Vol 16 No 7, July 2007
1009-1963/2007/16(07)/1996-07
Chinese Physics
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Polarization switching and synchronization of mutually coupled vertical-cavity surface-emitting semiconductor lasers*

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(Received 4 October 2006; revised manuscript received 31 January 2007)

Polarization switching (PS) dynamics and synchronization performances of two mutually coupled vertical-cavity surface-emitting lasers (VCSELs) are studied theoretically in this paper. A group of dimensionless rate equations is derived to describe our model. While analysing the PS characteristics, we focus on the effects of coupling rate and frequency detuning regarding different mutual injection types. The results indicate that the x-mode injection defers the occurrence of PS, while the y-mode injection leads the PS to occur at a lower current. Strong enough polarization-selective injection can suppress the PS. Moreover, if frequency detuning is considered, the effects of polarization-selective mutual injection will be weakened. To evaluate the synchronization performance, the correlation coefficients and output dynamics of VCSELs with both pure mode and mixed mode polarizations are given. It is found that performance of complete synchronization is sensitive to the frequency mismatch but it is little affected by mixed mode polarizations, which is opposite to the case of injection-locking synchronization.

Keywords: vertical-cavity surface-emitting semiconductor laser, polarization, mutual injection, synchronization

PACC: 4260, 4225J, 4280S

1. Introduction

Vertical-cavity surface-emitting semiconductor lasers (VCSELs) are extremely attractive in many practical applications, such as data communications, data storage and printing system, due to their prominent advantages over traditional edge-emitting lasers.[1−4] However, their cavity anisotropy makes the emissions from VCSELs commonly linearly polarized (LP) along one of two orthogonal directions, which is associated with crystalline or stress orientations.[5] The polarization switching (PS) between two LP modes is often observed when the injection current increases. Uncontrolled PS has a severe drawback, for example it can reduce the length of a fibre link.[6] Therefore, the PS control of VCSELs has received much attention and has been studied widely.

Optical injection and optical feedback have been suggested as effective methods to achieve the PS control. Pan *et al*[7] first observed PS characteristics of VCSELs with optical injection. After that, characteristics of bifurcation dynamics, low frequency fluctuations and multiple PS have been investigated in single mode VCSELs with consideration of polarizations.[8,9] Polarization behaviours of multitransverse-mode VCSELs with and without optical injections have also been investigated both theoretically and experimentally.[10] Panajotov *et al*[11] and Sciamanna *et al*[12] have investigated the polarization mode-hopping dynamics and the residence time distribution of mode-hopping. Hong *et al*[13] have demonstrated the polarization bistability phenomenon. Arteaga *et al*[6] have investigated the parametric dependence of polarization properties of VCSELs with an extremely short external cavity.

To our best knowledge, the previous studies on polarizations mainly concentrated on unidirectionally optical injections or self-injections (optical feedback). PS characteristics of VCSELs with mutual injections have not yet been studied and understood, nor has the synchronization of mutually coupled systems that take polarizations into consideration. On the other hand, the system of mutually coupled VCSELs is an excel-

*Project supported by the National Natural Science Foundation of China (Grant Nos 10174057 and 90201011), and the Foundation for Key Program of Ministry of Education, China (Grant No 2005-105148).
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lent and experimentally realizable example of coupled nonlinear oscillator, which plays an important role in high-power emission, noise reduction, and chaotic optical communications. The study of dynamics and synchronization of mutually coupled semiconductor lasers has been carried out.[14−17] Now the investigation on polarization dynamics of mutually coupled VCSELs is increasingly conducted. This is due to not only an interesting fundamental physical issue, but also practical applications, such as the PS control and chaos synchronization communications. So in this paper, we present a numerical simulation on the PS and synchronization characteristics of mutually coupled VCSELs.

The remainder of this paper is organized as follows. In Section 2, theoretical model is considered. The dimensionless rate equations that include two polarization modes are deduced to describe our model. In Section 3, the PS dynamics and synchronization performance are discussed. Conclusions are given in Section 4.

