bound-state can result in higher bit rate. Menyuk[6−9] has derived a pair of coupled nonlinear differential equations to describe pulse propagation in birefringent fibres and investigated the stability of soliton propagation. The collision of unchirped solitons has been studied,[10−13] but the collision of chirped solitons has been little investigated. However, optical pulses generally have frequency chirp which has great effect on pulse propagation.[15−23] The frequency chirp can be controlled by changing the input current of laser or changing the length of input fibre, etc. In the paper, we numerically study the effects of initial linear chirp on collision characteristics of two orthogonally polarized solitons in a linear birefringent fibre for $\beta_2 < 0$, and compare them with the collision characteristics of unchirped solitons.

2. Theoretical model of soliton collision in the birefringent fibre

The soliton propagation in birefringent fibres is described by the normalized coupled nonlinear Schrödinger equations (CNLS):[6,7]

$$\mathrm{i}\Big(\frac{\partial u}{\partial \xi} + \delta\frac{\partial u}{\partial \tau}\Big) + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + (|u|^2 + B|v|^2)u = 0, \quad (1a)$$

$$\mathrm{i}\Big(\frac{\partial v}{\partial \xi} - \delta\frac{\partial v}{\partial \tau}\Big) + \frac{1}{2}\frac{\partial^2 v}{\partial \tau^2} + (|v|^2 + B|u|^2)v = 0, \quad (1b)$$

where u and v are the normalized complex amplitudes of the two orthogonally polarized solitons in the CNLS

*Project supported by the Research Foundation of Education Department of Shandong Province, China (Grant No J05C09) and supported by the Research Foundation of Liaocheng University, China.

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system, ξ is propagation distance normalized to the soliton period, τ is the time normalized to T_0 (half-width of the input pulse), δ is the group velocity mismatch parameter and B is the cross-phase modulation (XPM) coefficient where $0 \leq B \leq 1$. u is a slow soliton and v is a fast one for $\delta > 0$. Equations (1) reduce to the uncoupled NLS system for $B = 0$,[6] and reduce to the Manakov equations and have analytical solutions for $B = 1$.[6,24−28] However, Eqs.(1) are unintegrable for $B = 2/3$ in the linear birefringent fibre.[6−13] We numerically study the collision characteristics of the chirped solitons by use of the split-step Fourier-method (SSFM) according to Eqs.(1) for $B = 2/3$. The two orthogonally polarized input solitons are, respectively,

$$u(\xi = 0, \tau) = \mathrm{sech}(\tau + \tau_0)\exp\Big(-\frac{\mathrm{i}C(\tau + \tau_0)^2}{2}\Big) \quad (2a)$$

and

$$v(\xi = 0, \tau) = \mathrm{sech}(\tau - \tau_0)\exp\Big(-\frac{\mathrm{i}C(\tau - \tau_0)^2}{2}\Big), \quad (2b)$$

where C is the linear chirp parameter, $2\tau_0$ is the initial interval between the solitons. The dispersion and the nonlinearity effects can be taken into account respectively according to the SSFM algorithm when the pulse propagates from ξ to $\xi + h$ (h being a small distance). Then, it is simple to deal with the pulse propagation by use of the SSFM. To suppress the edge effect, we set a larger computational region $\tau = (-80, 80)$ and set the number of sampling to 512.