2. Theoretical model

The configuration of our mutually coupled system is given in Fig.1. We should point out that only the case of x-mode injection is presented here. For the y-mode, the x-mode polarizer (XMP) should be replaced by the y-mode polarizer (YMP); for an isotropic injection, no polarizer is needed between VCSEL1 and VCSEL2. To describe mutually coupled VCSELs, two effects should be included in the foundational rate equations.

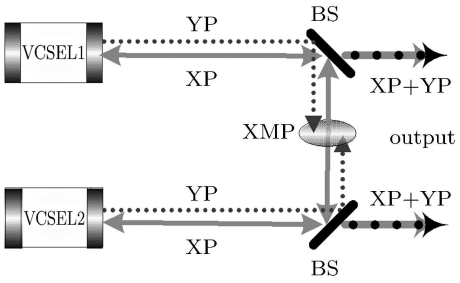


Fig.1. Schematic diagram of mutually coupled VCSELs; XMP: x-mode polarizer; BS: beam splitter; XP: x-polarization; YP: y-polarization.

They are the effects of polarization and mutual injection. Polarization properties of VCSELs have been explored theoretically using a number of models. The Lang–Kobayashi equations are shown to be highly accurate in describing the optical feedback and injection effects in semiconductor lasers.[18,19] They have been recently extended to being able to treat the PS phenomena in VCSELs.[12,13] We thus adopt this approach in the present work. Besides, the single mode rate equations for mutually coupled semiconductor lasers have been given in Refs.[15,16]. Combining these two aspects, and taking the dimensionless transformations: $\mu = \dfrac{g_X}{g_Y}$, $T = \dfrac{\gamma}{\gamma_e}$, $\eta_{X,Y} = \dfrac{k_{X,Y}}{\gamma}$, $s = t\gamma$, $\varepsilon = \tau\gamma$, $\theta = \dfrac{\Delta\omega}{\gamma}s$, $\phi_{1,2} = \dfrac{\omega_{1,2}}{\gamma}\varepsilon$, $F_{X,Y} = \sqrt{\dfrac{g}{2\gamma_e}}E_{X,Y}$, $P_{X,Y} = \dfrac{J}{q}\dfrac{g_{X,Y}}{2\gamma\gamma_e} - N_t\dfrac{g_{X,Y}}{2\gamma} - 0.5$, and $M_{X,Y} = \dfrac{G_{X,Y} - \gamma}{2\gamma}$, we can obtain the dimensionless rate equations describing mutually coupled VCSELs with polarized emissions as follows:

$$\frac{\mathrm{d}F_{X1,X2}}{\mathrm{d}s} = (1 + \mathrm{i}\alpha)M_{X1,X2}F_{X1,X2} + \eta_X F_{X2,X1}(s - \varepsilon)\exp[\mathrm{i}(\pm\theta - \phi_{2,1})], \tag{1}$$

$$\frac{\mathrm{d}F_{Y1,Y2}}{\mathrm{d}s} = (1 + \mathrm{i}\alpha)M_{Y1,Y2}F_{Y1,Y2} + \eta_Y F_{Y2,Y1}(s - \varepsilon)\exp[\mathrm{i}(\pm\theta - \phi_{2,1})], \tag{2}$$

$$T\frac{\mathrm{d}M_{X1,X2}}{\mathrm{d}s} = P_X - M_{X1,X2} - \mu\left[(1 + 2M_{X1,X2})\left|F_{X1,X2}\right|^2 - (1 + 2M_{Y1,Y2})\left|F_{Y1,Y2}\right|^2\right], \tag{3}$$

$$T\frac{\mathrm{d}M_{Y1,Y2}}{\mathrm{d}s} = P_Y - M_{Y1,Y2} - (1 + 2M_{X1,X2})\left|F_{X1,X2}\right|^2 - (1 + 2M_{Y1,Y2})\left|F_{Y1,Y2}\right|^2, \tag{4}$$

where subscripts '1' and '2' denote two mutually coupled lasers, respectively; 'X' and 'Y' represent x-polarization (XP) and y-polarization (YP) modes, respectively. From the physical point of view, F and M are the normalized electric field and the normalized excess carrier number respectively. Normalized power in the following discussion is defined as $|F_{X,Y}|^2$. α is the linewidth enhancement factor, γ is the cavity loss,

J is the bias current, γ_e is the carrier decay rate. k_X and k_Y are the coupling rates, τ is the injection delay, $\Delta\omega = 2\pi\Delta f = \omega_2 - \omega_1$ is the frequency detuning. $G_{X,Y} = g_{X,Y}(N - N_t)$ is the optical gain, where N_t is the transparency carrier number, and the gain indices, g_X and g_Y, have the follow relationships:

$$g_X = g\left[1 + \frac{g_0}{g}\left(1 - \frac{J}{J_0}\right)\right], \quad g_Y = g, \tag{5}$$

where g_0 is the difference-gain index, $J_0 = 1.4\,J_{th}$ is the solitary PS current, where J_{th} is the threshold current. In fact the solitary PS current can be predicted from the linear stability analysis of lasing solutions,[20] but here is taken as a typical experimental value[13] without losing generality.

On the right-hand sides in Eqs.(1) and (2), the first terms describe XP and YP electric fields in solitary condition, and the second terms represent contributions of mutual injections. It is worth noting that there exist three types of injections for mutual coupling of VCSELs: isotropic injection ($\eta_X = \eta_Y \neq 0$), x-mode injection ($\eta_X \neq 0$, $\eta_Y = 0$), and y-mode injection ($\eta_X = 0$, $\eta_Y \neq 0$). The last two types are jointly called polarization-selective injection. Practically, these injection types can be realized by adding a neutral density filter, an x mode polarizer, and a y-mode polarizer, respectively.

We should also point out that Eqs.(3) and (4) can be treated as one equation, since variables M_X and M_Y always satisfy the relationship

$$M_X = \mu(M_Y + 0.5) - 0.5. \tag{6}$$

Thus, we have obtained a theoretical model for mutually coupled VCSELs. In this model, the bias current is taken for example to control the difference in gain between XP and YP modes. This can also be extended to other parameters, such as thermal parameter, etc.

3. PS dynamics and synchronization of VCSELs

In this section, we describe the PS and synchronization characteristics of mutually coupled VCSELs. The theoretical model is described based on our previous work.[21,22] The values of parameters are mostly cited from Refs.[9, 12, 15]. Some are given directly in our discussion, and the others are listed in Table 1.

Table 1. Typical values of parameters in the simulation

Parameter	Symbol	Value
linewidth enhancement factor	α	3.5
threshold current	J_{th}	9.9 mA
cavity losses	γ	$282\,ns^{-1}$
differential gain	g_0	$0.8 \times 10^{-6}\,ns^{-1}$
gain index	g	$3.2 \times 10^{-6}\,ns^{-1}$
time delay	τ	4.75ns
carrier decay rate	γ_e	$0.3\,ns^{-1}$
transparency carrier concentration	N_t	1.2×10^8
lasing frequency	f_1	3.06×10^5 GHz

3.1. PS dynamics

First, we calculate the polarization-resolved L–I curves in the case of isotropic mutual injection as shown in Fig.2. Figure 2(a) corresponds to the solitary condition. The solid lines correspond to XP mode and the dot lines correspond to YP mode. The PS is observed when the current is increased. The XP mode switches to the YP mode at the switching point $J = 1.4\,J_{th}$. This is referred to as type-I switching. From the physical viewpoint, the increasing of current induces a redshift of both the gain spectrum and the cavity resonance. The redshift of the gain spectrum is faster than that of the cavity resonance, which might lead to a change of sign of the difference in gain between the two polarization modes and thus to a PS. In Figs.2(b)–2(d), the PS becomes more and more slowly with coupling rate increasing. The switching point keeps unchanged, because Eq.(1) is symmetry with respect to Eq.(2) at this point ($\mu = 1$, $M_X = M_Y$, and $\eta_X = \eta_Y$). In fact, due to cavity anisotropy the parameters between the two LP modes are not exactly the same. In this case, only one LP mode is favoured under the isotropic injection, and the switching point and switching times will change with increasing coupling rate.