3. Effect of initial linear chirp on collision characteristics

3.1. Effect of chirp on variation of the interval between the two solitons

The interval between the two solitons is defined as

$$\Delta\tau(\xi) = \tau^u_{\max}(\xi) - \tau^v_{\max}(\xi), \quad (3)$$

where $\tau^u_{\max}(\xi)$ and $\tau^v_{\max}(\xi)$ are the positions at which $|u(\xi, \tau)|$ and $|v(\xi, \tau)|$ reach maximum value, respectively. We can obtain $\tau^u_{\max}(\xi) = -\tau^v_{\max}(\xi)$ from Eqs.(1) and (2). The slow soliton precedes the fast one when $\Delta\tau(\xi) < 0$. The two solitons will separate when $\Delta\tau(\xi)$ always increases with the increase of propagation distance. If $\Delta\tau(\xi)$ varies slower and slower with propagation distance and becomes finally stabilized at $\delta = \delta_{\mathrm{th}}$, δ_{th} is known as the threshold value of soliton bound-state. The two orthogonally polarized solitons can be mutually bound for $\delta \leq \delta_{\mathrm{th}}$, and separate from each other for $\delta > \delta_{\mathrm{th}}$. The threshold value δ_{th} of soliton bound-state in the birefringent fibre can be determined by the variation of the interval with propagation distance. We assume that the two input solitons partially separate and $2\tau_0 = 1.25$. Figure 1 shows the interval between two solitons varying with propagation distance (soliton periods ξ) in the case of $2\tau_0 = 1.25$. The dashed curve is for $\delta = 0.832$ and $C = 0.5$, and the solid curve for $\delta = 0.615$ and $C = 0$, and the dotted curve for $\delta = 0.46$ and $C = -0.5$. It is found from Fig.1 that $\delta_{\mathrm{th}} = 0.615$ for $C = 0$, $\delta_{\mathrm{th}} = 0.832$ for $C = 0.5$ and $\delta_{\mathrm{th}} = 0.46$ for $C = -0.5$. It shows that initial chirp changes the threshold value δ_{th} of soliton bound-state in the birefringent fibre. The positive chirp enhances the nonlinear effect and makes the threshold value δ_{th} larger, the negative one makes the threshold value δ_{th} smaller. This is important to applications in optical switches.

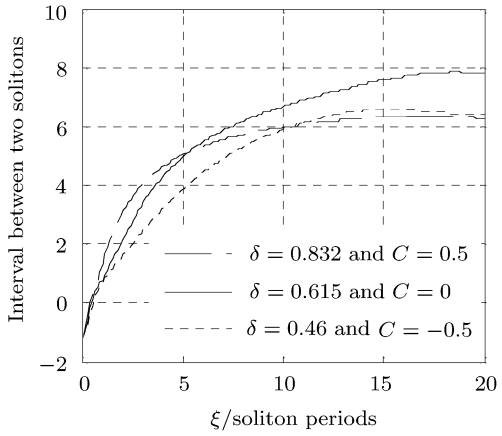


Fig.1. Variation of the interval between two solitons with propagation distance in the case of $2\tau_0 = 1.25$. The dashed curve is for $\delta = 0.832$ and $C = 0.5$, solid curve is for $\delta = 0.615$ and $C = 0$, dotted curve is for $\delta = 0.46$ and $C = -0.5$.

3.2. Effect of chirp on evolution of solitons mixing

The soliton mixing parameter $m(\xi)$ for describing collision characteristics is defined as

$$m(\xi) = \frac{\int_{-\infty}^{+\infty} |u(\xi, \tau)||v(\xi, \tau)|\mathrm{d}\tau}{\int_{-\infty}^{+\infty} |u(\xi = 0, \tau)|^2\mathrm{d}\tau}. \quad (4)$$

The $m(\xi)$ shows that the mixing degree of two solitons, which can vary from 0 to 1. Two identical solitons overlap exactly when $m(\xi) = 1$, completely separate when $m(\xi) = 0$, and trap each other to form a bound-state when $m(\xi)$ approaches 1. Figure 2 shows the

mixing parameter $m(\xi)$ of linear chirped solitons varying with the normalized propagation distance (soliton periods ξ) for $\delta = 0.7$ and $2\tau_0 = 1.25$. The curves (1–5) are for $C = -0.5$, -0.3, 0, 0.3 and 0.5, respectively. It is found from curve 4 that two solitons with $C = 0.3$ partially separate and $m = 0.78$ at $\xi = 0$, the fast soliton catches up with the slow one and $m \approx 1$ at $\xi \approx 0.5$. Then, they separate again and $m \approx 0.6$ at $\xi \approx 1$. The chirped soliton traps a small part of the other one and m increases with the increase of the propagation distance. Finally, the two chirped solitons are mutually bound and m oscillates about 0.9 for $\xi > 4$.

Two linear chirped solitons can be bound together with the increase of the propagation distance for $0.2 \le C \le 1$ when $\xi > 4$. The larger the value of C, the earlier the solitons are bound. Two chirped or unchirped solitons are separated from each other with the increase of the propagation distance for $-1 \le C < 0.2$ when $\xi > 4$.