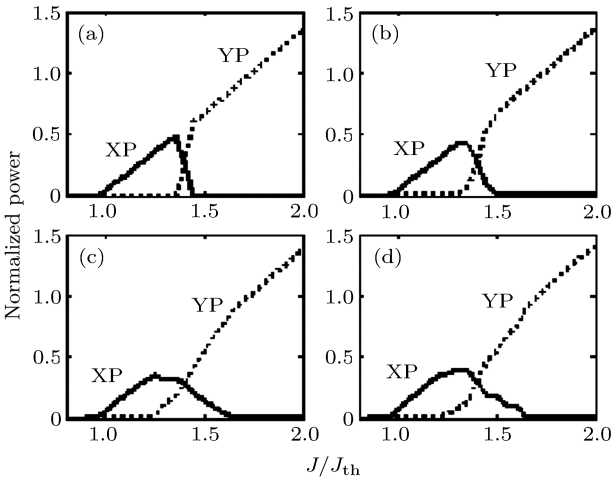


Fig.2. Polarization resolved L–I curve for the case of isotropic mutual injection when frequency detuning is zero. $\eta_X = \eta_Y =$ 0, 0.018, 0.035 and 0.053 are for sub-diagrams (a)–(d) respectively.

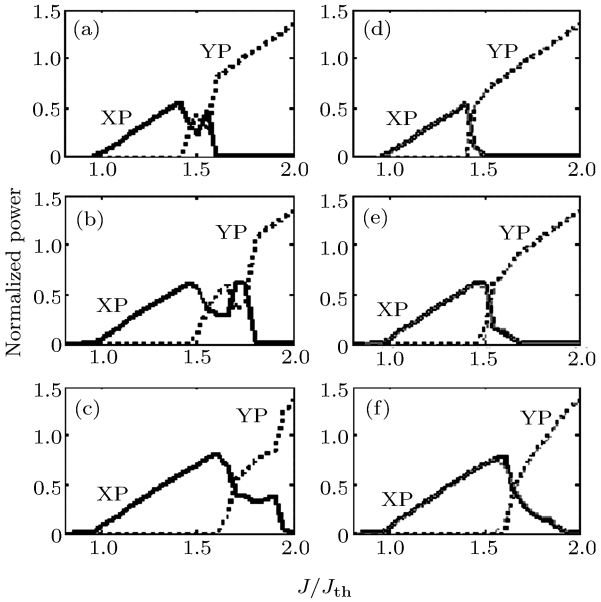


Fig.3. L–I curves for the case of x-mode mutual injection. In the left column, $\Delta f = 0$; In the right column, $\Delta f = 2.5$ GHz. $\eta_Y = 0$ and $\eta_X = 0.017$, 0.035 and 0.053 are for sub-diagrams (a)–(c) respectively, so are for sub-diagrams (d)–(f).

For the case where the frequency detuning is considered and other parameters are kept the same as those adopted in Fig.2, the L–I curves are not much different from those in Fig.2, so we do not present them here. It seems that the frequency detuning has little influence on the PS dynamics of VCSELs with the isotropic mutual injection.

Next, we present the PS dynamics in the cases of x-mode and y-mode mutual injections as shown in Fig.3 and Fig.4. For the x-mode injection, switching points appear at higher values of current when coupling rate is increased. We can see the multiple PS in Figs.4(a) and 4(b), which is also called the 'channel' effect.[12] The mechanism for multiple PS is that the increasing of current favours one polarization mode, while the increasing of coupling rate favours the other mode. These two effects dominate each other alternatively when coupling rate and injection current take certain values. These have been explained in detail in our previous work.[23] For the y-mode injection, switching points appear at lower values of current with increasing coupling rate. When the coupling rate is large enough, the PS disappears and only the y-mode exists (see Figs.4(a)–4(c)). Comparing the second columns with the first columns in Fig.3 and Fig.4, we find that the multiple PSs are suppressed. It seems that the frequency detuning can weaken the influence from the polarization-selective mutual injections.

In addition, it should be pointed out that the above L–I curves are obtained from the output of laser 1. Comparing the L–I curves of laser 2 with those of laser 1, we find that they are almost the same. So the frequency detuning cannot change the power distributions of individual polarization modes between two mutually coupled VCSELs.

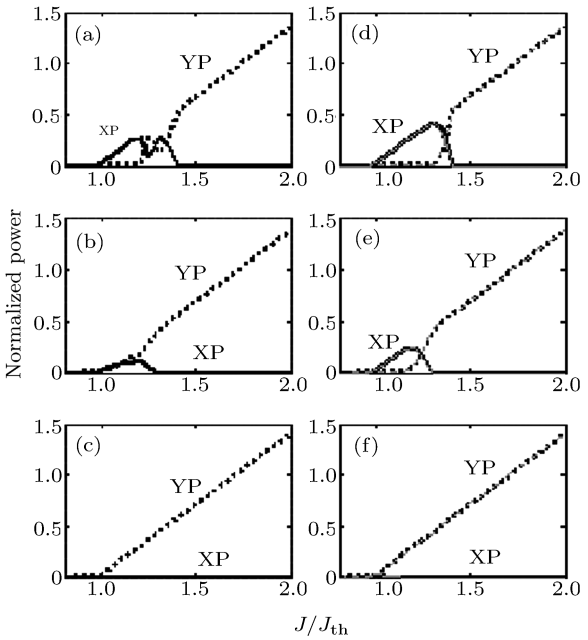


Fig.4. L–I curves for the case of y-mode mutual injection. In the left column, $\Delta f = 0$; In the right column, $\Delta f = 2.5$ GHz. $\eta_X = 0$ and $\eta_Y = 0.017$, 0.035 and 0.053 are for sub-diagrams (a)–(c) respectively, so are for sub-diagrams (d)–(f).

We can conclude that strong enough polarization-selective mutual injections can suppress the PS (the x-mode injection favours the XP mode, and the y-mode injection favours the YP mode), which is similar to the case of optical feedback or unidirectionally optical injection. With coupling rate increasing, both the total powers of LP modes and the power for each indi-

vidual LP mode for the two coupled lasers are equal, even if frequency detuning is nonzero.

3.2. Synchronization performances

In Section 3.1, we can see that the emissions from VCSELs occur in pure XP mode, pure YP mode, or mixed mode, depending on the value of current, coupling rate and injection types. In this part, the x-mode mutual injection is taken for example to study the influences of output polarizations on synchronization performances. Synchronization performance is quantitated by correlation coefficient as expressed as

$$\rho(\Delta t) = \frac{\langle [P_1(t) - \langle P_1(t)\rangle][P_2(t+\Delta t) - \langle P_2(t)\rangle]\rangle}{\left\langle |P_1(t) - \langle P_1(t)\rangle|^2 \right\rangle^{1/2} \left\langle |P_2(t+\Delta t) - \langle P_2(t)\rangle|^2 \right\rangle^{1/2}}. \tag{7}$$

Here, $\langle\ \rangle$ means the time average, $P_{1,2}$ are the normalized output powers, and Δs is the time shift. According to this definition, it is obvious that the larger the value of ρ, the better the performance of synchronization.