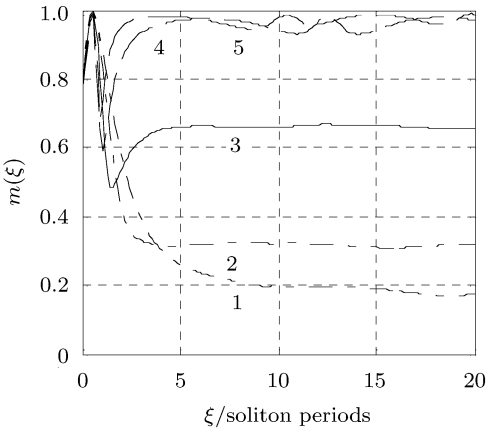


Fig.2. Variation of the parameter $m(\xi)$ of the solitons with propagation distance ξ for $\delta = 0.7$ and $2\tau_0 = 1.25$. The curves (1–5) are respectively for $C = -0.5$, -0.3, 0, 0.3 and 0.5.

3.3. Effect of chirp on variation of frequency centroid

The frequency centroids of the two solitons are defined as

$$\omega^u_{\text{cent}}(\xi) = \frac{\int_{-\infty}^{+\infty} \omega |u(\xi,\omega)|^2 \mathrm{d}\omega}{\int_{-\infty}^{+\infty} |u(\xi,\omega)|^2 \mathrm{d}\omega}, \tag{5a}$$

$$\omega^v_{\text{cent}}(\xi) = \frac{\int_{-\infty}^{+\infty} \omega |v(\xi,\omega)|^2 \mathrm{d}\omega}{\int_{-\infty}^{+\infty} |v(\xi,\omega)|^2 \mathrm{d}\omega}. \tag{5b}$$

We can obtain $\omega^u_{\text{cent}}(\xi) = -\omega^v_{\text{cent}}(\xi)$ from Eqs.(1), (2) and (5). Figure 3 shows the frequency centroids $\omega^u_{\text{cent}}(\xi)$ and $\omega^v_{\text{cent}}(\xi)$ of the solitons with different chirp parameter C varying with the normalized propagation distance for $\delta = 0.7$ and $2\tau_0 = 1.25$. Solid curves (1–5) show $\omega^u_{\text{cent}}(\xi)$ for $C = -0.5$, -0.3, 0, 0.3 and 0.5, dashed curves (6–10) show $\omega^v_{\text{cent}}(\xi)$ for $C = -0.5$, -0.3, 0, 0.3 and 0.5, respectively. It is found from Fig.3 that the frequency centroid $\omega^u_{\text{cent}}(\xi)$ of slow soliton shows to bathochromic shift before collision and changes to hypsochromic shift after collision because of the interaction between the birefringent effect and nonlinearity. Frequency centroid $\omega^v_{\text{cent}}(\xi)$ of fast soliton acts in the opposite way. As a result, the group velocities of the solitons change, the slow soliton becomes fast and the fast one becomes slow. Finally, the two chirped solitons propagate at same velocity for $C \ge 0.2$. Frequency shifts increase with the increase of chirp parameter C for $-1 \le C < 0.2$, and have a clear oscillatory structure for $C \ge 0.2$.

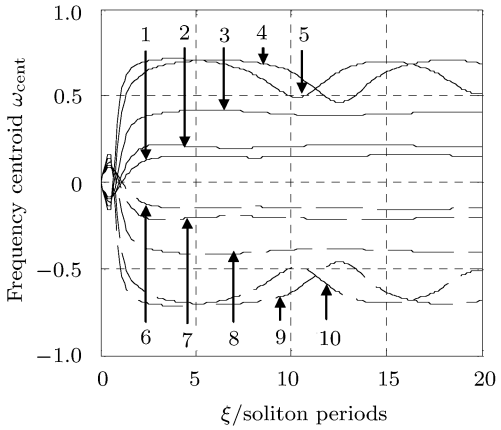


Fig.3. Variation of frequency centroid of the solitons with propagation distance for $\delta = 0.7$ and $2\tau_0 = 1.25$. The solid curves (1–5) show $\omega^u_{\text{cent}}(\xi)$ for $C = -0.5$, -0.3, 0, 0.3 and 0.5, dashed curves (1–5) show $\omega^v_{\text{cent}}(\xi)$ for $C = -0.5$, -0.3, 0, 0.3 and 0.5 respectively.