Figure 5 presents the waveforms of normalized output powers of the two lasers in a time domain. Figures 5(a)–5(c) correspond to zero frequency detuning,

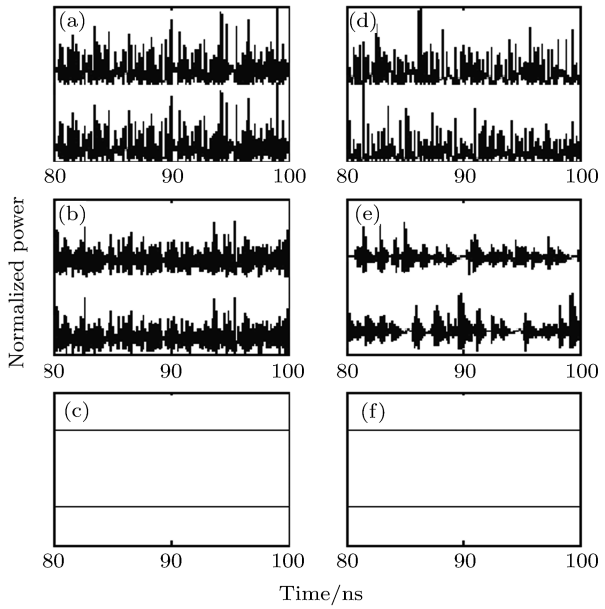


Fig.5. Normalized powers of the two lasers in the time domain under the x-mode mutual injection with $\eta = 0.035$. In the left column, $\Delta f = 0$; In the right column, $\Delta f = 2.5$ GHz. Sub-diagrams (a)–(c) show the laser emissions occurring in pure XP mode, mixed mode and pure YP mode respectively, so do sub-diagrams (d)–(f).

and Figs.5(d)–5(f) correspond to nonzero frequency detuning. Figures 5(a)–5(c) show that the emissions from the two lasers occur in pure XP mode, mixed mode, and pure YP mode respectively, so do Figs.5(d)–5(f). When the frequency detuning is zero, the two lasers show completely synchronized chaotic waveforms if their emissions occur in pure XP mode or mixed mode. There are no fluctuations in the outputs if their emissions appear in pure YP mode. When the frequency detuning is considered, the complete synchronization is destroyed, and the two waveforms synchronize with each other, but with a positive or negative time delay (sometimes laser 1 drives laser 2, and sometimes laser 2 drives laser 1) (seeing Figs.5(d) and 5(e)).

The correlation coefficient versus time shift corresponding to Figs.5(a), 5(b), 5(d) and 5(e) are given in Fig.6.

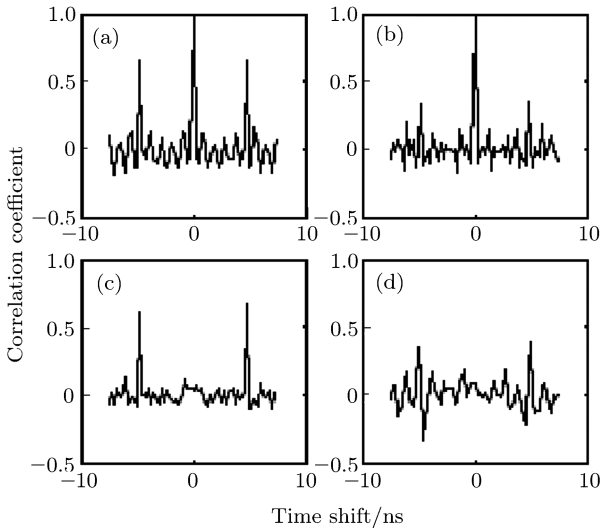


Fig.6. Correlation coefficient as a function of time shift with $\eta = 0.035$. In (a) and (b), $\Delta f = 0$; In (c) and (d), $\Delta f = 2.5$ GHz. (a) and (c) pure mode emission; (b) and (d) mixed mode emission.