3.4. Effect of chirp on variation of temporal FWHM

T^u_{FWHM} and T^v_{FWHM} are respectively the temporal full width at half maximum (FWHM) of the two solitons. We obtain $T^u_{\text{FWHM}} = T^v_{\text{FWHM}}$ from Eqs.(1) and (2). Figure 4 shows the variation of temporal FWHM T^u_{FWHM} with propagation distance for $\delta = 0.7$ and $2\tau_0 = 1.25$. The curves (1–5) are for $C = -1$, -0.5, 0, 0.5 and 1, respectively. It can be found from numerical results that the pulse initially compresses near $\xi = 1$ for $0 \le C \le 1$ mainly because of cross-phase modulation (XPM), although positive chirp also enhances the effect. The temporal FWHM performs damped

oscillation with the propagation distance. The period and amplitude of the oscillation increase with the increase of $|C|$ for $-1 \leq C \leq 0.2$ and $0.6 \leq C \leq 1$, decrease with the increase of C for $0.2 < C < 0.6$. For a given magnitude of initial chirp, oscillatory amplitude of temporal FWHM for positive chirp is greater than that for negative one. It shows that the effect of positive chirp on temporal FWHM is greater than that of negative one. The variation of temporal FWHM is consistent with the prediction of the chirped solitons.

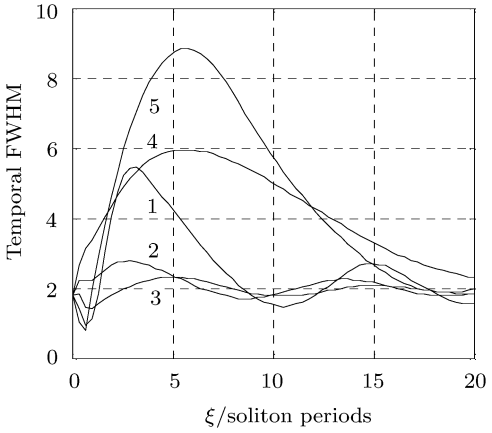


Fig.4. Variation of temporal FWHM with propagation distance for $\delta = 0.7$ and $2\tau_0 = 1.25$. The curves (1–5) are respectively for $C = -1$, -0.5, 0, 0.5 and 1.

3.5. Effect of chirp on evolution of temporal waveform

It can be obtained from Eqs.(1) and (2) that $u(\xi, \tau) = v(\xi, -\tau)$. Figure 5 shows that temporal waveform of the slow soliton $|u(\xi, \tau)|$ varying with propagation distance for $C = 0.5$, $\delta = 0.7$ and $2\tau_0 = 1.25$. That of the fast soliton $|v(\xi, \tau)|$ is its mirror image about $\tau = 0$. It can be found from numerical results that the chirped solitons with $C = 0.5$ are initially separated mainly because of initial position. Then, they are firstly compressed, and are gradually bound together after collision. The edges of temporal waveform produce obvious dispersive waves mainly because of XPM and initial chirp. The peak of temporal waveform oscillates with the propagation distance. The period and amplitude of the oscillation for the chirped case are greater than those for the unchirped case, and they vary with the increase of $|C|$. Variation of the peak with propagation distance is opposite to that of temporal FWHM. Therefore, the peak of output temporal waveform can be controlled by changing the initial chirp. It is useful to applications in the optical switches. The radiation of dispersive waves is expected to exist because the Eqs.(1) are unintegrable. The larger the value of $|C|$ is, the stronger the dispersive waves near the edges of the chirped solitons are.

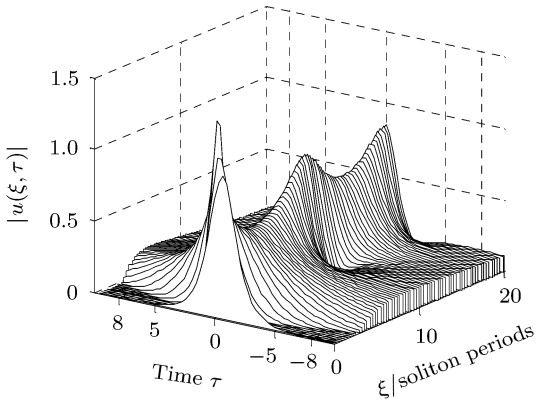


Fig.5. Evolution of temporal waveform of $|u(\xi, \tau)|$ with propagation distance for $C = 0.5$, $\delta = 0.7$ and $2\tau_0 = 1.25$.

3.6. Effect of chirp on the relationship between $\delta_{\rm th}$ and $2\tau_0$

It is considered that the two solitons are completely separated for $|2\tau_0| > 5$. The fast soliton precedes the slow one and their interaction can be neglected for $2\tau_0 < -5$. The slow soliton precedes the fast one for $2\tau_0 > 5$, and their interaction is the same as that for $2\tau_0 = 5$. Figure 6 shows the effect of linear chirp on the relationship between the threshold value $\delta_{\rm th}$ and initial interval $2\tau_0$ between two solitons. The solid curve shows the relationship for $C = 0$, dashed curve shows that for $C = 0.5$, dotted curve shows that for $C = -0.5$. It is found from Fig.6 that the threshold value is determined by initial interval and chirp of two solitons. In the case of $C = 0$, it obviously decreases with the increase of initial interval for $|2\tau_0| \leq 3$, is maximum for $\tau_0 = 0$, and remains almost unchanged for $\tau_0 > 3$. The variations are consistent with those in Refs.[11-13]. In the case of $C = 0.5$, the threshold value is obviously greater than that of unchirped case for $-1 < 2\tau_0 \leq 2.1$, much less than that for $2.1 < 2\tau_0 \leq 5$, and slightly less than that for $-4 < 2\tau_0 \leq -1$. In the case of $C = -0.5$, it is slightly larger than that of the unchirped case for $-5 \leq 2\tau_0 < 0$, and much less than that for $0 \leq 2\tau_0 \leq 5$. The maximum value of $\delta_{\rm th}$ shifts to the region of $\tau_0 > 0$ for $C > 0$, shifts to that of $\tau_0 < 0$ for $C < 0$.

It is shown that the initial chirp change the threshold value $\delta_{\rm th}$ of solitons trapping. The effect of initial positive chirp on the change of threshold value $\delta_{\rm th}$ is greater than that of negative chirp. In designing optical switches one should therefore consider the effect of initial chirp.

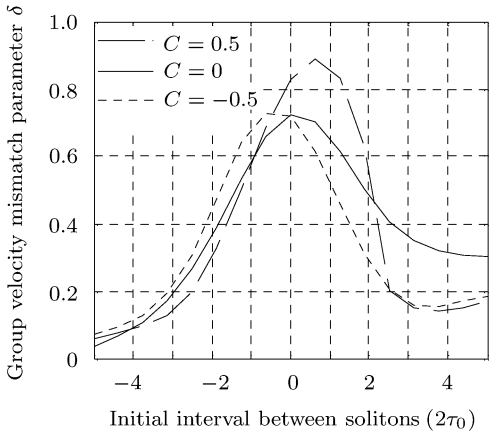


Fig.6. The relationship between $\delta_{\rm th}$ and initial interval $2\tau_0$ between two solitons. The solid curve shows the relationship for $C = 0$, dashed curve shows that for $C = 0.5$, dotted curve shows that for $C = -0.5$.

4. Conclusion

The collision characteristics of the orthogonally polarized solitons with initial linear frequency chirp are numerically studied in the linear birefringent fibre for $\beta_2 < 0$. It is found that initial chirp changes the threshold value $\delta_{\rm th}$ of solitons forming the bound-state in the birefringent fibre. The effect of initial positive chirp on the threshold value is greater than that of negative chirp.

In the case of $\delta = 0.7$ and $2\tau_0 = 1.25$, the two solitons are mutually bound for $0.2 \leq C \leq 1$, and they do not form into bound state for $-1 \leq C < 0.2$. Frequency shifts increase with the increase of chirp parameter C for $-1 \leq C < 0.2$, and have oscillatory structure for $C \geq 0.2$. The temporal FWHM performs damped oscillation with the propagation distance. The period and amplitude of the oscillation increase with the increase of $|C|$ for $-1 \leq C \leq 0.2$ and $0.6 \leq C \leq 1$, decrease with the increase of C for $0.2 < C < 0.6$. The effect of positive chirp on temporal FWHM is greater than that of negative chirp. The peak of temporal waveform oscillates with the propagation distance. The period and amplitude of the oscillation for chirped case are greater than those for the unchirped case, and they vary with the increase of $|C|$. The peak of output temporal waveform can be controlled by changing the initial chirp. This is useful for applications in optical switches.

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