Figure 6(a) shows the emissions from the two lasers occurring in pure x-mode. Three peaks are found, and the largest one at $\Delta t = 0$ corresponds to the case

of complete synchronization. The other two peaks at $\Delta t = \pm 4.75$ ns correspond to the direct injection (injection-locking synchronization). When the laser emissions occur in mixed mode, the two peaks at $\Delta t = \pm 4.75$ ns decrease obviously (see Fig.6(b) and 6(d)). When the frequency detuning is considered, the peak at $\Delta t = 0$ disappears (see Figs.6(c) and 6(d)). It is easy to understand that the complete (injection-locking) synchronization is sensitive (insensitive) to frequency detuning. But why is the injection-locking synchronization easily affected by the coexistence of polarization modes? This can be explained from experimental results in Ref.[14]: the dynamics of XP modes are caused by mutual injection, and they are well synchronized in both synchronization regimes. The dynamics of YP modes are induced by anticorrelated oscillations with the X components, and they are not well synchronized in the injection-locking regime, which reduces the synchronization degree of the total power.

To study the robustness of complete synchronization and injection-locking synchronization, Fig.7 shows the peak correlation coefficient versus frequency detuning. Comparing Fig.7(a) with Fig.7(b), we can obtain a similar conclusion to that from Fig.6, i.e. the complete synchronization is sensitive to frequency mismatch, but it is little affected by mixed mode emission; the case is opposite to the injection-locking synchronization. In practical implementations, the frequency mismatch always exists, so the injection-locking synchronization is preferred for chaotic communications.

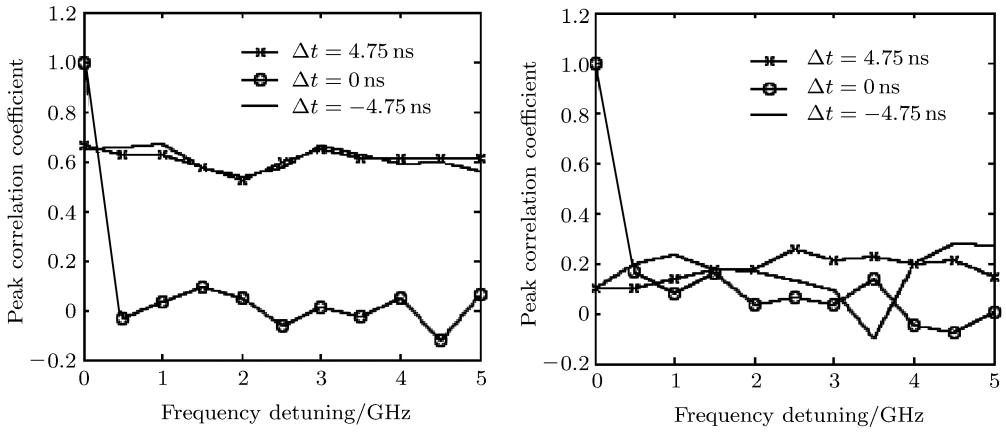


Fig.7. Peak correlation coefficient versus frequency detuning with $\eta = 0.035$. (a) pure XP mode emission, (b) mixed mode emission.

4. Conclusions

In this paper, we report on an investigation of the PS and synchronization characteristics of mutually coupled VCSELs. The dimensionless rate equations that include two polarization modes are given for the first time to describe polarization dynamics in VCSELs. The x-mode injection can defer the occurrence of PS, while the y-mode injection can lead the PS to occur at a lower current. Strong enough polarization-selective mutual injection can suppress the PS. These influences are similar to the cases of optical feedback and unidirectionally injection, and are consistent with the previous experimental observations.[8,12−14] Frequency detuning cannot change the distributions of total powers and powers of individual polarization modes between the two coupled VCSELs, but it can weaken the influence of polarization-selective mutual injections, which is different from the cases of optical feedback and unidirectionally injection. Meanwhile, the effects of output polarizations on synchronization performance of mutually coupled VCSELs are discussed. When the lasers emissions occur in mixed mode, the injection-locking synchronization is difficult to realize. So to improve the synchronization performance, we should choose an appropriate bias current, injection rate and coupling type to keep VCSELs emitting in pure polarization mode. In fact, the polarization mode competition and optical injection can also make complex the nonlinear dynamics in mutually coupled VCSELs, which should be investigated in future studies.

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